

# A New “Skin Thickness” in High-spin Isomers as a Probe for EOS of Spin-polarized Matter

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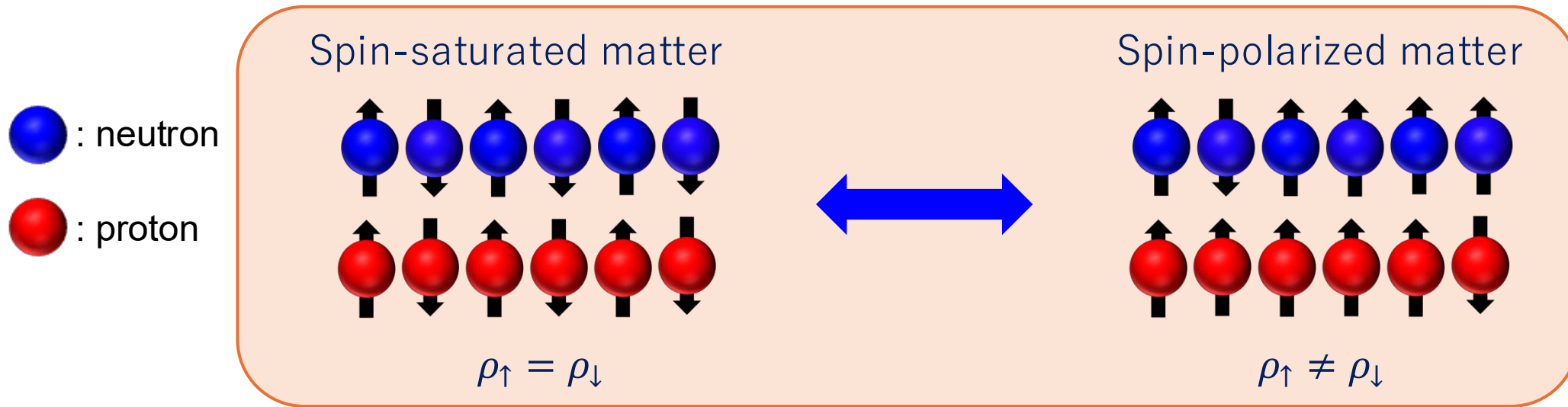
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# Outline

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- 1. Motivation – Spin-polarized matter –**
2. The relativistic point-coupling model
3. Spin slope parameter
4. New “skin thickness” in high-spin isomers
5. Summary

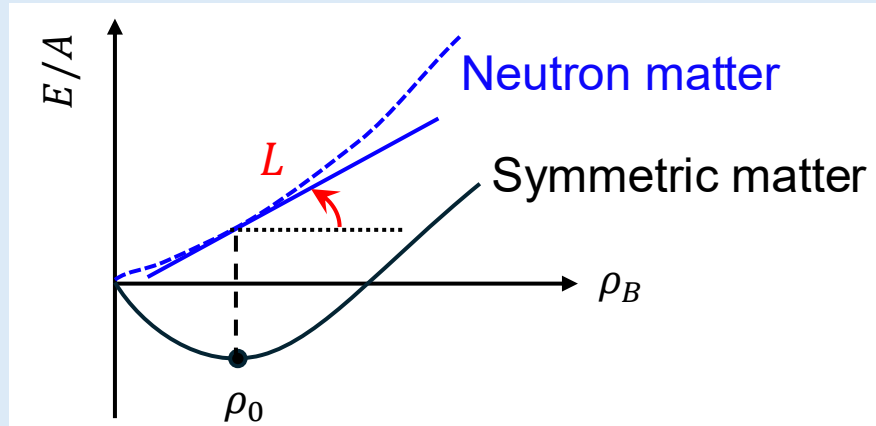


- The ferromagnetic phase in high-density region ▶ Origin of the magnetic field of **magnetar**  
I. Vidana, I. Bombaci, Phys. Rev. C 66, 045801 (2002)  
T. Maruyama, T. Tatsumi, Nucl. Phys. A 693 (2001) 710-730
- Strong magnetic field in the remnant of neutron star merger ▶ Possible formation of **spin-polarized matter**  
B. D. Metzger, et al., Astrophys. J. Lett. 856, 101 (2018)
- Changes in particle fraction due to spin polarization ▶ Affecting **neutron star cooling**.  
N. H. Khoa, N. H. Tan, D. T. Khoa, Phys. Rev. C 105, 065802 (2022)



EOS for spin-polarized matter has been increasingly important.

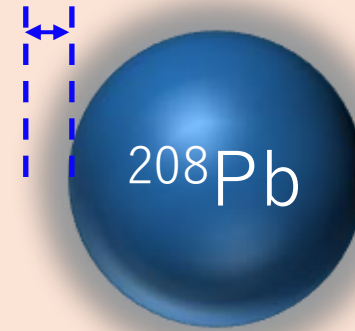
## Slope parameter $L$



Linear  
correlation



## Neutron skin thickness $\Delta r_{np}$



$$Z = 82$$

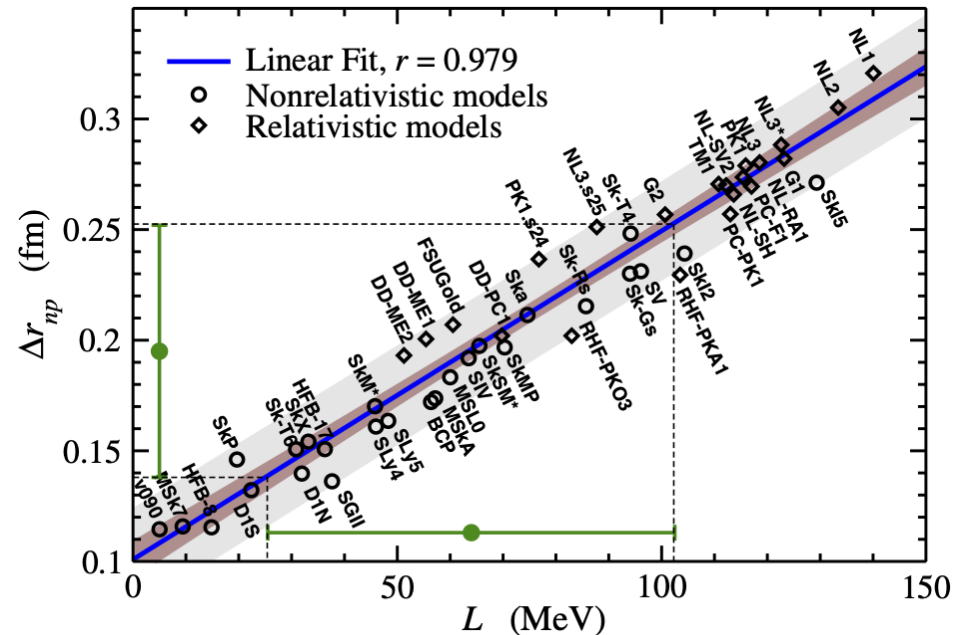
$$N = 126$$

PREX

D. Adhikari, et al.,  
Phys. Rev. Lett. 126,  
172502 (2021)

p elastic scattering

J. Zenihiro, et al.,  
Phys. Rev. C 82,  
044611 (2010)



## Current State

Neutron skin thickness is an experimental probe to constrain the EOS for asymmetric nuclear matter.



Almost no analogous probes for constraining the EOS for **spin-polarized matter** has been considered.

## Our goals

- Proposing a method to constrain the EOS for spin-polarized matter by terrestrial nuclear experiments.

## This Study

- Calculation of the EOS for spin-polarized matter in the relativistic model, which can naturally treat the spin d.o.f. of nucleons
- Correlation between the slope parameter and **the spin slope parameter** (details to follow)
- Exploring a new skin “thickness” correlated with **the spin slope parameter**

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$$\begin{aligned}\mathcal{L}_{\text{int}} = & -\frac{1}{2}\alpha_S(\bar{\psi}\psi)^2 - \frac{1}{2}\alpha_{tS}(\bar{\psi}\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_V(\bar{\psi}\gamma^\mu\psi)^2 - \frac{1}{2}\alpha_{tV}(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_{PS}(\bar{\psi}\gamma_5\psi)^2 - \frac{1}{2}\alpha_{tPS}(\bar{\psi}\gamma_5\vec{\tau}\psi)^2 \\ & - \frac{1}{2}\alpha_{PV}(\bar{\psi}\gamma_5\gamma^\mu\psi)^2 - \frac{1}{2}\alpha_{tPV}(\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_T(\bar{\psi}\sigma^{\mu\nu}\psi)^2 - \frac{1}{2}\alpha_{tT}(\bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi)^2\end{aligned}$$

Mean Field approx.

$$\psi = \sum_{\mathbf{p},s,t} [u(\mathbf{p},s,t)a_{\mathbf{p},s,t}e^{-i\mathbf{p}\cdot\mathbf{x}} + \cancel{v(\mathbf{p},s,t)b_{\mathbf{p},s,t}^\dagger e^{i\mathbf{p}\cdot\mathbf{x}}}] \quad (\text{no sea approximation})$$

$$\begin{aligned}\langle [\bar{\psi}(\mathcal{O}\Gamma)_i\psi]^2 \rangle &= \sum_{\alpha,\beta} \int_{V_{p_F}} dp^3 dq^3 [\bar{u}(\mathbf{p},\alpha)(\mathcal{O}\Gamma)_i u(\mathbf{p},\alpha) \times \bar{u}(\mathbf{q},\beta)(\mathcal{O}\Gamma)_i u(\mathbf{q},\beta) \quad \mathcal{O} \in \{1, \tau_3\} \\ &\quad - \bar{u}(\mathbf{p},\alpha)(\mathcal{O}\Gamma)_i u(\mathbf{q},\beta) \times \bar{u}(\mathbf{q},\beta)(\mathcal{O}\Gamma)_i u(\mathbf{p},\alpha)] \quad \Gamma \in \{1, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\} \\ &\quad \text{Fierz transf.} \quad \blacktriangleright \Lambda_{ij} \bar{u}(\mathbf{p},\alpha)(\mathcal{O}\Gamma)_j u(\mathbf{p},\alpha) \times \bar{u}(\mathbf{q},\beta)(\mathcal{O}\Gamma)_j u(\mathbf{q},\beta) \\ &= (\delta_{ij} - \Lambda_{ij}) \left[ \sum_{\alpha} \int_{V_{p_F}} dp^3 \bar{u}(\mathbf{p},\alpha)(\mathcal{O}\Gamma)_j u(\mathbf{p},\alpha) \right]^2\end{aligned}$$

## Total Energy Density

$$\mathcal{E} = \langle \mathcal{H} \rangle = \mathcal{E}_{\text{kin}} + \frac{1}{2} \alpha_S \rho_S^2 + \frac{1}{2} \alpha_{ts} \rho_{ts}^2 + \frac{1}{2} \alpha_V \rho_V^2 + \frac{1}{2} \alpha_{tV} \rho_{tV}^2 - \frac{1}{2} \alpha_{PV} \rho_{PV}^2 - \frac{1}{2} \alpha_{tPV} \rho_{tPV}^2 + \alpha_T \rho_T^2 + \alpha_{tT} \rho_{tT}^2$$

$$\rho_S = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} \bar{u}(p, s, t) u(p, s, t) , \quad \rho_V = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) u(p, s, t)$$

$$\rho_{PV} = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) \gamma^0 \gamma^3 \gamma_5 u(p, s, t) = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) \Sigma_3 u(p, s, t) ,$$

$$\Sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$\rho_T = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) \gamma_0 \sigma^{12} u(p, s, t) = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) \gamma_0 \Sigma_3 u(p, s, t) ,$$

$u(p, s, t)$  : positive-energy plane wave spinor

## Total Energy Density

$$\mathcal{E} = \langle \mathcal{H} \rangle = \mathcal{E}_{\text{kin}} + \frac{1}{2} \alpha_S \rho_S^2 + \frac{1}{2} \alpha_{tS} \rho_{tS}^2 + \frac{1}{2} \alpha_V \rho_V^2 + \frac{1}{2} \alpha_{tV} \rho_{tV}^2 - \frac{1}{2} \alpha_{PV} \rho_{PV}^2 - \frac{1}{2} \alpha_{tPV} \rho_{tPV}^2 + \alpha_T \rho_T^2 + \alpha_{tT} \rho_{tT}^2$$

Independent parameters are only  $\alpha_S, \alpha_V, \alpha_{tS}, \alpha_{tV}, \alpha_T$ , ( because  $\text{rank}(1 - \Lambda) = 5$  )

and other coupling constants are determined by coefficients of Fierz transf. :

$$\begin{aligned} \alpha_{tT} &= \frac{1}{18} (-\alpha_S + 3\alpha_{tS} + 2\alpha_V - 6\alpha_{tV} + 6\alpha_T), \\ \alpha_{PV} &= \frac{1}{3} (2\alpha_S + 3\alpha_{tS} + 2\alpha_V + 3\alpha_{tV} + 6\alpha_T), \\ \alpha_{tPV} &= \frac{1}{9} (2\alpha_S + 3\alpha_{tS} + 5\alpha_V - 6\alpha_{tV} + 6\alpha_T). \end{aligned}$$

The four coupling constants  $\alpha_S, \alpha_V, \alpha_{tS}$  and  $\alpha_{tV}$  are density-dependent :

$$\alpha_i(\rho_B) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \times \alpha_i(\rho_0) \quad \left( x = \frac{\rho_B}{\rho_0} \right) \quad i = S, V, tS, tV$$

Dirac eq. for nucleons ( $s$ : spin index,  $t$ : isospin index)

$$[\mathbf{p} \cdot \boldsymbol{\alpha} + (M + \alpha_S \rho_S + \tau_3 \alpha_{tS} \rho_{tS}) \beta - (\alpha_{PV} \rho_{PV} + \tau_3 \alpha_{tPV} \rho_{tPV}) \Sigma_3 + 2(\alpha_T \rho_T + \tau_3 \alpha_{tT} \rho_{tT}) \beta \Sigma_3] u(p, s, t) = (E - \alpha_V \rho_V - \alpha_{tV} \rho_{tV}) u(p, s, t)$$

$$\Sigma_3 = \gamma^0 \gamma^3 \gamma_5 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad \rho_S = \langle \bar{\psi} \psi \rangle, \quad \rho_V = \langle \psi^\dagger \psi \rangle (= \rho_B), \quad \rho_{PV} = \langle \psi^\dagger \Sigma_3 \psi \rangle, \quad \rho_T = \langle \psi^\dagger \beta \Sigma_3 \psi \rangle$$

We have two positive energy solutions ▶  $u(p, s = +1, t)$  and  $u(p, s = -1, t)$

$$\Delta_t = \frac{(\rho_{s=+1,t} - \rho_{s=-1,t})}{\rho_t}, \quad \rho_{s=\pm 1,t} = \int \frac{d^3 p}{(2\pi)^3} u^\dagger(p, s = \pm 1, t) u(p, s = \pm 1, t)$$

However,  $u(p, s, t)$  is not an eigenvector of  $\Sigma_3$ . (when  $\mathbf{p} = 0$ , it is an eigenvector of  $\Sigma_3$ )

▶  $\langle \Sigma_3 \rangle$  is true spin polarization rate, and  $\Delta \neq \langle \Sigma_3 \rangle$

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EOS for symmetric nuclear matter (using PCF-PK1)

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

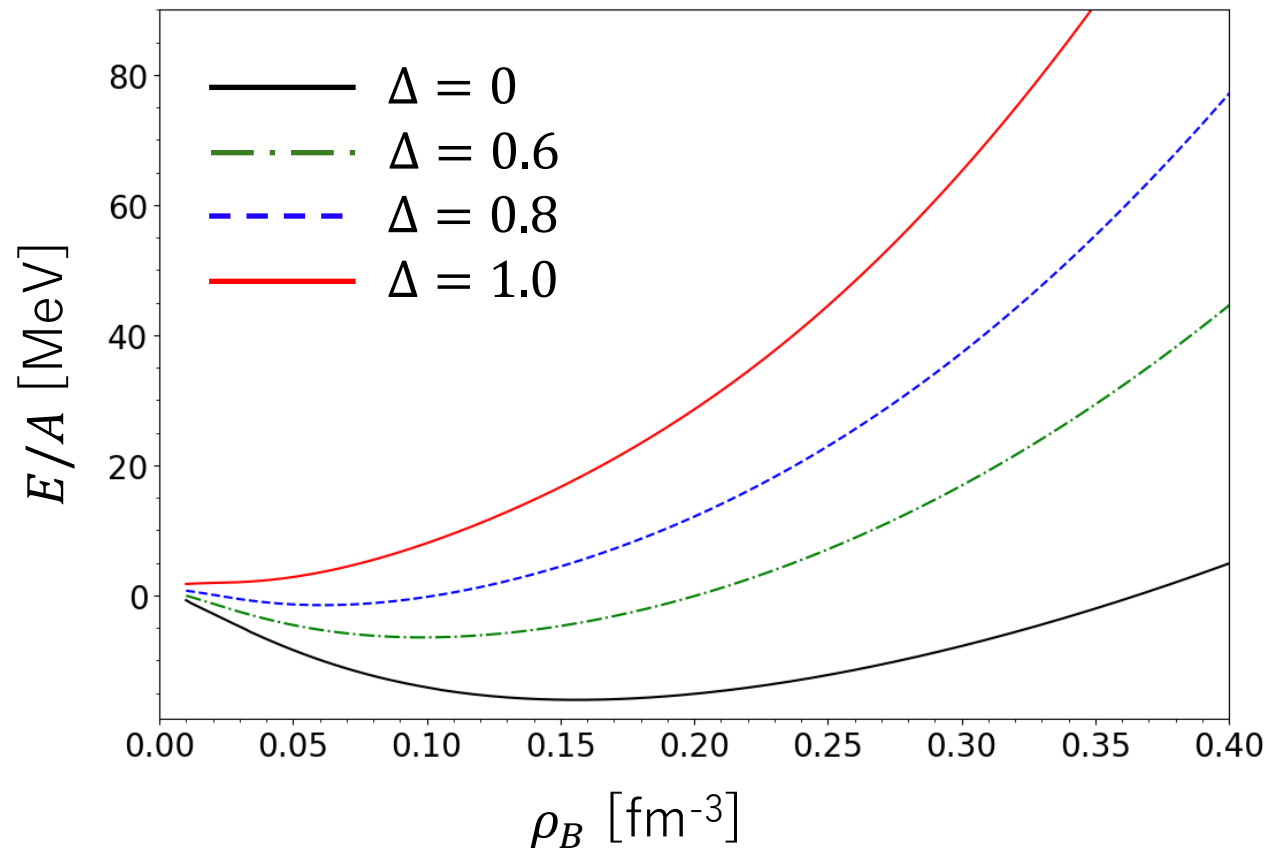
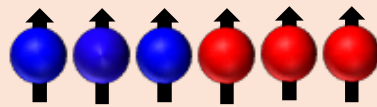
$$\delta \equiv (\rho_n - \rho_p)/\rho_B, \quad \Delta_t \simeq (\rho_{\uparrow,t} - \rho_{\downarrow,t})/\rho_t$$

For simplicity, we assume  $|\Delta_n| = |\Delta_p|$

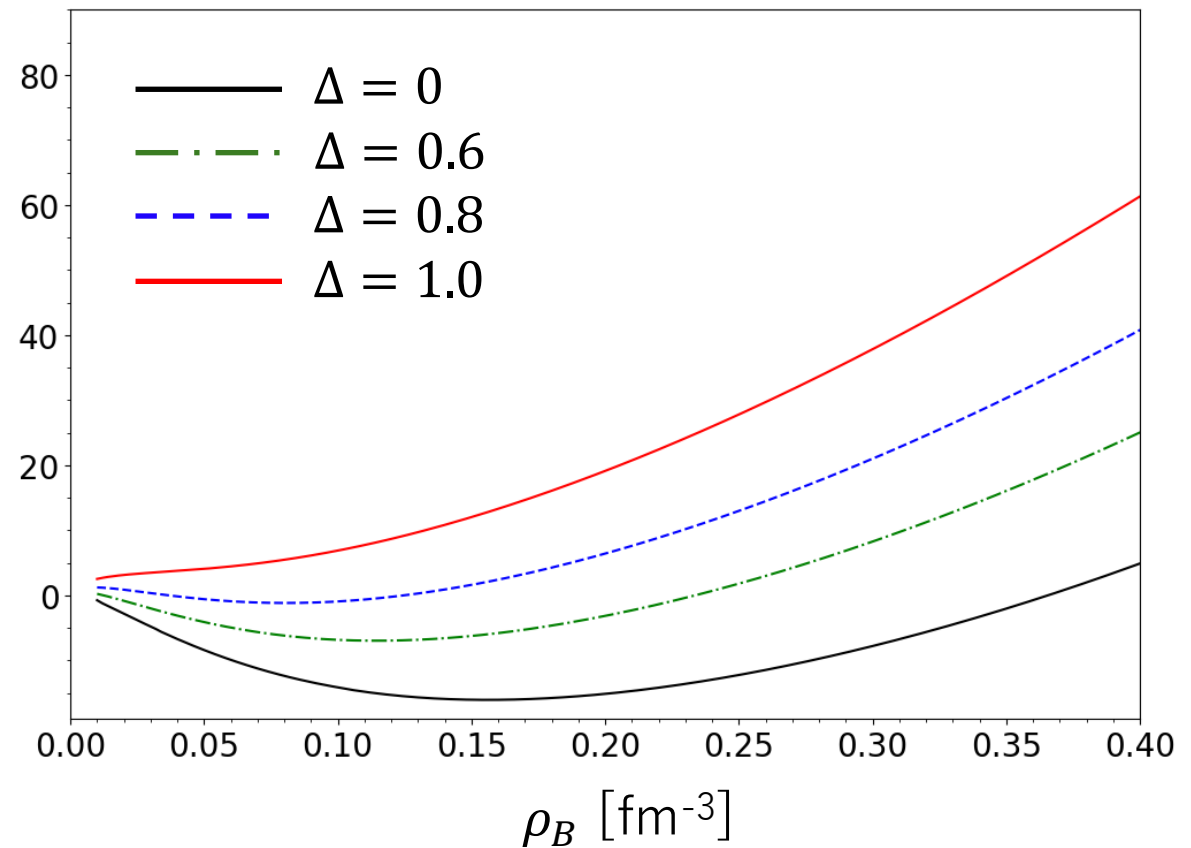
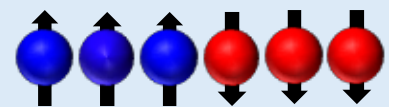


$$E(\rho_{\uparrow,n}, \rho_{\downarrow,n}, \rho_{\uparrow,p}, \rho_{\downarrow,p}) = E(\rho_B, \delta, \Delta)$$

IS(Isoscalar):  $\Delta_n = \Delta_p = \Delta$



IV(Isovector):  $\Delta_n = -\Delta_p = \Delta$



## Slope parameter

$$\frac{E}{A}(\rho, \delta) \cong \frac{E}{A}(\rho, \delta = 0) + \underline{S(\rho)}\delta^2$$

Symmetry energy

$$L = 3\rho_0 \left. \frac{\partial S(\rho)}{\partial \rho} \right|_{\rho=\rho_0}$$

## Spin slope parameter

$$\frac{E}{A}(\rho, \langle \Sigma_3 \rangle, \delta) \cong \frac{E}{A}(\rho, \langle \Sigma_3 \rangle = 0, \delta) + \underline{W(\rho, \delta)}\langle \Sigma_3 \rangle^2$$

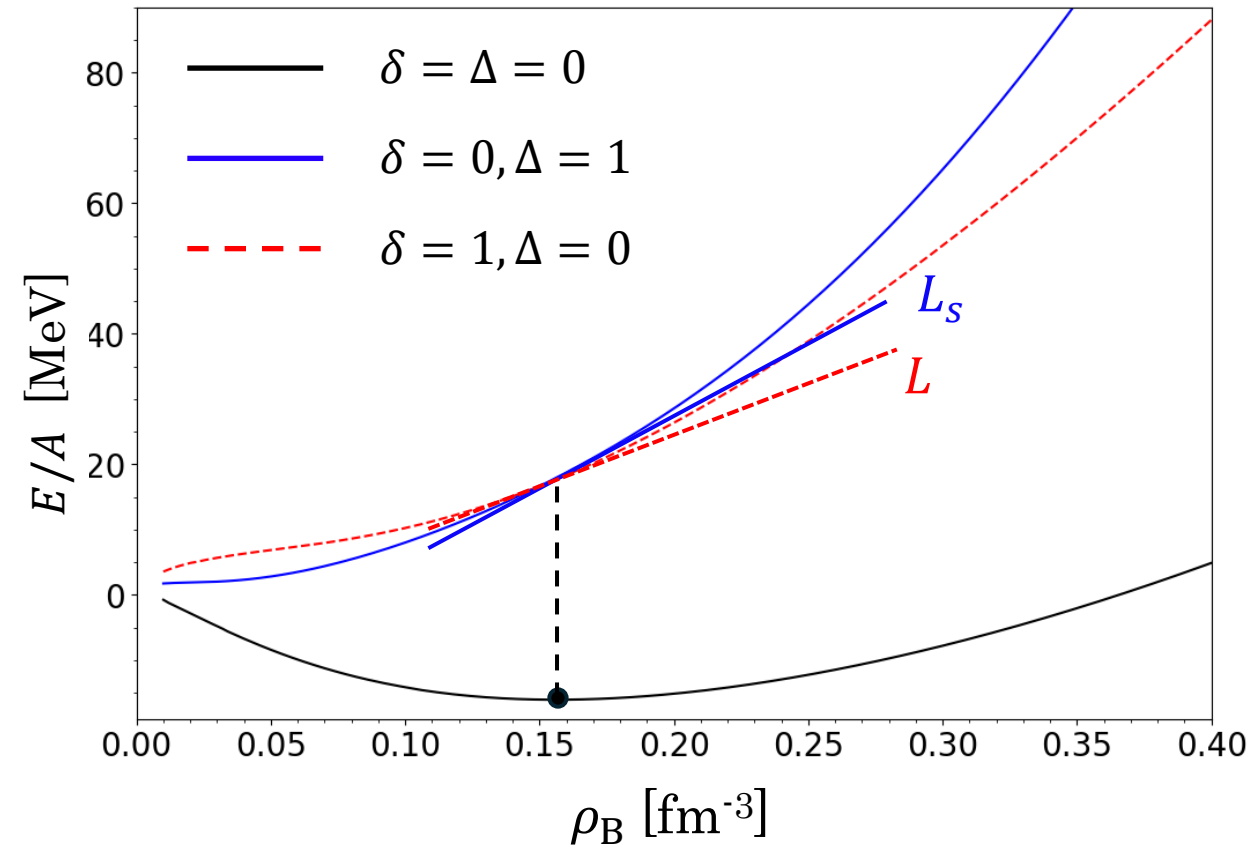
Spin symmetry energy

$$\Sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$L_s = 3\rho_0 \left. \frac{\partial W(\rho)}{\partial \rho} \right|_{\rho=\rho_0}$$

$$\delta \equiv (\rho_n - \rho_p)/\rho_B, \quad \Delta_\tau \simeq (\rho_{\uparrow,\tau} - \rho_{\downarrow,\tau})/\rho_\tau$$

$$\Delta \equiv |\Delta_n| = |\Delta_p| \quad (\neq \langle \Sigma_3 \rangle)$$

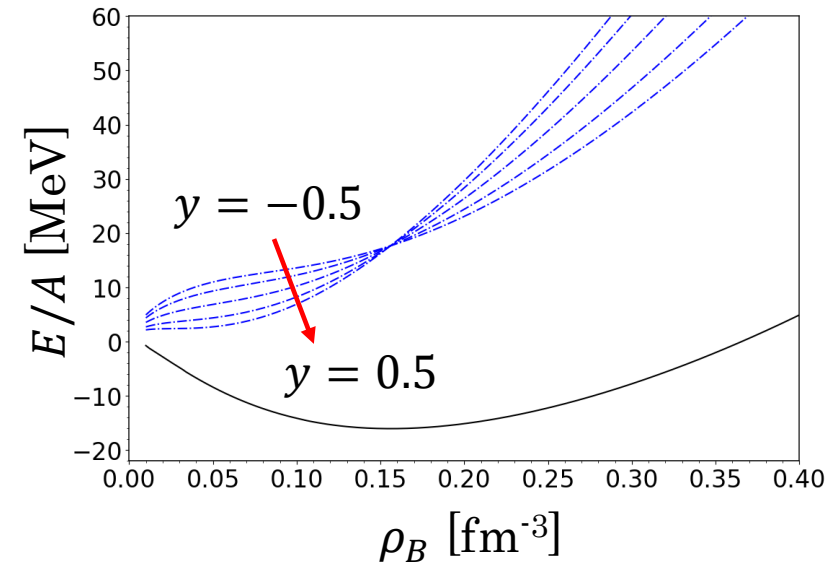
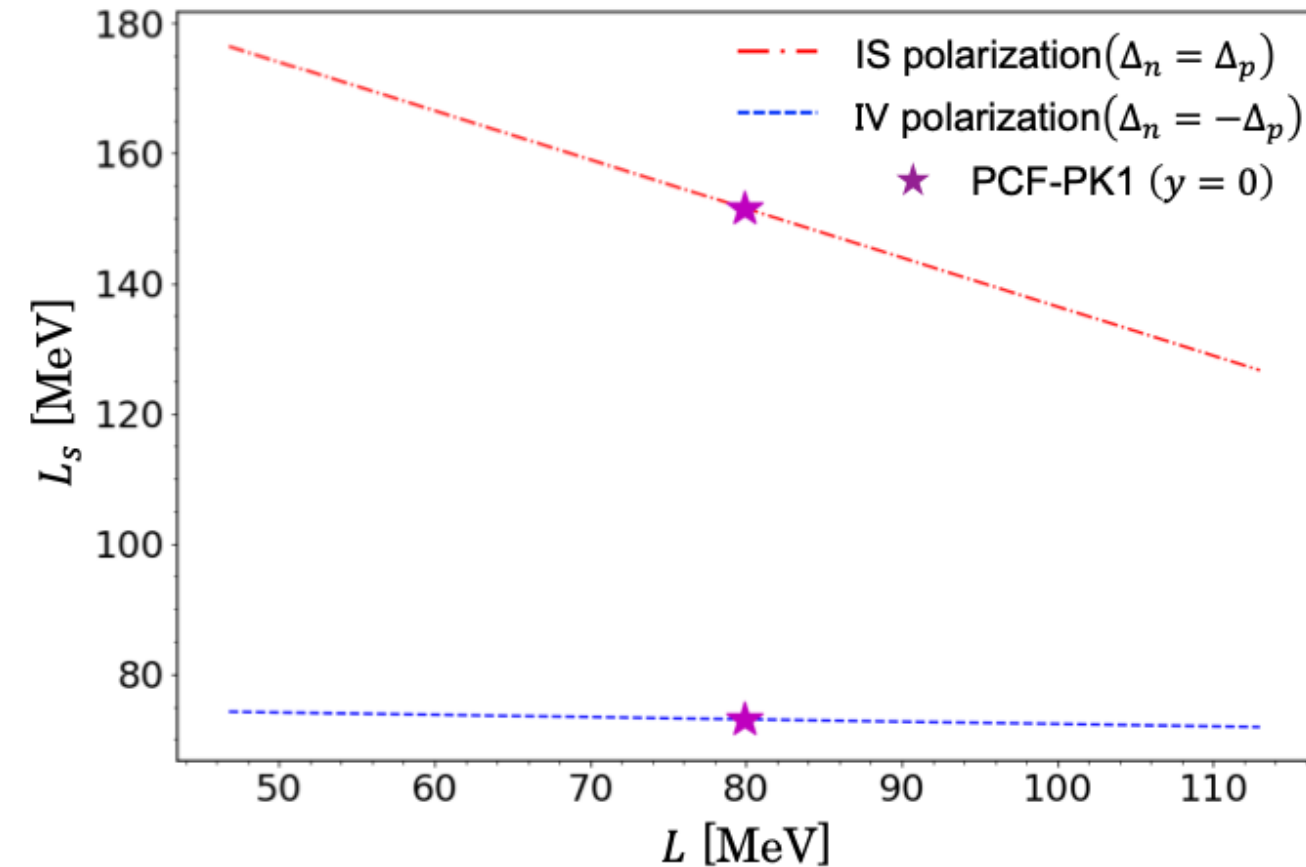


► We investigated the correlation between  $L$  and  $L_s$ .

Introducing a new variable  $y$ :

$$\alpha_{tV}(\rho) \rightarrow (1 - y)\alpha_{tV}(\rho) + y\alpha_{tV}(\rho_0)$$

cf. T. Inakura, H. Nakada, Phys. Rev. C 92, 064302 (2015)



- In the case of IS polarization  $\Delta_n = \Delta_p = \Delta$ ,  $L_s$  and  $L$  are linearly correlated.
- In the case of IV polarization  $\Delta_n = -\Delta_p = \Delta$ ,  $L_s$  is nearly independent of  $L$ .

$$\alpha_{tV}(\rho) \rightarrow (1 - y)\alpha_{tV}(\rho) + y\alpha_{tV}(\rho_0)$$

$$\alpha_{tT} = \frac{1}{18}(-\alpha_S + 3\alpha_{tS} + 2\alpha_V - 6\alpha_{tV} + 6\alpha_T),$$

$$\alpha_{PV} = \frac{1}{3}(2\alpha_S + 3\alpha_{tS} + 2\alpha_V + 3\alpha_{tV} + 6\alpha_T),$$

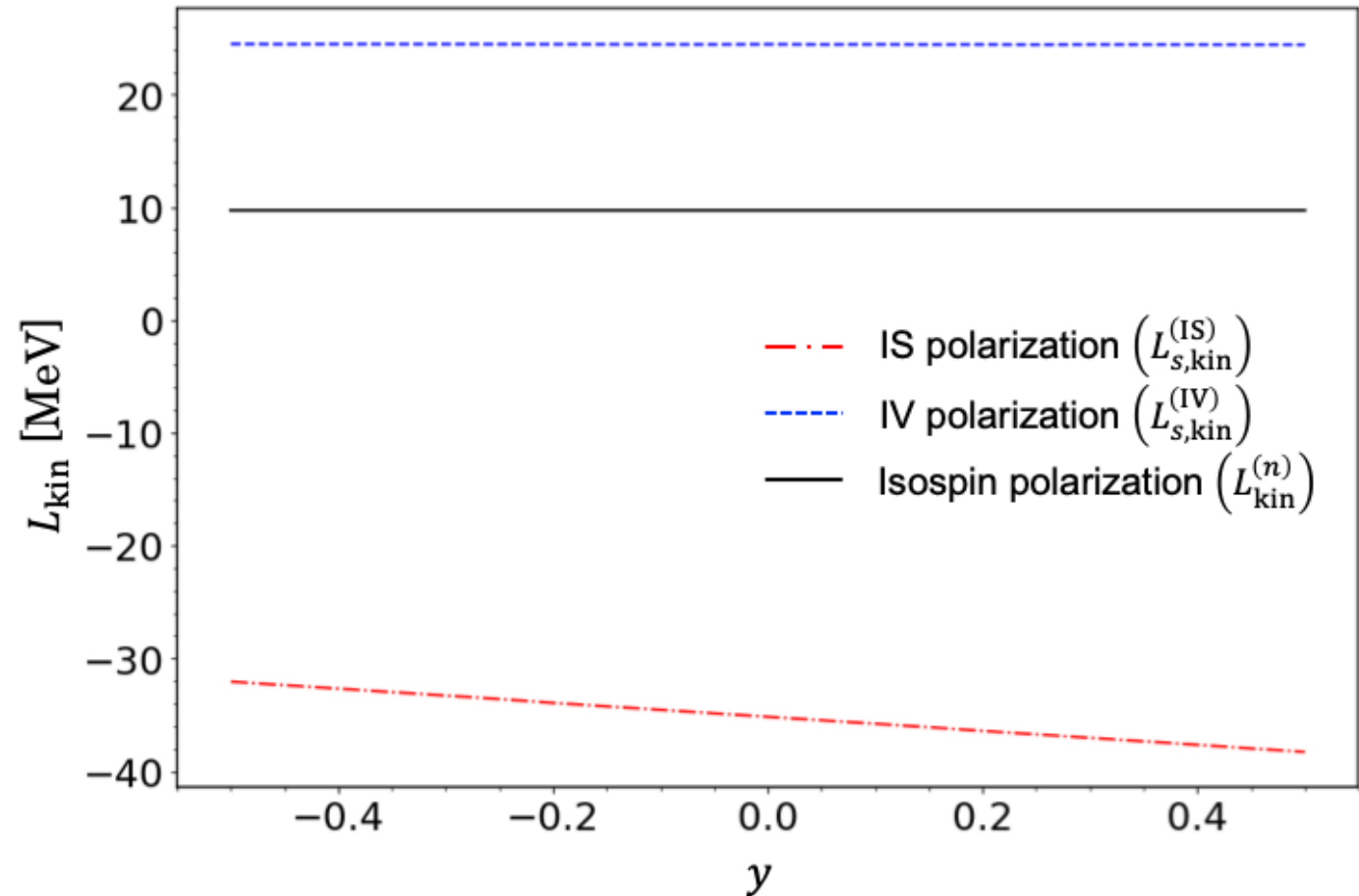
$$\alpha_{tPV} = \frac{1}{9}(2\alpha_S + 3\alpha_{tS} + 5\alpha_V - 6\alpha_{tV} + 6\alpha_T).$$

$$L + L_S^{\text{IS}} \simeq \text{const.} \quad \blacktriangleright \quad L \simeq -L_S^{\text{IS}} + \text{const.}$$

$$* \quad L + L_S^{\text{IS}} \simeq L_{\text{kin},n} + L_{\text{kin}}^{\text{IS}} - (\alpha_S + \alpha_V)\rho_0$$

$$L_S^{\text{IV}} \simeq \text{const.}$$

$$* \quad L_S^{\text{IV}} = L_{\text{kin}}^{\text{IV}} - \frac{\rho_0}{2}(\alpha_S + \alpha_V)$$



Correlations between  $L$  and  $L_S$  arise from the **relativistic structure** of the model

# Outline

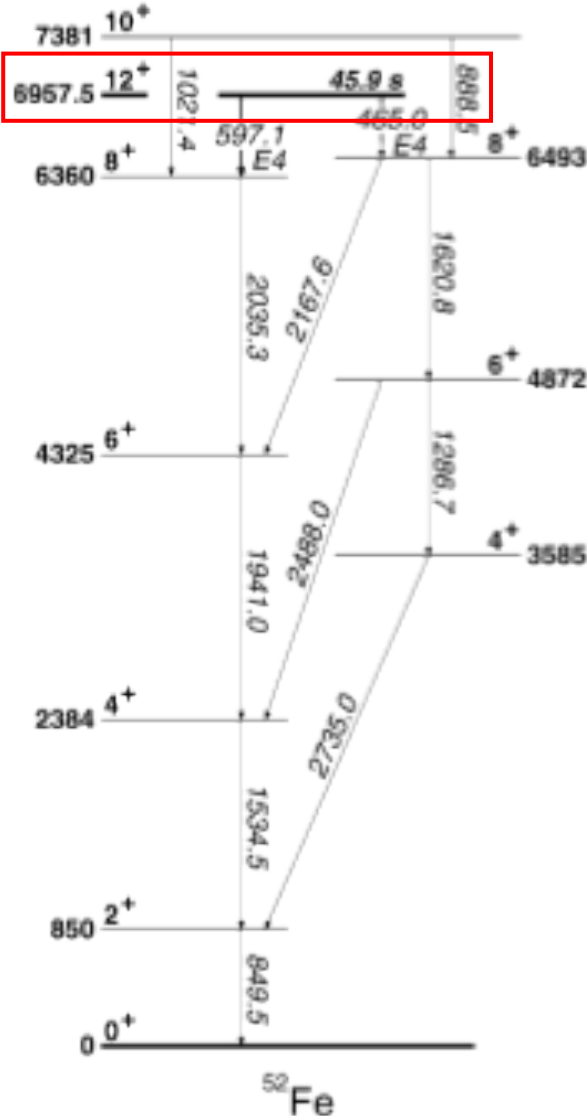
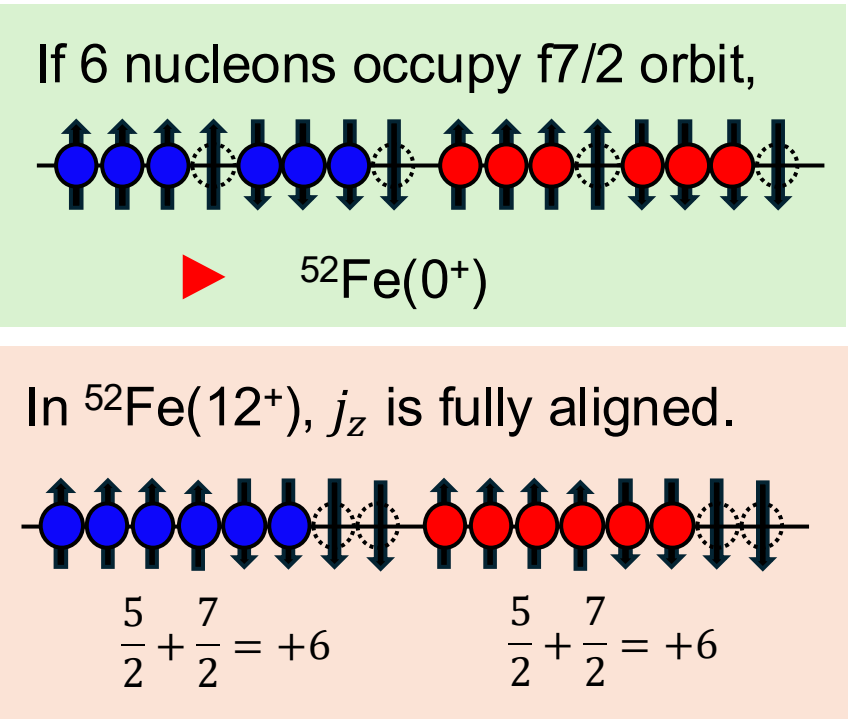
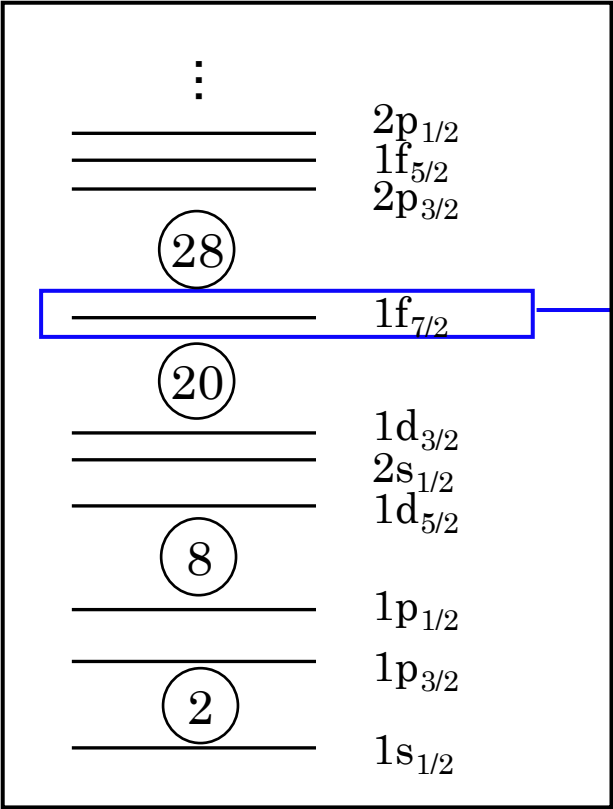
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# Nuclear Isomer and $12^+$ Excited State of $^{52}\text{Fe}$ ( $N = Z = 26$ )

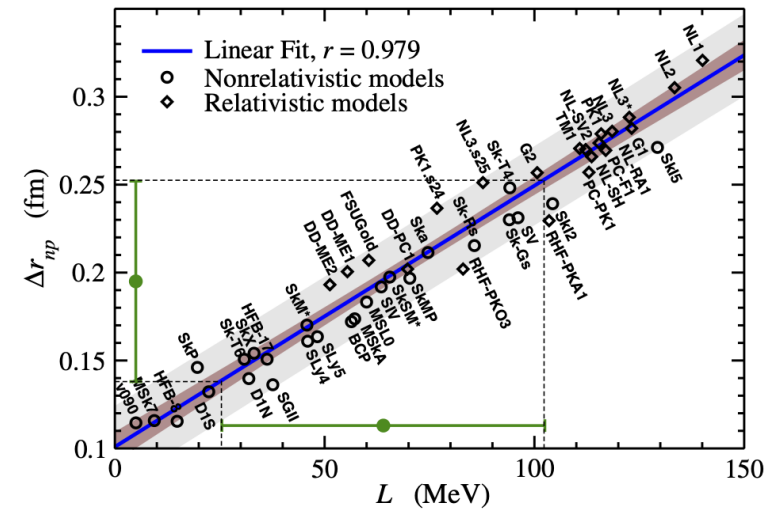
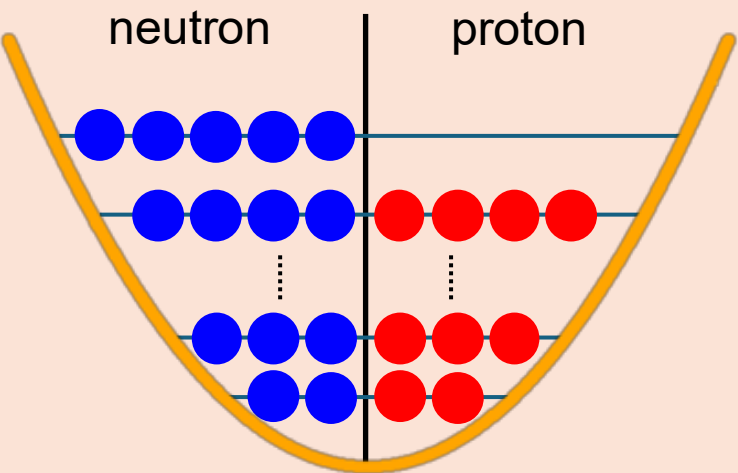
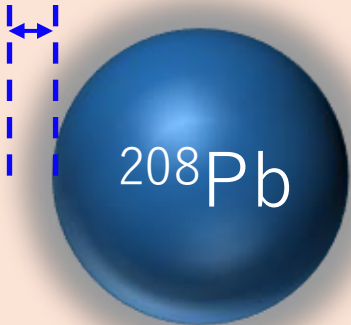
Nuclear isomer : excited state with a long lifetime

Lifetime of  $12^+$  excited state of  $^{52}\text{Fe}$  is 45.9s. ▶ Isomer with high spin



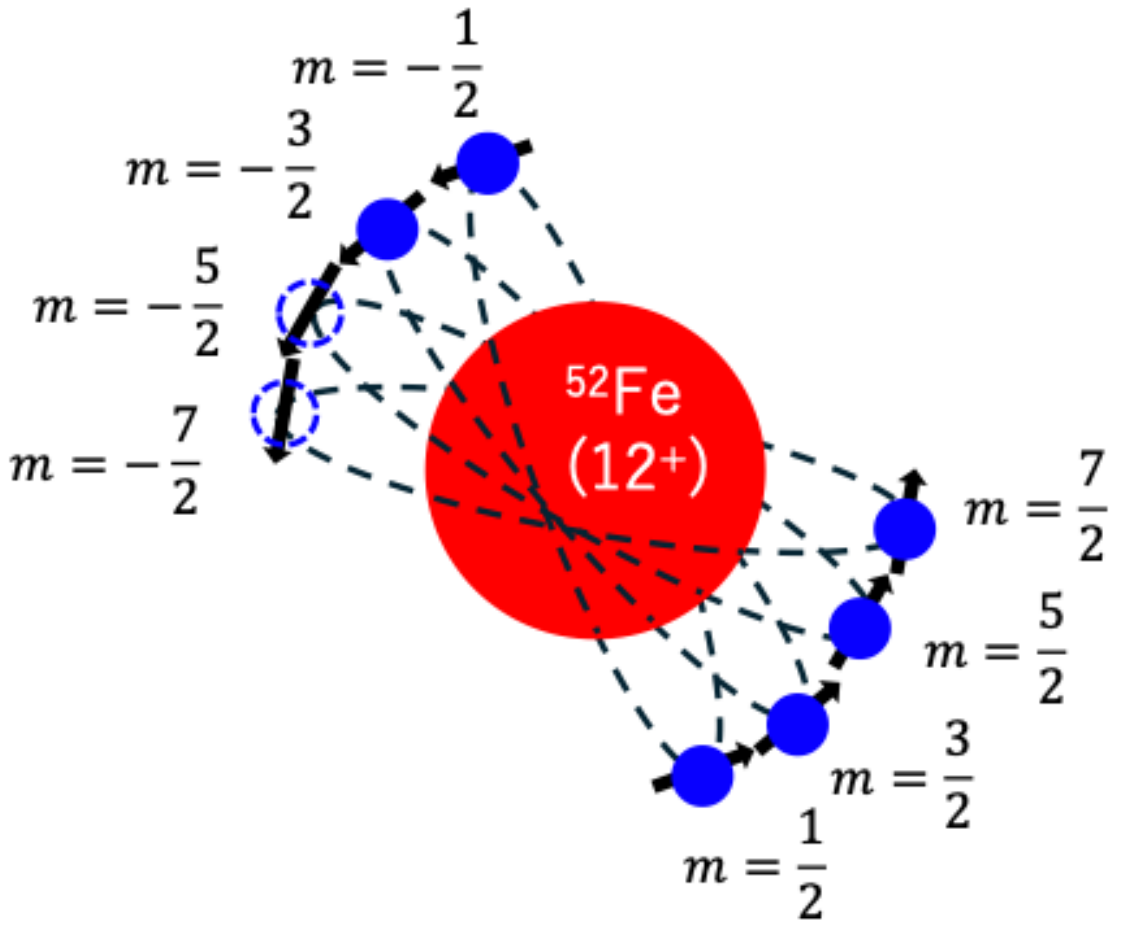
# Why High-spin Isomers?

Neutron skin thickness



$0^+$  ground state : spherical HFB calculation with separable pairing interaction

$12^+$  isomer state : HF calculation for  $^{52}\text{Fe}$  with the fixed configuration



- The configurations in  $f_{7/2}$  orbit are fixed by blocking  $m = -5/2$  and  $m = -7/2$  orbits.

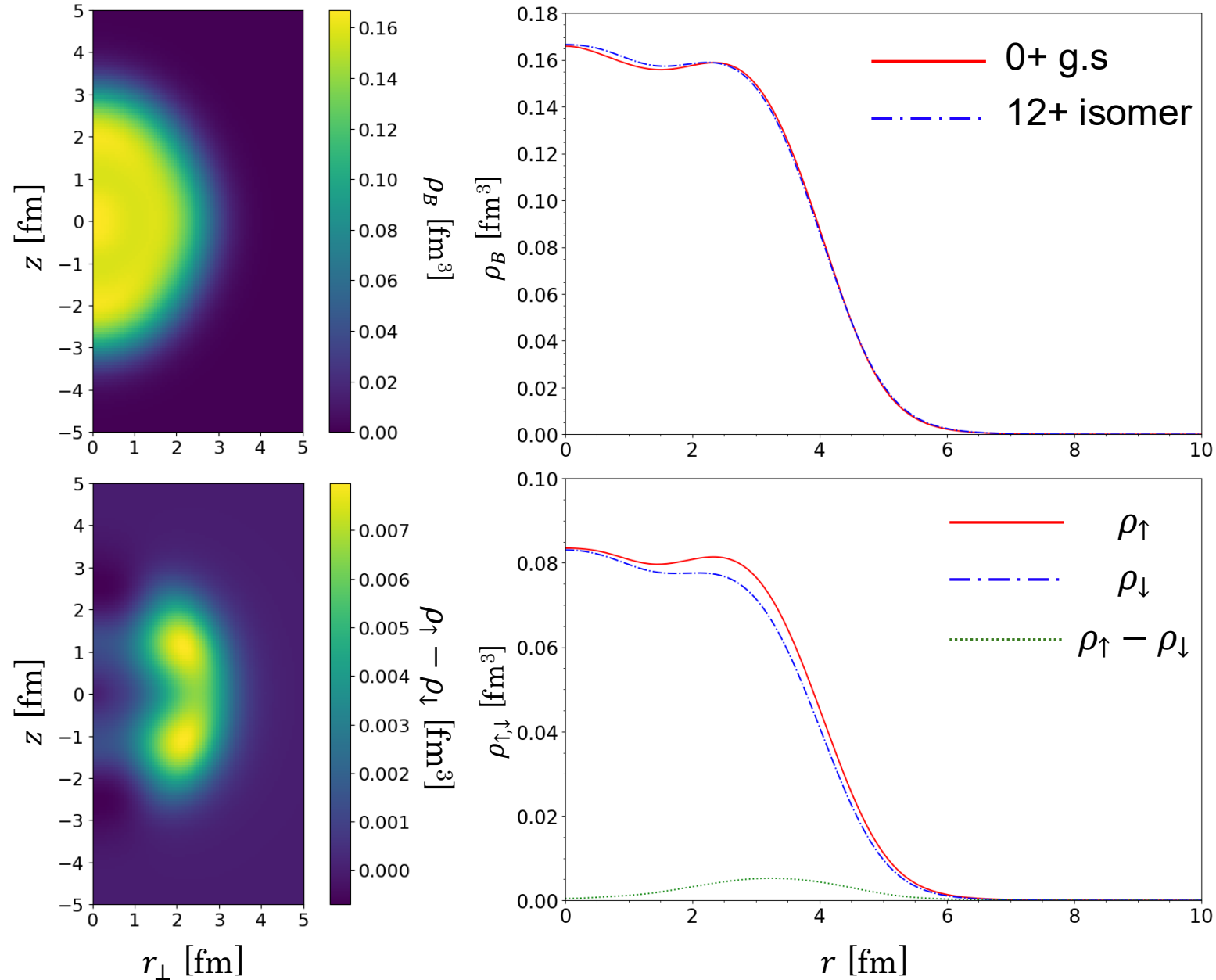


The total angular momentum is 6 for both neutrons and protons.

- Pairing correlations are neglected in calculation of  $12^+$  isomer state.

# Nucleon and Spin Density of $^{52}\text{Fe}(12+)\text{ State}$

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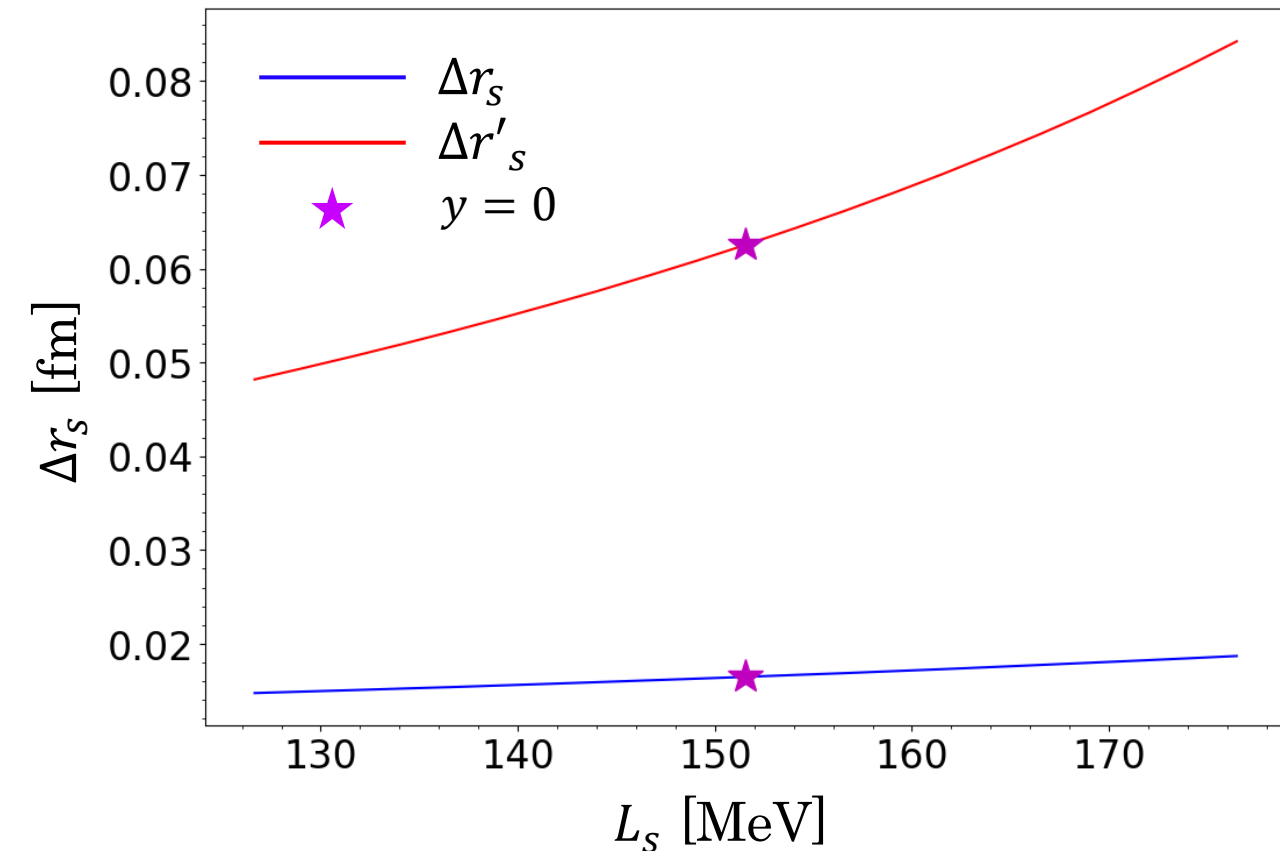


Spin densities are localized near the surface!

Introducing a new variable  $y$ :

$$\alpha_{tV}(\rho) \rightarrow (1 - y)\alpha_{tV}(\rho) + y\alpha_{tV}(\rho_0)$$

cf. T. Inakura, H. Nakada, Phys. Rev. C 92, 064302 (2015)



Two definitions of the "spin skin thickness"

1.  $\Delta r_s = (\text{rad. of } 12^+ \text{ isomer}) - (\text{rad. of } 0^+ \text{ g. s.})$

2.  $\Delta r'_s = r_+ - r_-$

$r_{\pm}$  : rms radius of  $\rho_{\pm} = \langle \psi^{\dagger} (1 \pm \Sigma_3) \psi \rangle / 2$ ,  
the spin-up (-down) nucleon density.

$\Delta r'_s$  is determined only by the properties of  $12^+$  state of  $^{52}\text{Fe}$ .

$\Delta r_s$  : positive but weak correlation

cf.  $\Delta R_{\text{exp}} \approx 0.003 \text{ fm}$

A. R. Vernon, et al., PRL 134 252501 (2025)

$\Delta r'_s$  : positive and stronger correlation

However, how to observe?

# Summary

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- ❑ Correlation between slope parameter and spin slope parameter
  - a negative correlation in the case of  $\Delta_n = \Delta_p = \Delta$
  - $L_s$  is nearly independent of  $L$  in the case of  $\Delta_n = -\Delta_p = \Delta$
- ❑ Correlation between spin skin thickness  $\Delta r_s$  ( $\Delta r'_s$ ) and spin slope parameter  $L_s$ 
  - $\Delta r_s$  : the difference in radii between  $0^+$  ground state and  $12^+$  isomer state
  - $\Delta r'_s$  : the difference in radii between spin-up and spin-down nucleon in  $12^+$  state
  - a positive but weak correlation between  $\Delta r_s$  and  $L_s$
  - a positive and stronger correlation between  $\Delta r'_s$  and  $L_s$
- ❑ Future work
  - Calculating high-spin isomer state of other nuclei
  - Including the pairing correlation in the calculation of high-spin isomers