Interaction cross sections and the investigation of deformation effects in neutron-rich Zr isotopes

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Outline

① Motivation

Shape transition in Zr isotopes

② Measurements

interaction cross sections → matter radii

③ Interpretation

Structure & reaction

④ Fluctuations in nuclear reactions

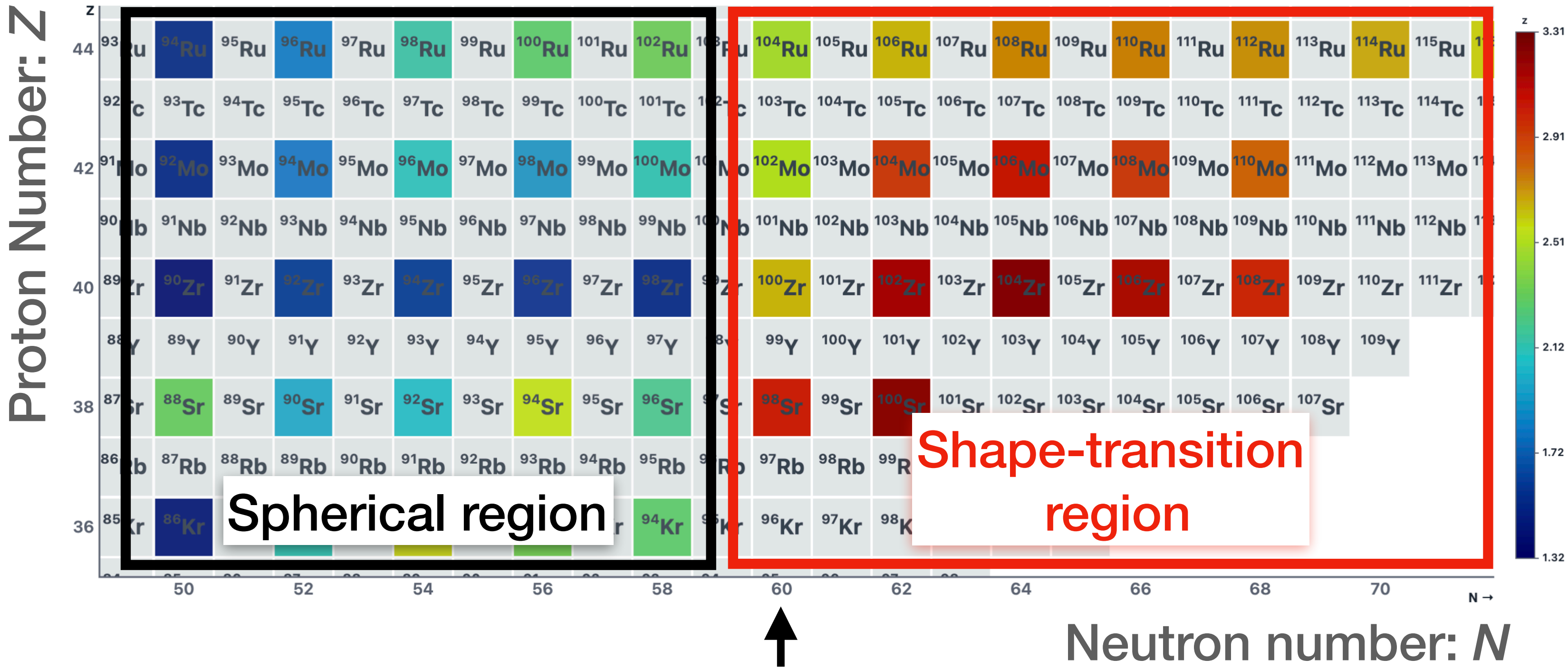
From Glauber to Good-Walker picture

⑤ Future

Toward nuclear shape imaging

Shape transition around Zr isotopes

Nuclear chart of $R_{4/2}$ (E(4+)/E(2+))

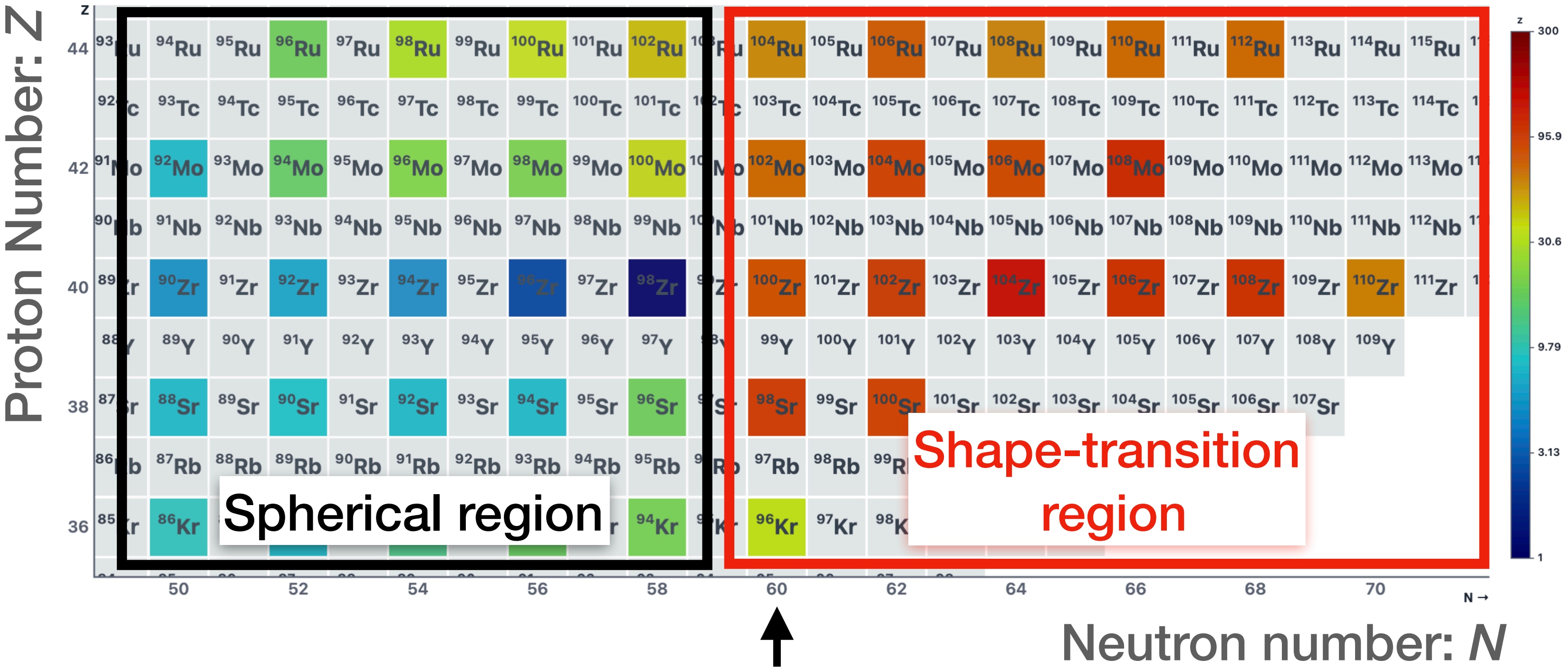


N = 60: transition boundary

Nudat3

Shape transition around Zr isotopes

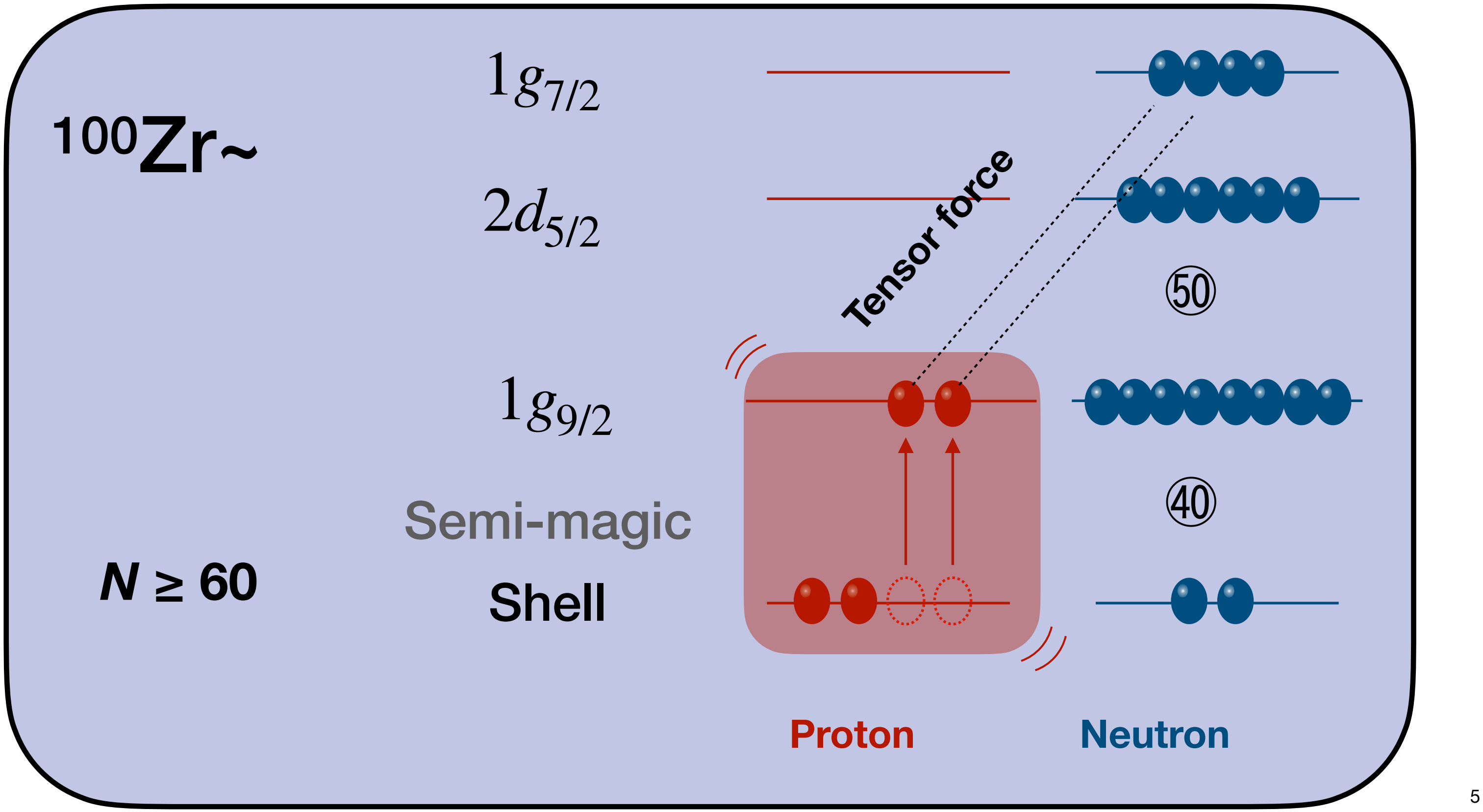
Nuclear chart of B(E2) w.u.



N = 60: transition boundary

Byul Moon, "Triaxial deformation of neutron-rich Zr nuclei explored by high-resolution in-beam γ-ray spectroscopy", Phys. Lett. B 858 (2024) 139047

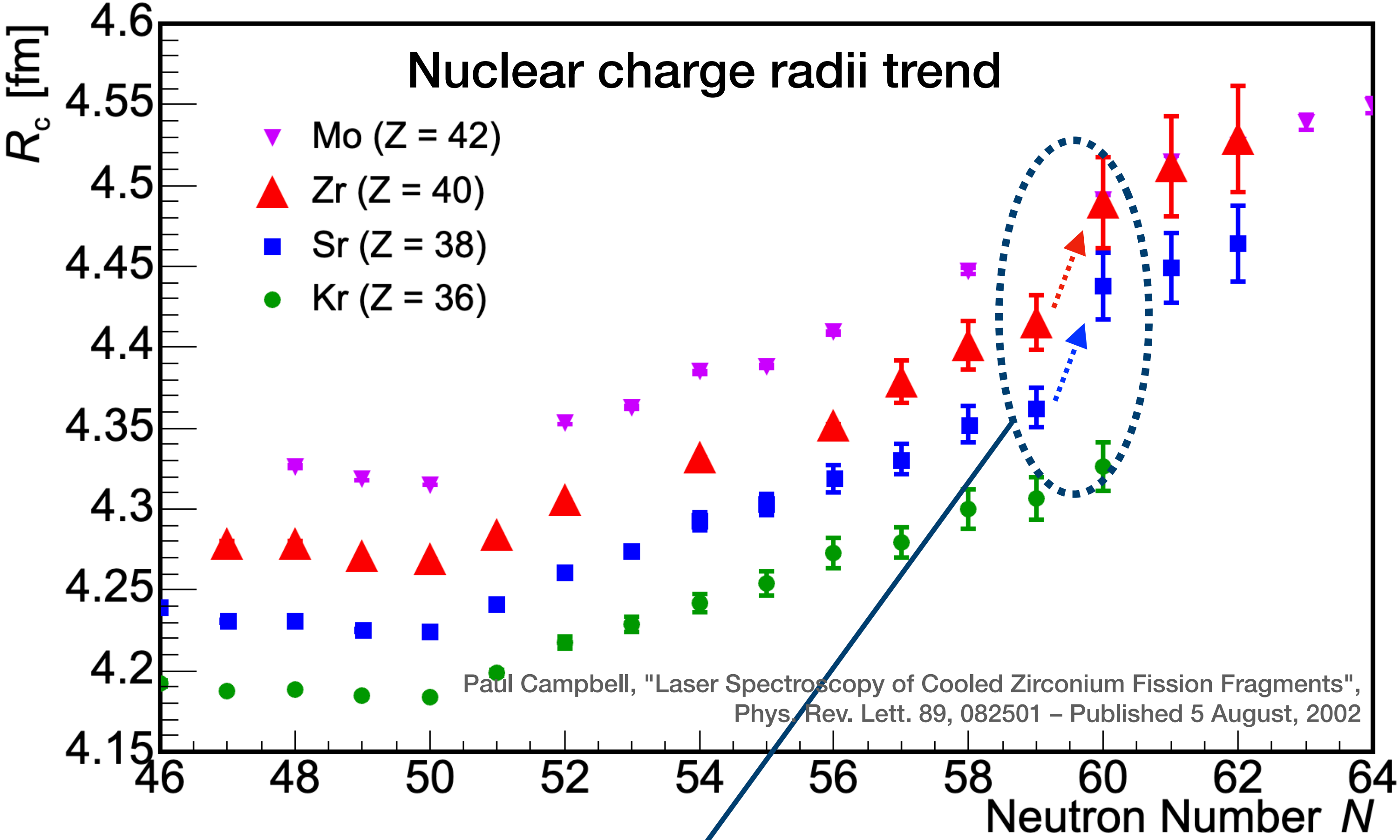
Shape transition around Zr isotopes



Tensor of $n_{1g7/2}$ - $p_{1g9/2}$ drives the proton shell configuration drastically changing

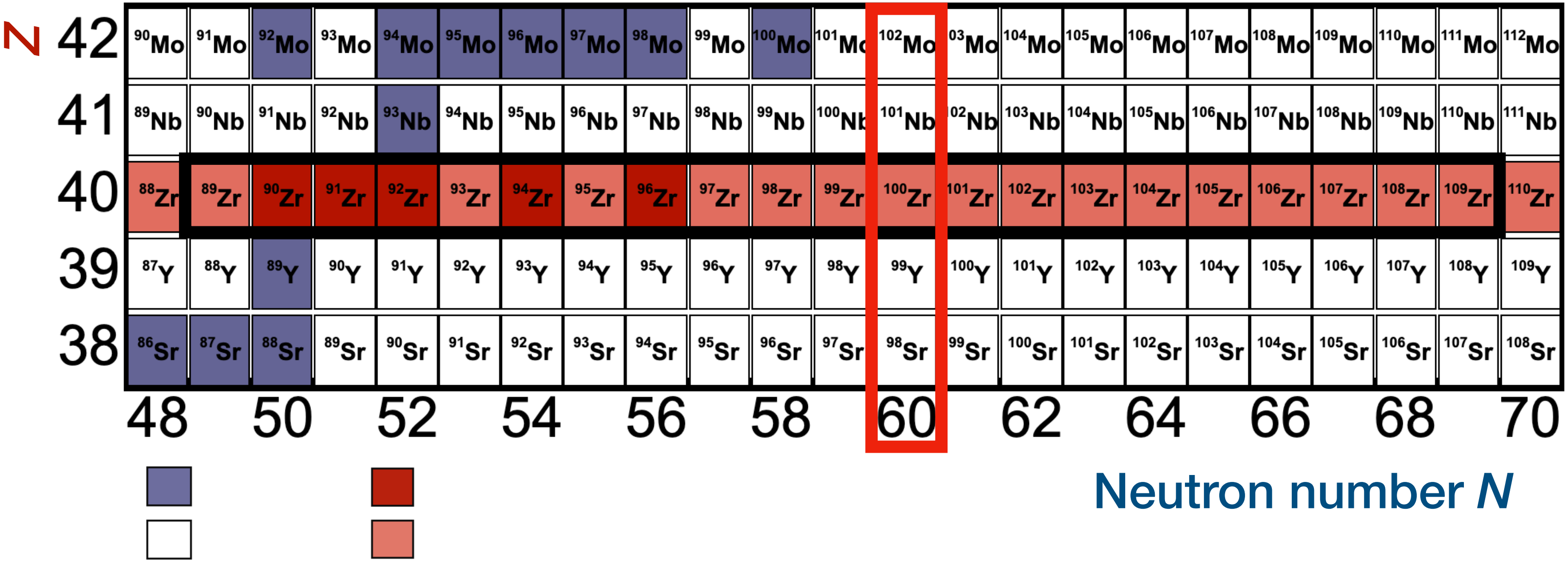
Tomoaki Togashi, "Quantum Phase Transition in the Shape of Zr isotopes", Phys. Rev. Lett. 117, 172502 – Published 17 October, 2016

Sudden charge-radius increase at N = 60



Charge-radius jump reflects the shape transition

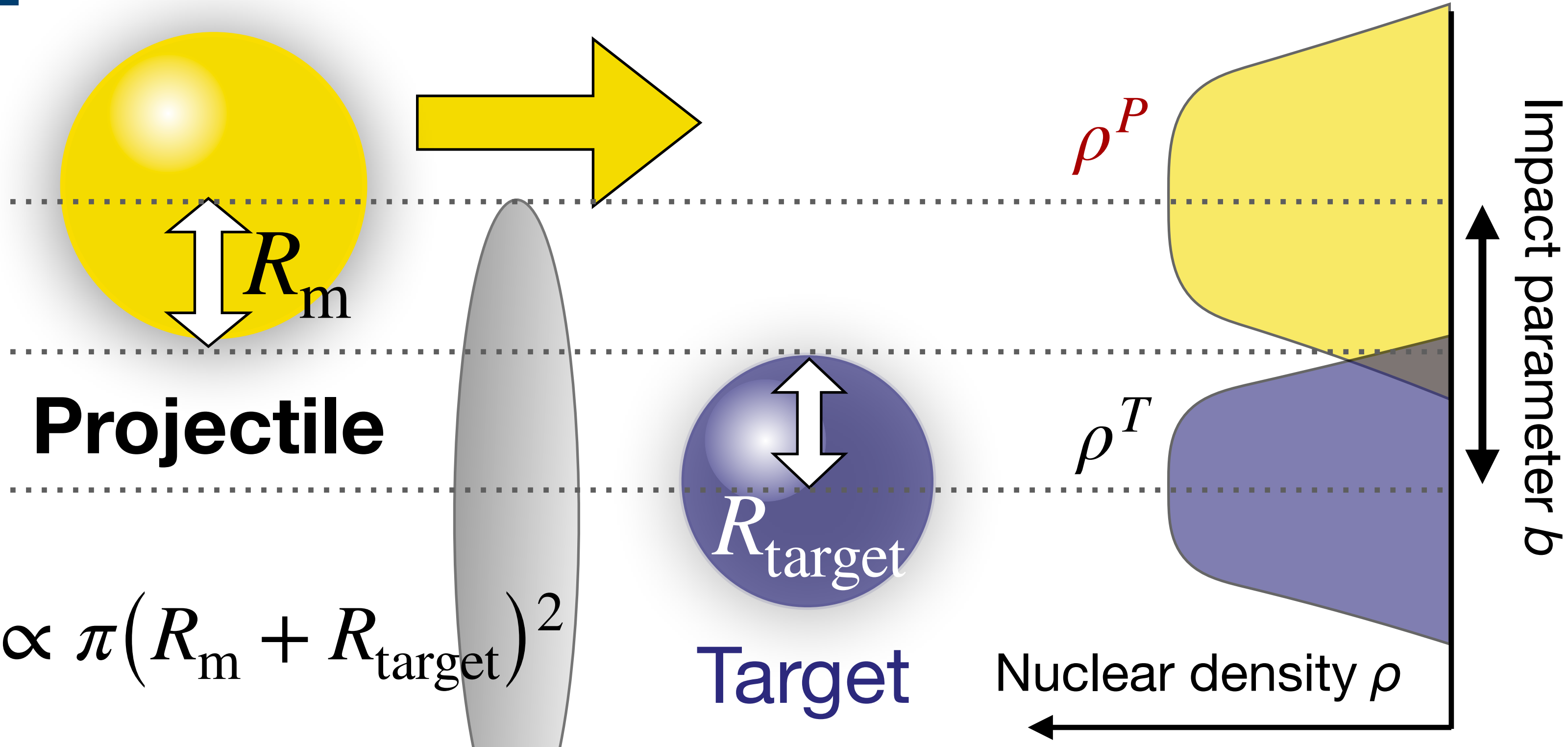
Why measure matter radii ?



- ▶ Proton distributions are well established (R_{ch}, $B(E2)$, etc.)
- ▶ **Neutron distributions remain poorly known**

→ **Matter radii provide direct probe of neutron distributions**

Reaction cross section

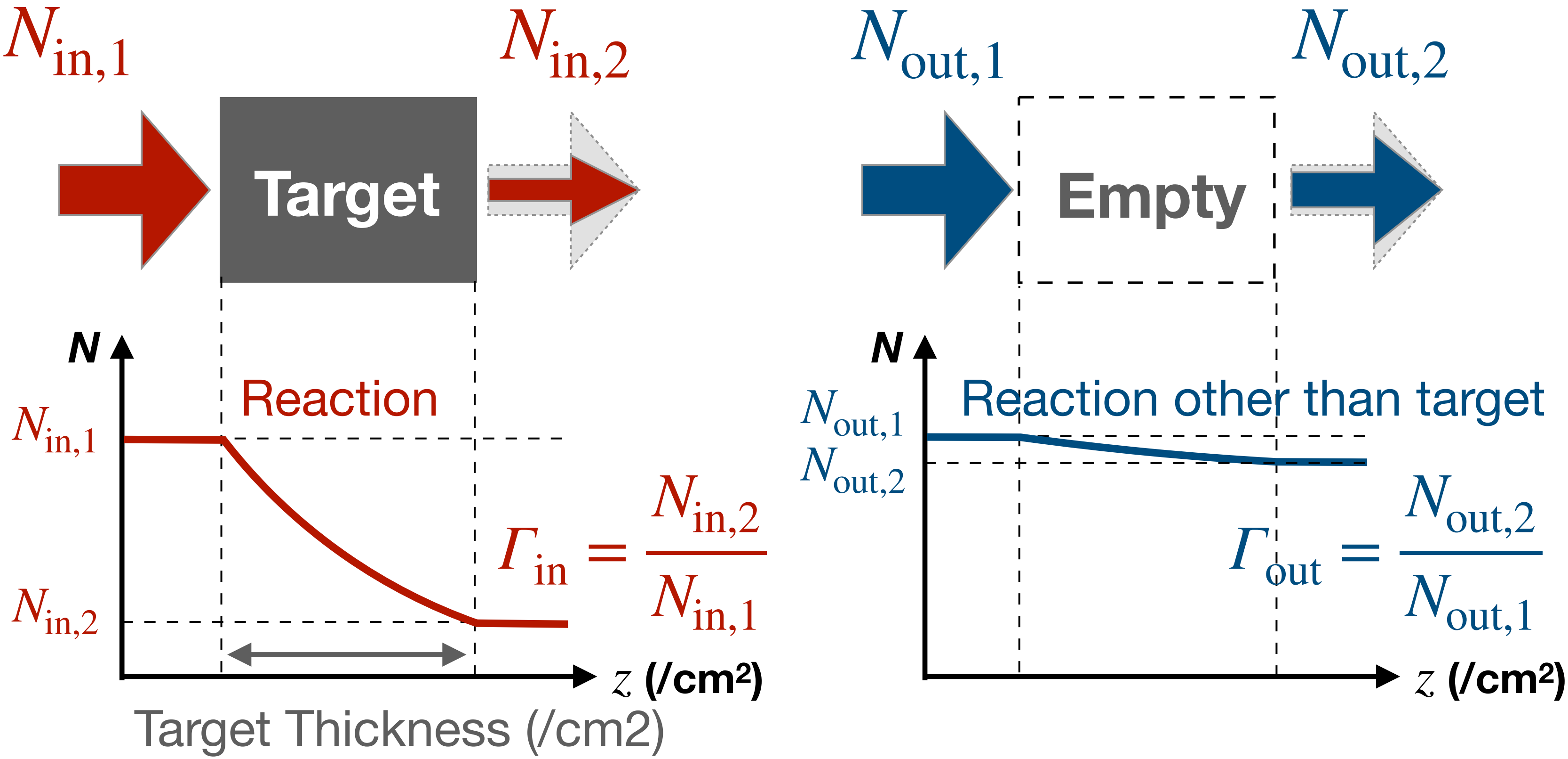


$$\sigma_{\mathrm{R}} \propto \pi\left(R_{\mathrm{m}} + R_{\mathrm{target}}\right)^2$$

$$\sigma_{\mathrm{R}}(E) = \int d\mathbf{b} \left[1 - \exp\left(- \int d\mathbf{s} \sum_{\mathrm{i,j=(p,n)}} \sigma_{\mathrm{ij}}(E)\, \rho^{\mathrm{P}(i)}(\mathbf{s})\, \rho^{\mathrm{T}(j)}(\mathbf{s}-\mathbf{b}) \right) \right]$$

Glauber Theory NN scattering x-section Projectile Target

Reaction cross section



Interaction cross section: $\sigma_{\mathrm{I}} = -\frac{1}{n_{\mathrm{t}}}\ln\left(\frac{\Gamma_{\mathrm{in}}}{\Gamma_{\mathrm{out}}}\right)$

σ_{R} vs σ_{I}

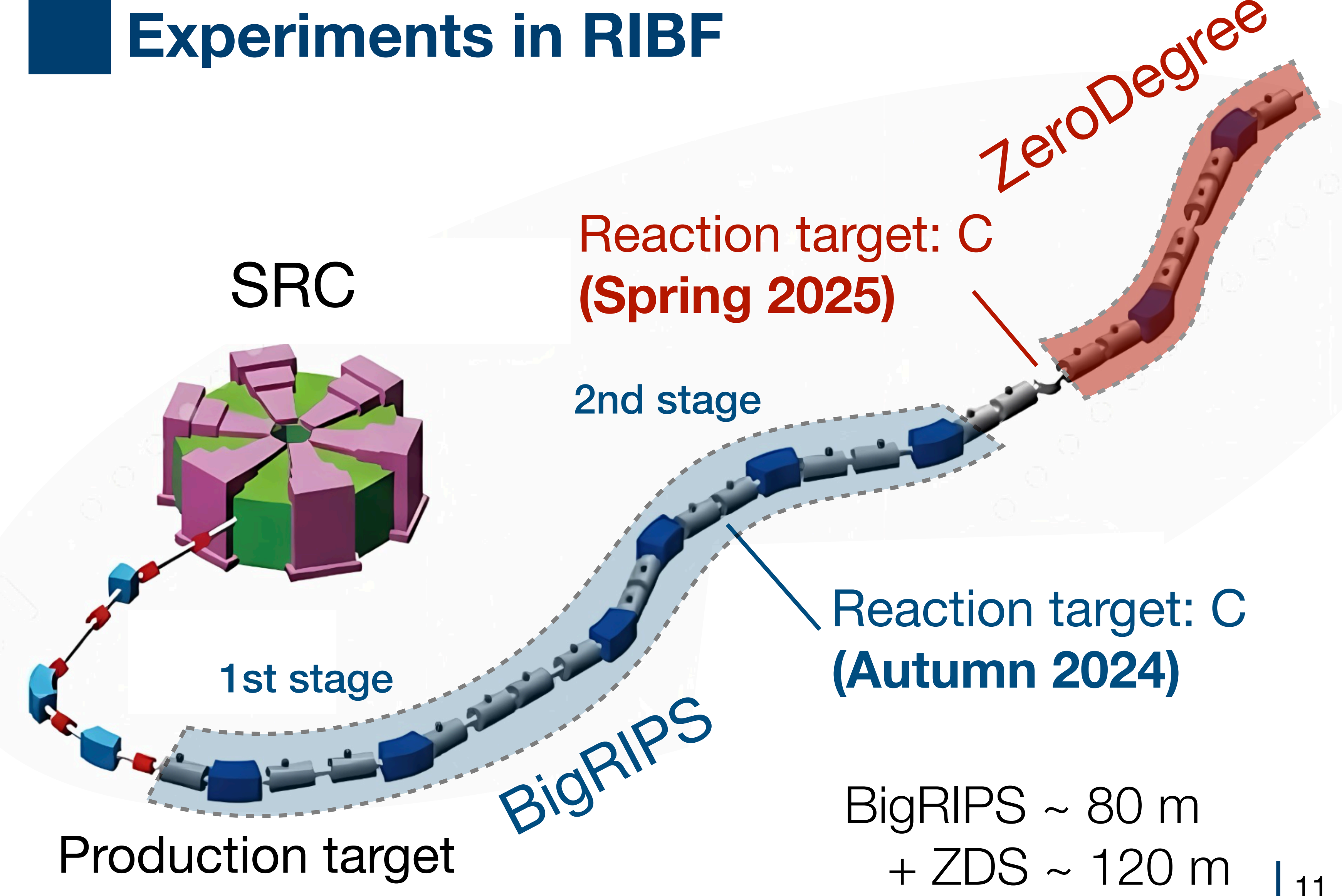
$$\sigma_{\mathrm{R}} = \sigma_{\mathrm{tot}} - \sigma_{\mathrm{elastic}}$$

Measurement is difficult because inelastic transitions
To bound states must be identified

$$\sigma_{\mathrm{I}} = \sigma_{\mathrm{R}} - \sigma_{\mathrm{inelastic\ to\ b.s.}}$$

Measurement is easy because it only requires identifying the surviving projectile.

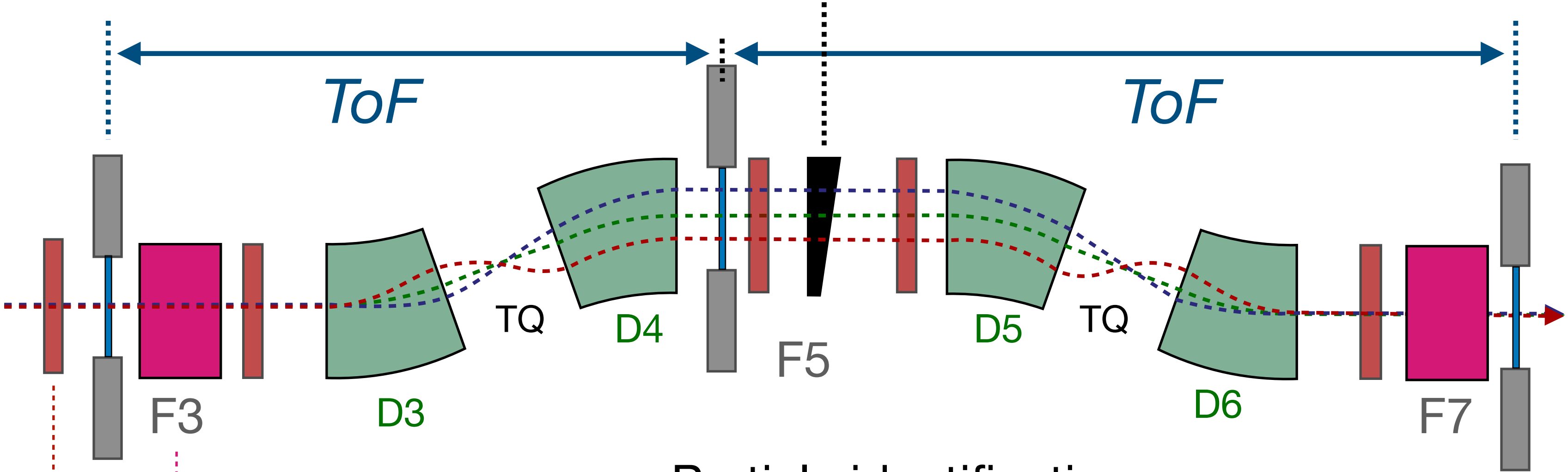
Experiments in RIBF



Experimental setup

Plastic Detector
Time of flight (ToF)

Carbon target
(t_{ave} ~ 1.0, 1.5 g/cm2)



Particle identification
by $B\rho - ToF - \Delta E$

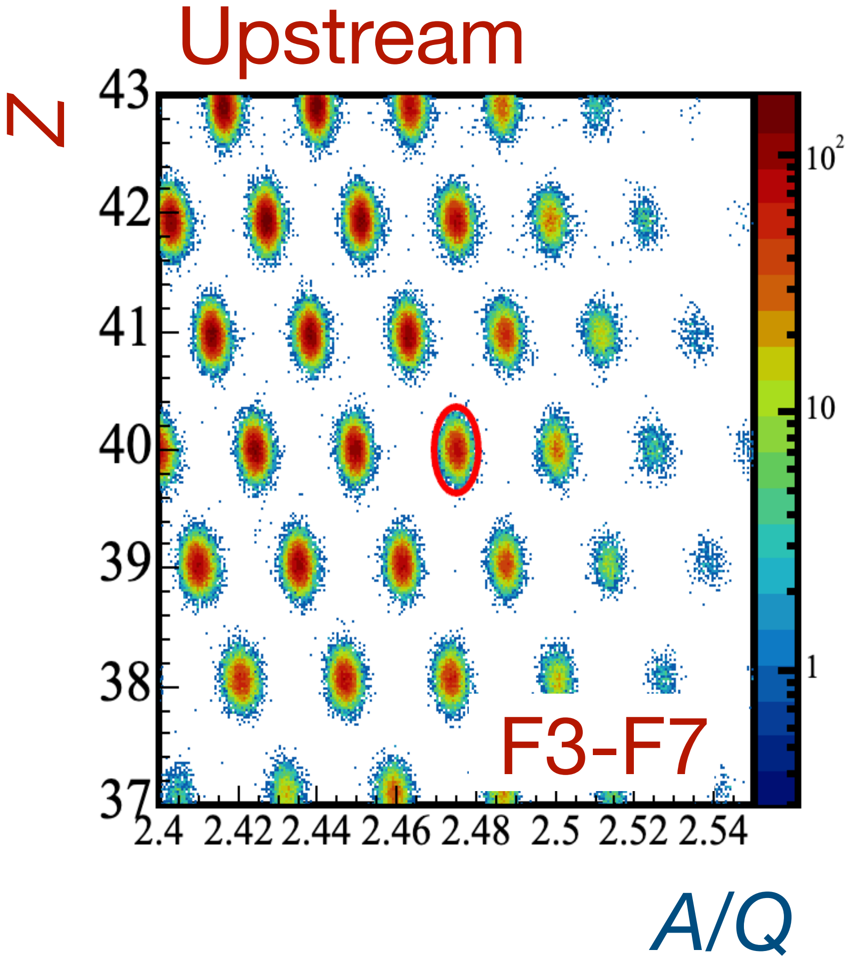
Ionization Chamber:
ΔE measurement

Position Detector PPAC:
Momentum meas.

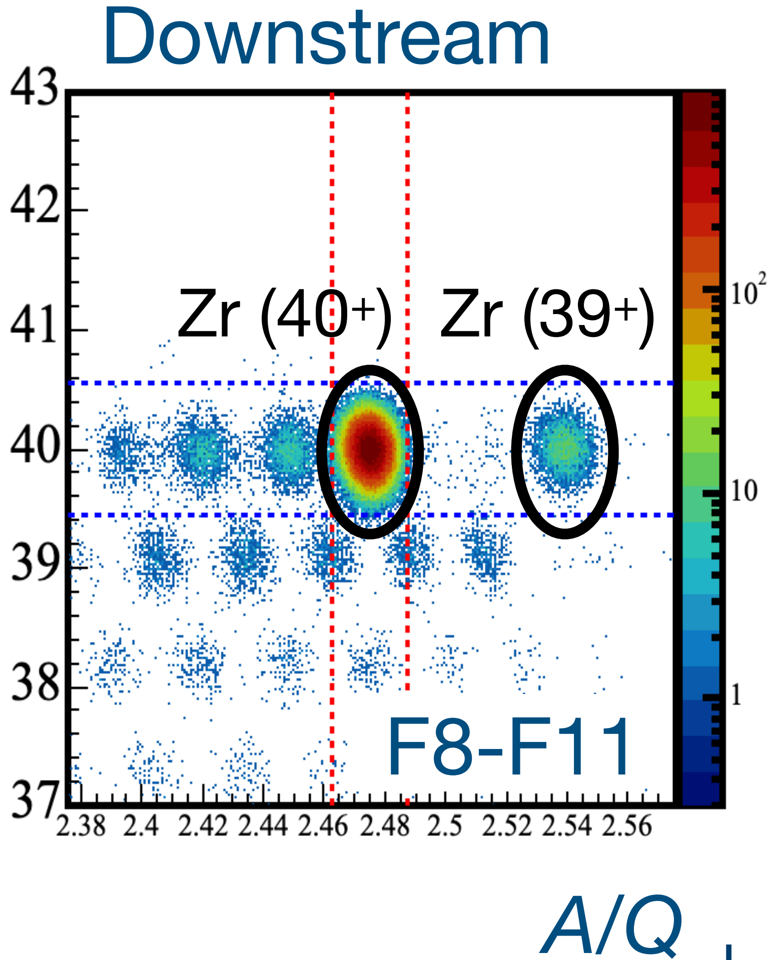
- $ToF \propto 1/v$
- $\Delta E \propto Z^2/v^2$
- $B\rho \propto Av/Q$

⇨ ***A/Q* and *Z***

Analysis



Upstream
Z
43
42
41
40
39
38
37
2.4
2.42
2.44
2.46
2.48
2.5
2.52
2.54
10^2
10
1
F3-F7
A/Q



Downstream
43
42
41
40
39
38
37
Zr (40+)
Zr (39+)
2.38
2.4
2.42
2.44
2.46
2.48
2.5
2.52
2.54
2.56
10^2
10
1
F8-F11
A/Q

Results

[redacted]

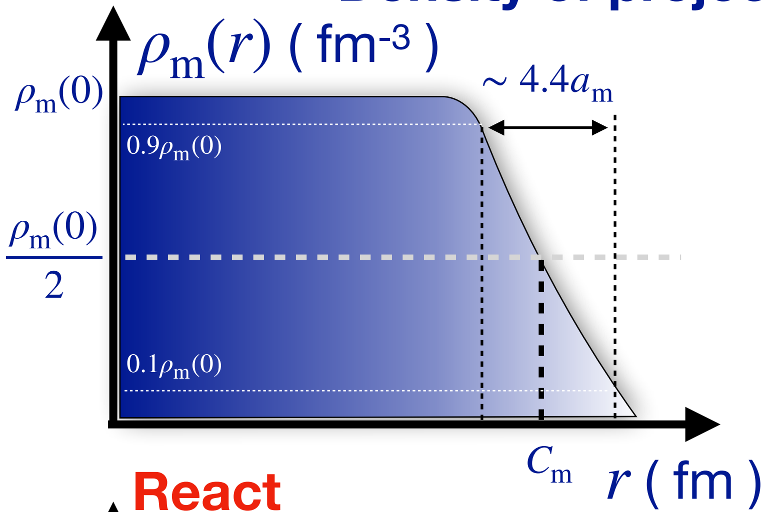
Results

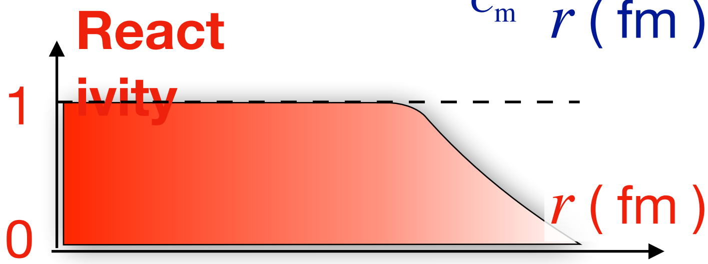
[redacted]

Matter radii extraction

$$\sigma_{\mathrm{R}}(E) = \int d\mathbf{b} \left[1 - \exp\left(- \int d\mathbf{s} \sum_{\mathrm{i,j=(p,n)}} \sigma_{\mathrm{ij}}(E)\, \rho^{\mathrm{P}(i)}(\mathbf{s})\, \rho^{\mathrm{T}(j)}(\mathbf{s}-\mathbf{b}) \right) \right]$$

Density of projectile nucleus **^{12}C density distribution**





$$\rho_{\mathrm{m}}(r) = \frac{\rho_{0,\mathrm{m}}}{1 + \exp\left(\frac{r - C_{\mathrm{m}}}{a_{\mathrm{m}}}\right)}$$

2-parameter-Fermi (2pF) type

Is assumed

$$R_{\mathrm{m}} = \sqrt{\langle r_{\mathrm{m}}^2 \rangle} = \sqrt{\frac{\int_0^\infty r^2 \rho_{\mathrm{m}}(r)\, 4\pi r^2\, dr}{\int_0^\infty \rho_{\mathrm{m}}(r)\, 4\pi r^2\, dr}}$$

Results

[redacted]

Results

Interpretation

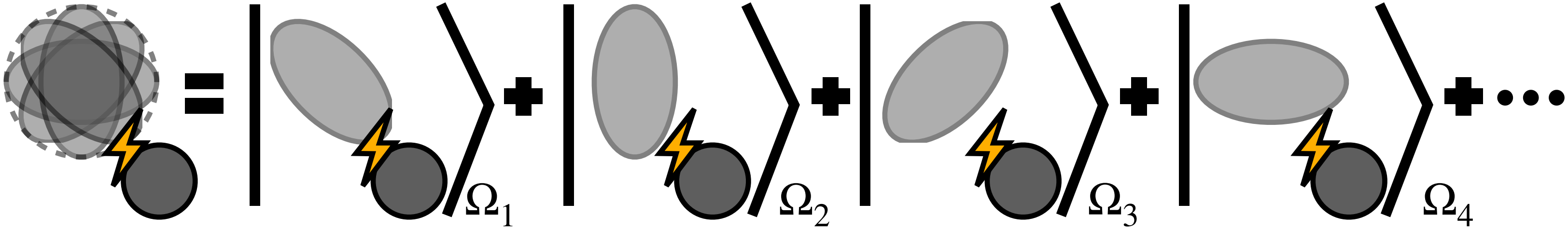
① Proton and neutron deformation is different

② Reaction model

$$P_{\text{b.s.}}(b) = \sum_{J=0,2,4,\cdots} \sum_{M} \left| \langle JM | S(b) | 0^+ \rangle \right|^2$$
$$= \sum_{J,M} \langle 0^+ | S^\dagger(b) | JM \rangle \langle JM | S(b) | 0^+ \rangle$$
$$= \langle 0^+ | S^\dagger(b) \left(\sum_{J,M} | JM \rangle \langle JM | \right) S(b) | 0^+ \rangle$$
$$= \langle 0^+ | S^\dagger(b) S(b) | 0^+ \rangle \,.$$

$$P_{\text{reaction}} = 1 - P_{\text{elastic}} = 1 - |\langle 0^+ | S | 0^+ \rangle|^2$$

$$P_{\text{interaction}} = 1 - P_{\text{transition to b.s.(incl. g.s.)}} = 1 - \langle 0^+ | S^\dagger S | 0^+ \rangle$$



Reaction calculations must be performed for each orientation.

Re-extraction of matter radius

[redacted]

projectile mass number A

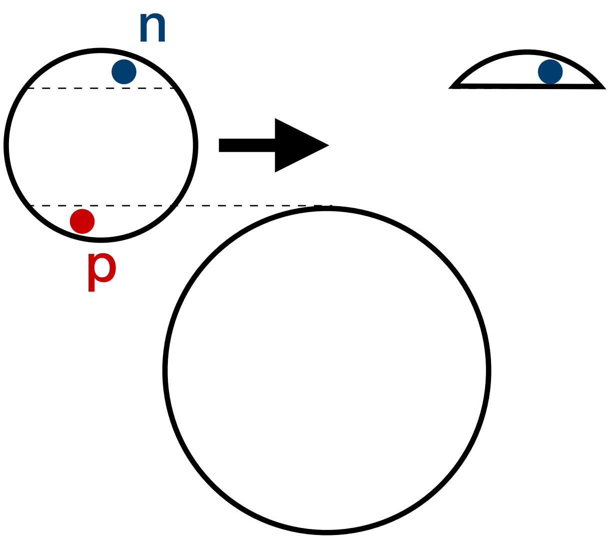
Inelastic to bound states

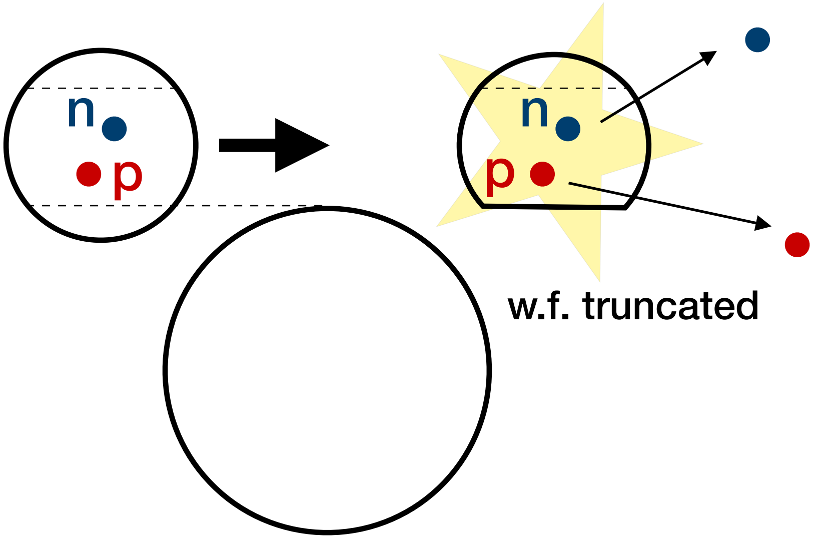
$$P_{\text{inelastic to b.s.}} = P_{\text{reaction}} - P_{\text{interaction}}$$
$$= \langle S^2 \rangle - \langle S \rangle^2$$

This is exactly the variance of S

What is the physical meaning of it ??

Deuteron breakup in Glauber theory





Stripping

Classical picture

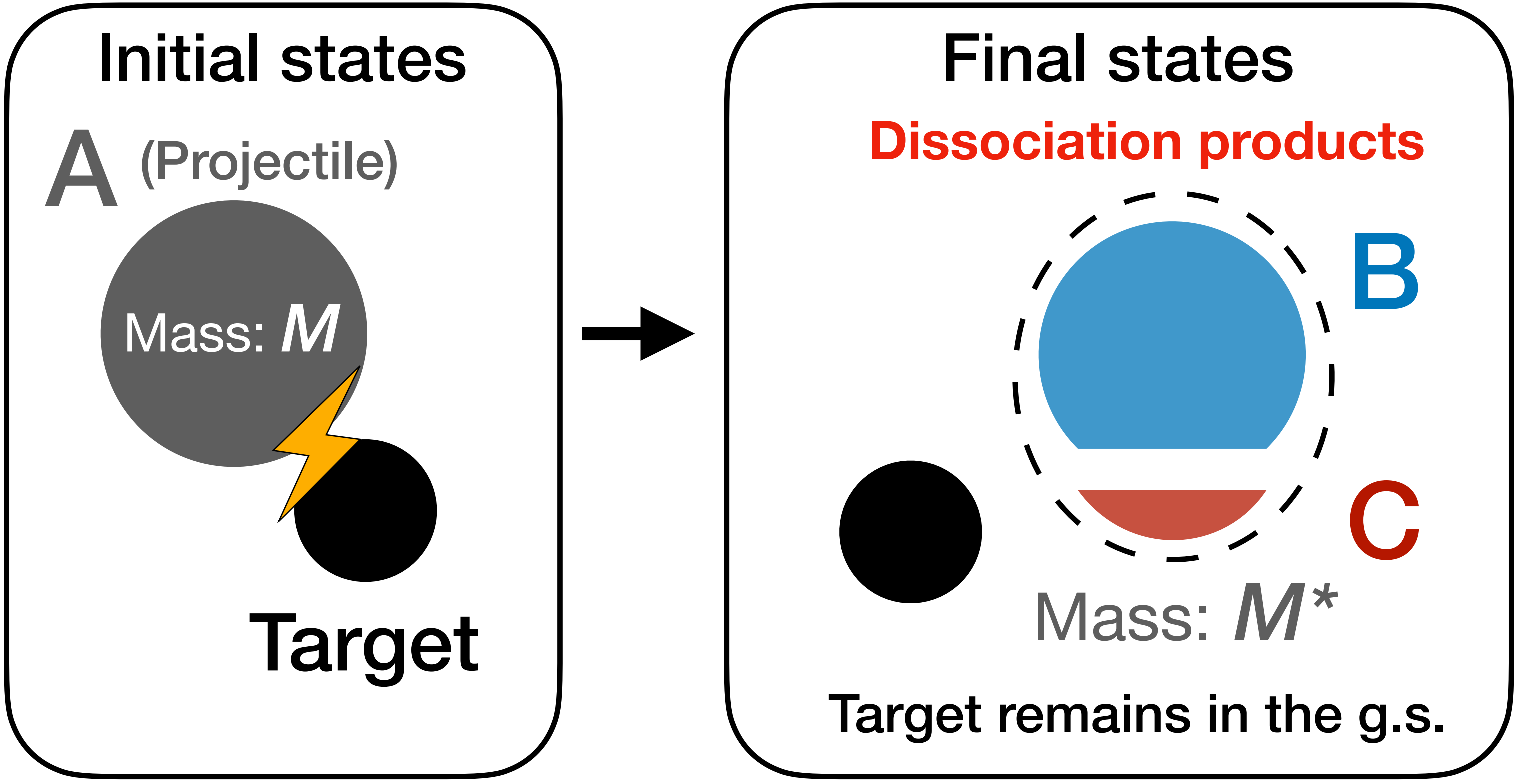
Diffraction Dissociation

Quantum effect:

① Wave function is truncated
② Deuteron is excited
③ Dissociation occurs

Roy J. Glauber, "Deuteron Stripping Processes at High Energies", Physical Review 99, 1515 (1955)

Good-Walker diffractive dissociation



$$q_{\parallel} = \frac{M^{*2} - M^2}{2p}$$

$q_{\parallel}$: Target's longitudinal momentum transfer

$q_{\perp}$ (Transverse momentum transfer) is assumed to be small enough

Good-Walker diffractive dissociation

$$|I\rangle = e^{ikz}\sum_i c_i |C_i\rangle$$

$|I\rangle$: Initial state

C_i : attenuation eigenstates in nuclear matter

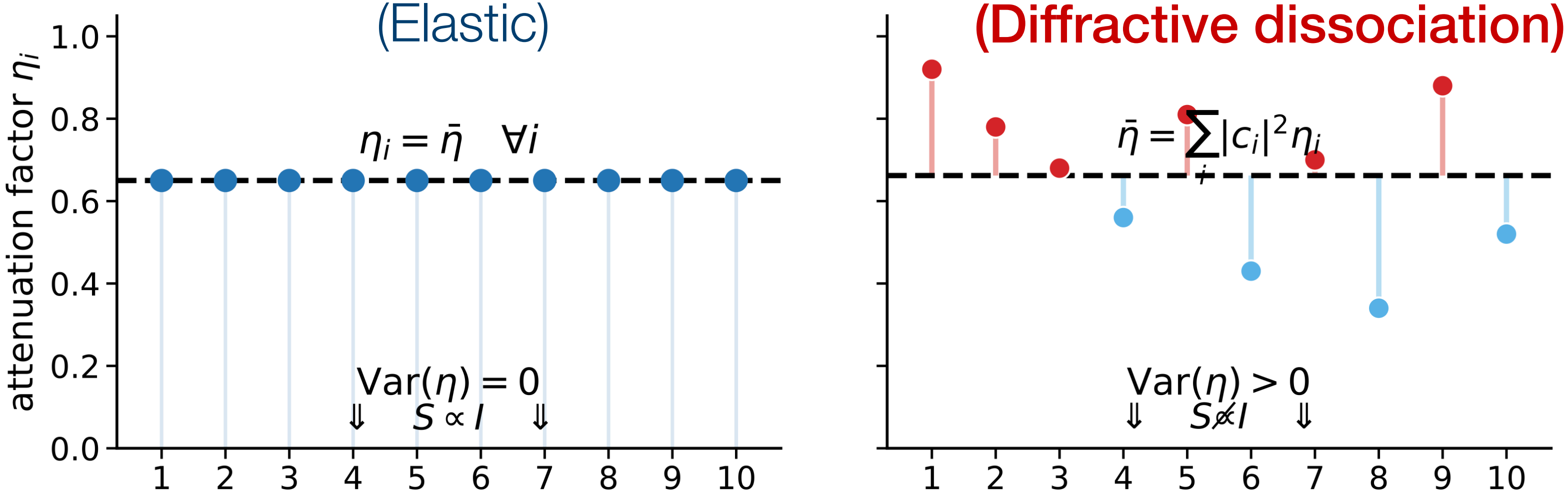
$$|T\rangle = e^{ikz}\sum_i c_i\eta_i |C_i\rangle$$

$|T\rangle$: transmitted state

η_i : attenuation factor

Coherent scattering (Elastic)

Incoherent scattering (Diffractive dissociation)



No fluctuation

Fluctuation

Good-Walker diffractive dissociation

Decomposition

$$|S\rangle \equiv |I\rangle - |T\rangle = \sum_i c_i(1-\eta_i)|C_i\rangle$$

$|S\rangle$: scattered wave

$$= (1-\bar{\eta})|I\rangle + \sum_i (\bar{\eta}-\eta_i)c_i|C_i\rangle$$

Coherent term
(Elastic scattering)

Incoherent term
(Diffractive dissociation)

$$\sigma_{\mathrm{coh}} \propto \left|\langle 0|A|0\rangle\right|^2 = (1-\bar{\eta})^2$$

Scattering Amplitude

$$\sigma_{\mathrm{DD}} \propto \sum_{f\neq 0} \left|\langle f|A|0\rangle\right|^2$$

(Using completeness)

$$= \langle 0|A^\dagger A|0\rangle - \left|\langle 0|A|0\rangle\right|^2$$

$$= \langle \eta^2\rangle - \langle \eta\rangle^2$$ Variance of η

Hannu I. Miettinen, "Diffraction scattering and the parton structure of hadrons", Physical Review D 18, 1696 (1978)

Configuration fluctuation

① Event-by-event config.

$$C_k = \{N, b_1, y_1, \ldots, b_N, y_N\}$$

② Scatt. Amp. for each config.

$$\eta_k \to F_k = F(C_k)$$

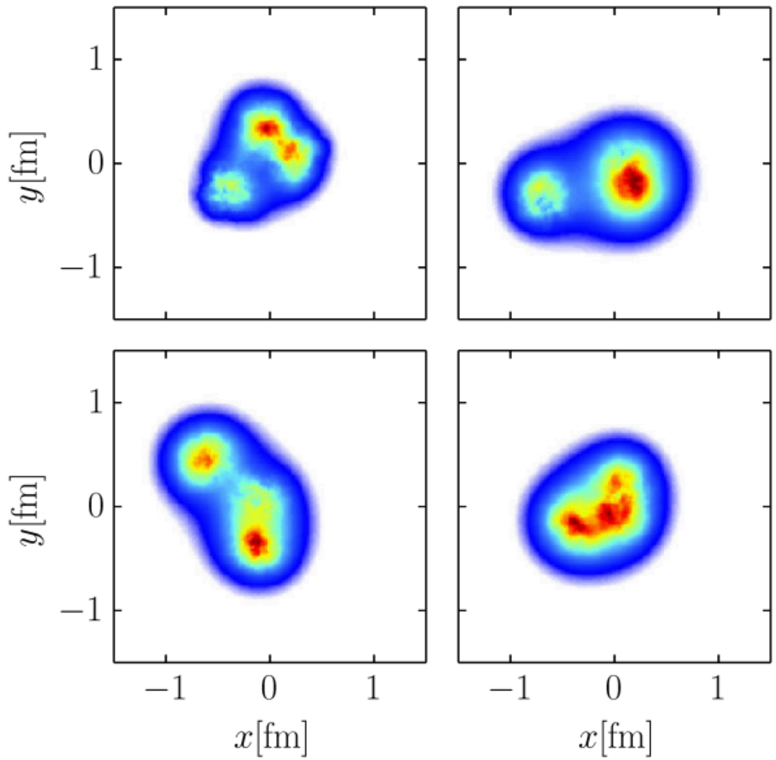
③ Diffractive dissociation

$$\boxed{\sigma_{\mathrm{DD}} \propto \langle F^2 \rangle - \langle F \rangle^2 = \mathrm{Var}(F)}$$

$$= \mathrm{Var}\big(F(\{b_i, y_i\})\big)$$

Variance of the parton config.

Examples of event-by-event config.



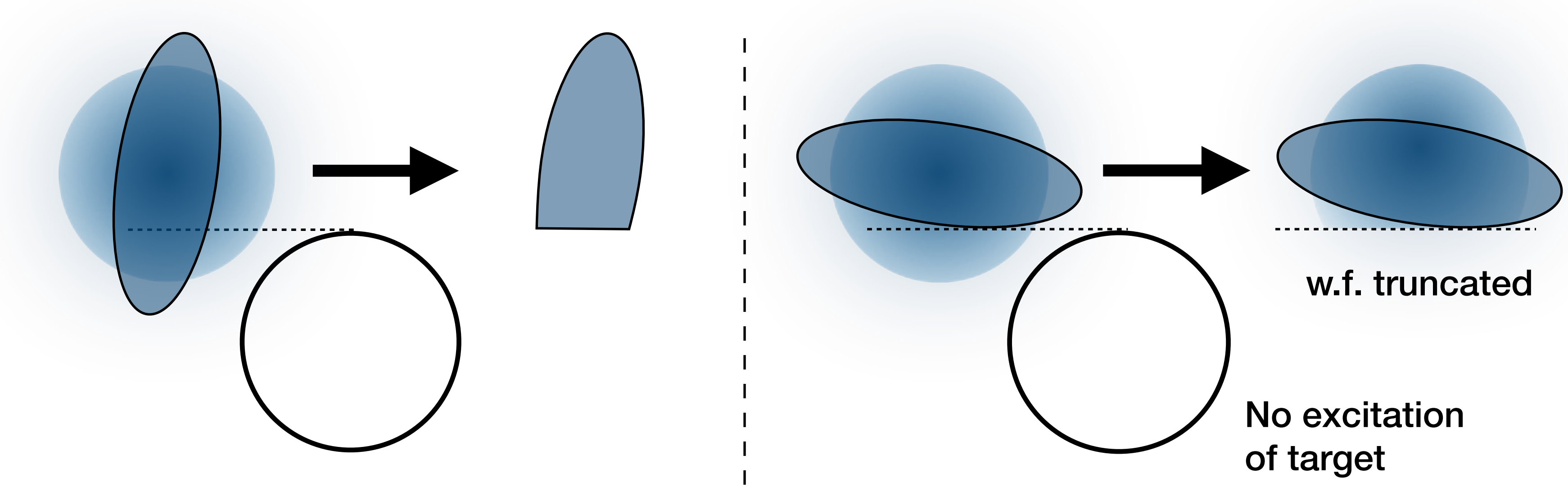
➡ σ_{DD} is sensitive to fluctuations of configurations

Heikki Mäntysaari, "Review of proton and nuclear shape fluctuations at high energy", Rep. Prog. Phys. 83 (2020) 082201

Orientation fluctuation

$$|0^+\rangle = \int d\Omega \, c_\Omega \, |\Phi_\Omega\rangle$$

0^+ g.s. is a superposition of intrinsically deformed states with different orientations Ω

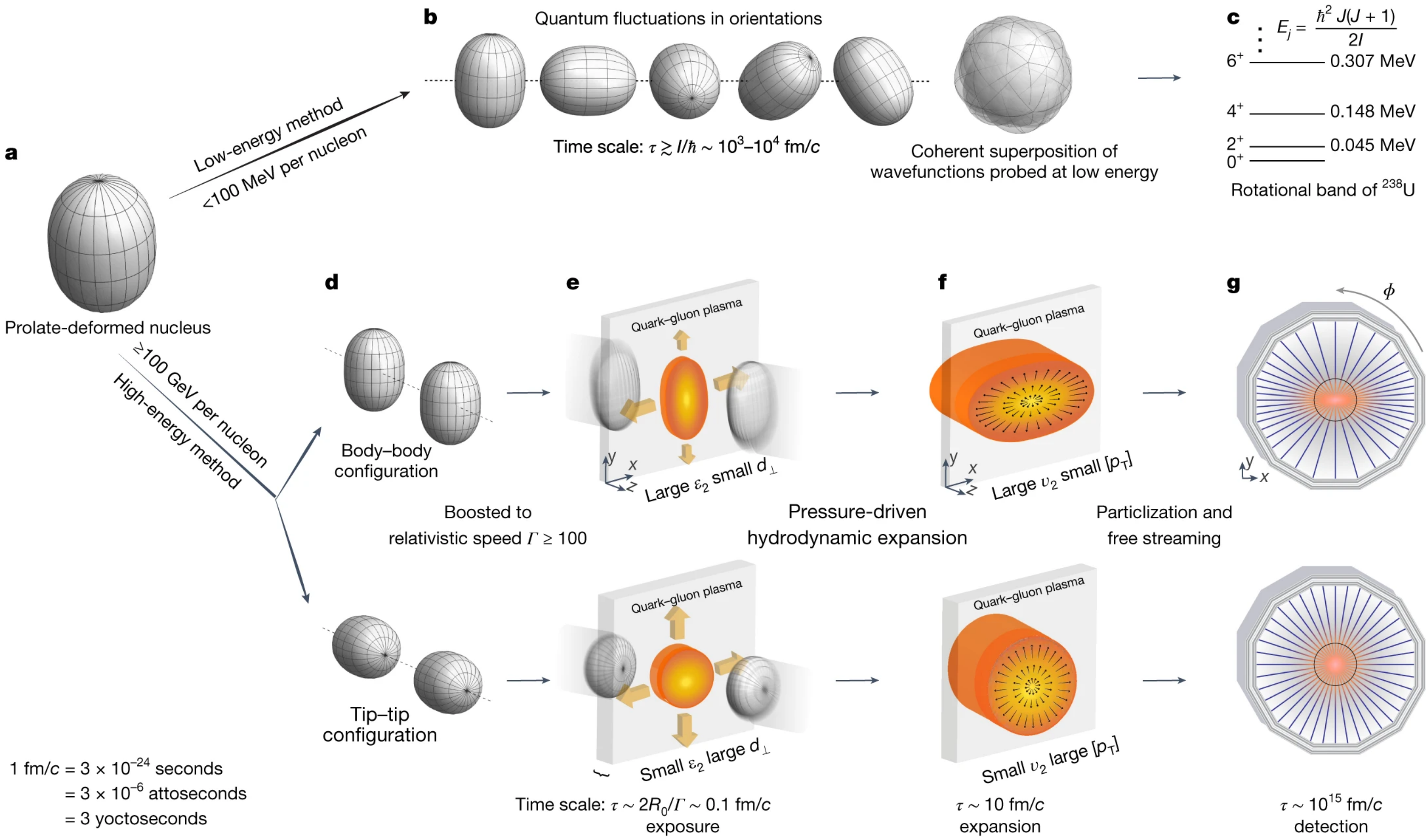


Stripping / fragmentation

Rotational excitation

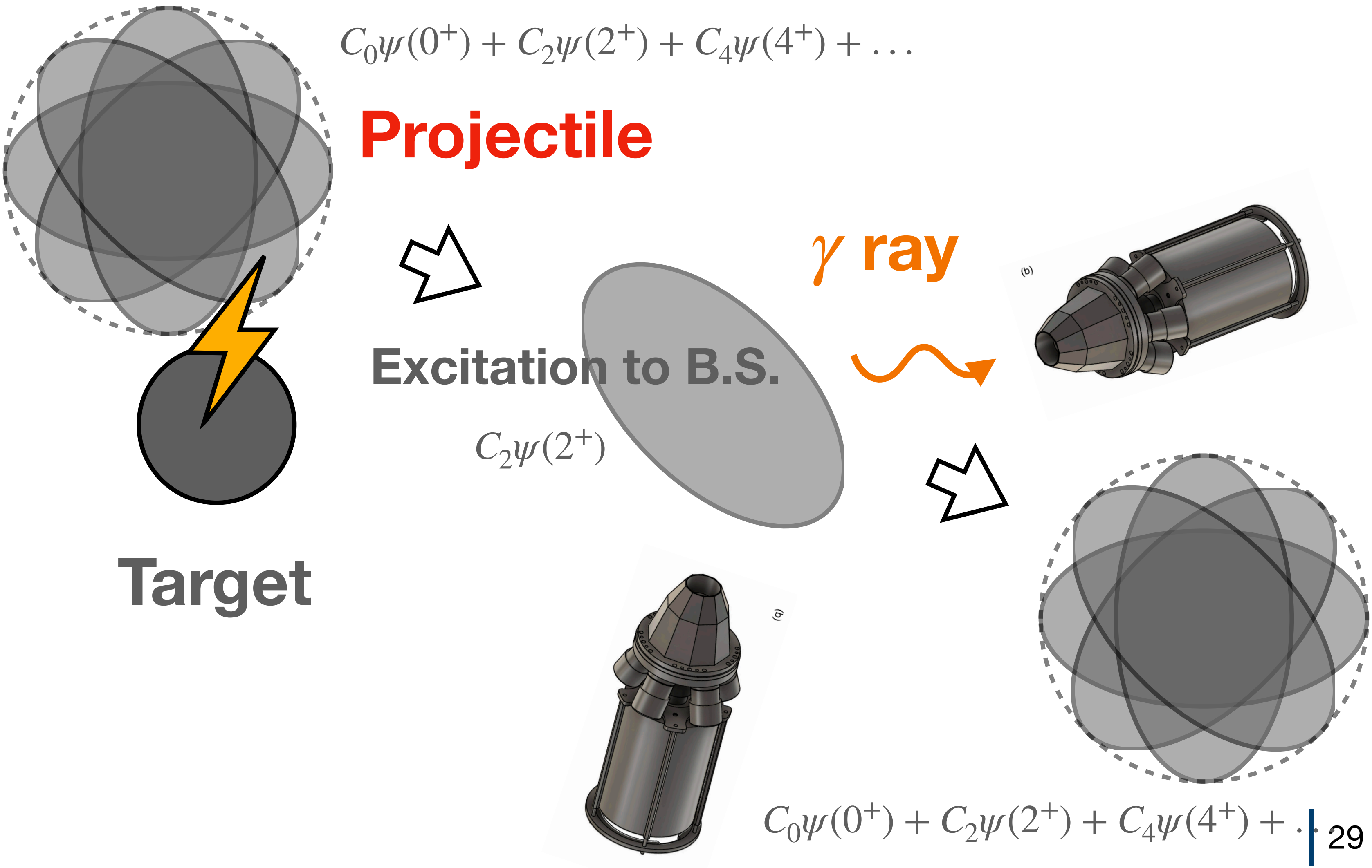
$$\sigma_{\mathrm{rot}} = \int d^2b \left[\left\langle |S_\Omega(b)|^2 \right\rangle_\Omega - \left| \left\langle S_\Omega(b) \right\rangle_\Omega \right|^2 \right]$$

Nuclear shape imaging in H.E. HIC



STAR Collaboration, "Imaging shapes of atomic nuclei in high-energy nuclear collisions", Nature 635, pages67–72 (2024)

Nuclear shape imaging in L.E. HIC



SUMMARY

89-109Zr interaction cross section measured

Jump due to deformation was not seen

Complementary σ_{inel} can explain the discrepancy between
BE2 and charge radii measurements

σ_{inel} : variance of S -> sensitive to fluctuations of orientation

Backup

Inelastic effects

$$\underbrace{\left(1 - \langle S \rangle^2\right)}_{P_{\rm reac}} - \underbrace{\left(1 - \langle S^2 \rangle\right)}_{P_{\rm abs}}$$

$$\begin{aligned}
P_{\rm band}(b) &= \sum_{J=0,2,4,\cdots} \sum_{M} \left| \langle JM \,|\, S(b) \,|\, 0^+ \rangle \right|^2 \\
&= \sum_{J,M} \langle 0^+ \,|\, S^\dagger(b) \,|\, JM \rangle \langle JM \,|\, S(b) \,|\, 0^+ \rangle \\
&= \langle 0^+ \,|\, S^\dagger(b) \left(\sum_{J,M} |\, JM \rangle \langle JM \,| \right) S(b) \,|\, 0^+ \rangle \\
&= \langle 0^+ \,|\, S^\dagger(b) S(b) \,|\, 0^+ \rangle \,.
\end{aligned}$$

Good-Walker diffractive dissociation

$$E^2 - p^2 = M^2$$

$$E^2 - (p - q_{\|})^2 = M^{*2}$$

$$p^2 - (p - q_{\|})^2 = M^{*2} - M^2$$

$$2pq_{\|} - q_{\|}^2 = M^{*2} - M^2$$

$$q_{\|} \ll p \quad \Rightarrow \quad q_{\|}^2 \simeq 0$$

$$q_{\|} \simeq \frac{M^{*2} - M^2 + q_{\perp}^2}{2p}$$

Good-Walker diffractive dissociation

$q_{\|}R_A \ll 1$ → **Longitudinal coherence condition**

The longitudinal phase variation across the target nucleus is negligible

$$R_A \simeq r_0 A^{1/3}\,, \quad r_0 \sim \frac{1}{m_\pi}$$

$$q_{\|} \ll \frac{1}{R_A} \sim \frac{m_\pi}{A^{1/3}}$$

Target excitation can be in g.s.

Good-Walker diffractive dissociation

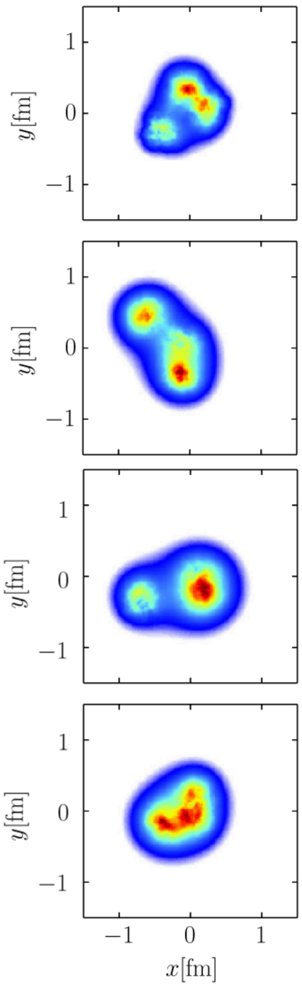
$$
\begin{aligned}
|S\rangle &\equiv |I\rangle - |T\rangle \\
&= \sum_i c_i(1-\eta_i)|C_i\rangle \\
&= (1-\bar{\eta})|I\rangle \quad + \quad \sum_i (\bar{\eta}-\eta_i)c_i|C_i\rangle
\end{aligned}
$$

Coherent **Diffractive dissociation**

$$\sigma_{\mathrm{coh}} \propto \left|\langle 0|S|0\rangle\right|^2 \qquad \sigma_{\mathrm{DD}} \propto \sum_{f\neq 0} \left|\langle f|S|0\rangle\right|^2$$

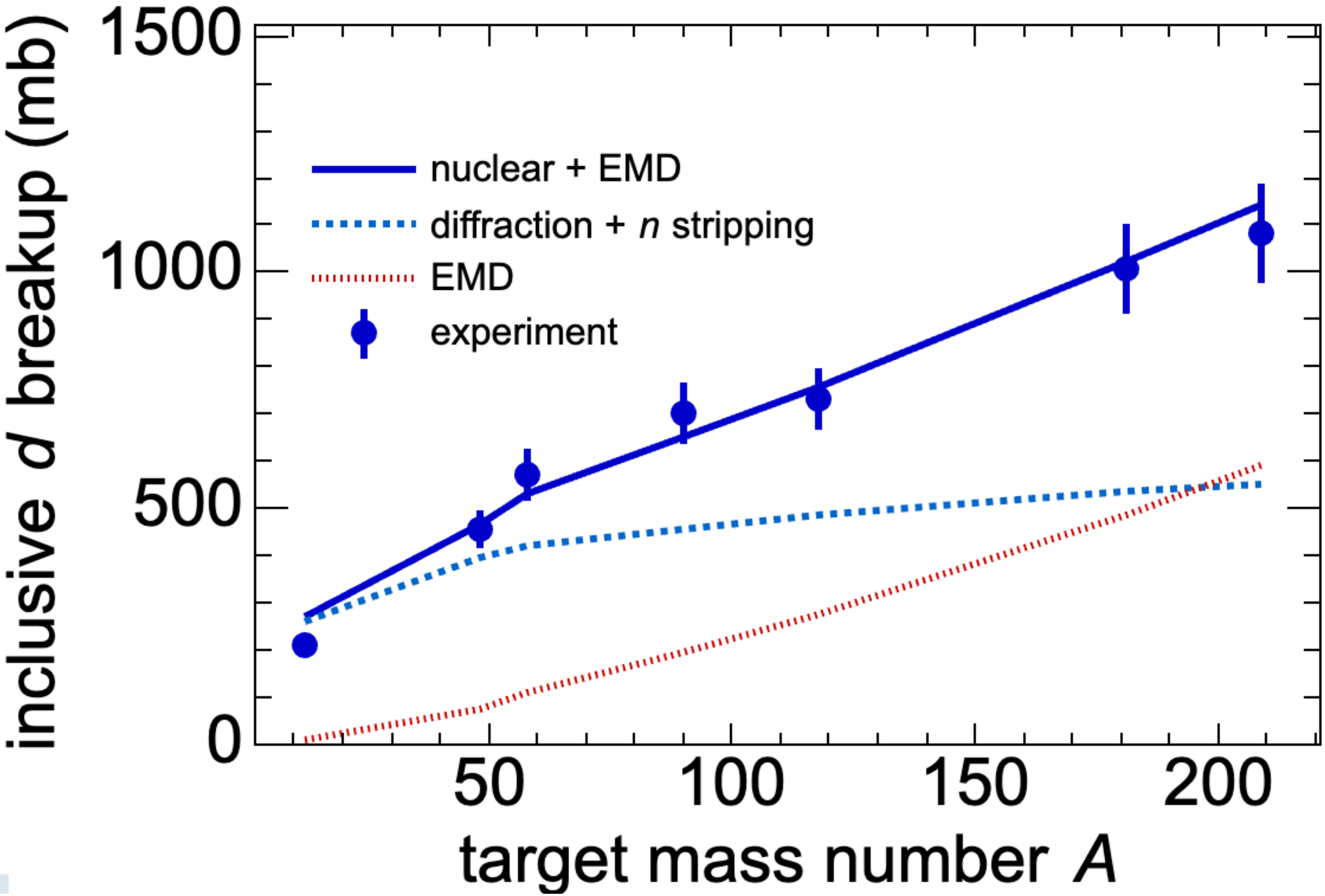
(Using completeness)

$$
\begin{aligned}
&= \langle 0|S^\dagger S|0\rangle - \left|\langle 0|S|0\rangle\right|^2 \\
&= \langle \eta^2\rangle - \langle \eta\rangle^2 \\
&= \underbrace{\left\langle F^2(\{b_i, y_i\})\right\rangle - \left\langle F(\{b_i, y_i\})\right\rangle^2}_{\text{configuration fluctuation}}
\end{aligned}
$$

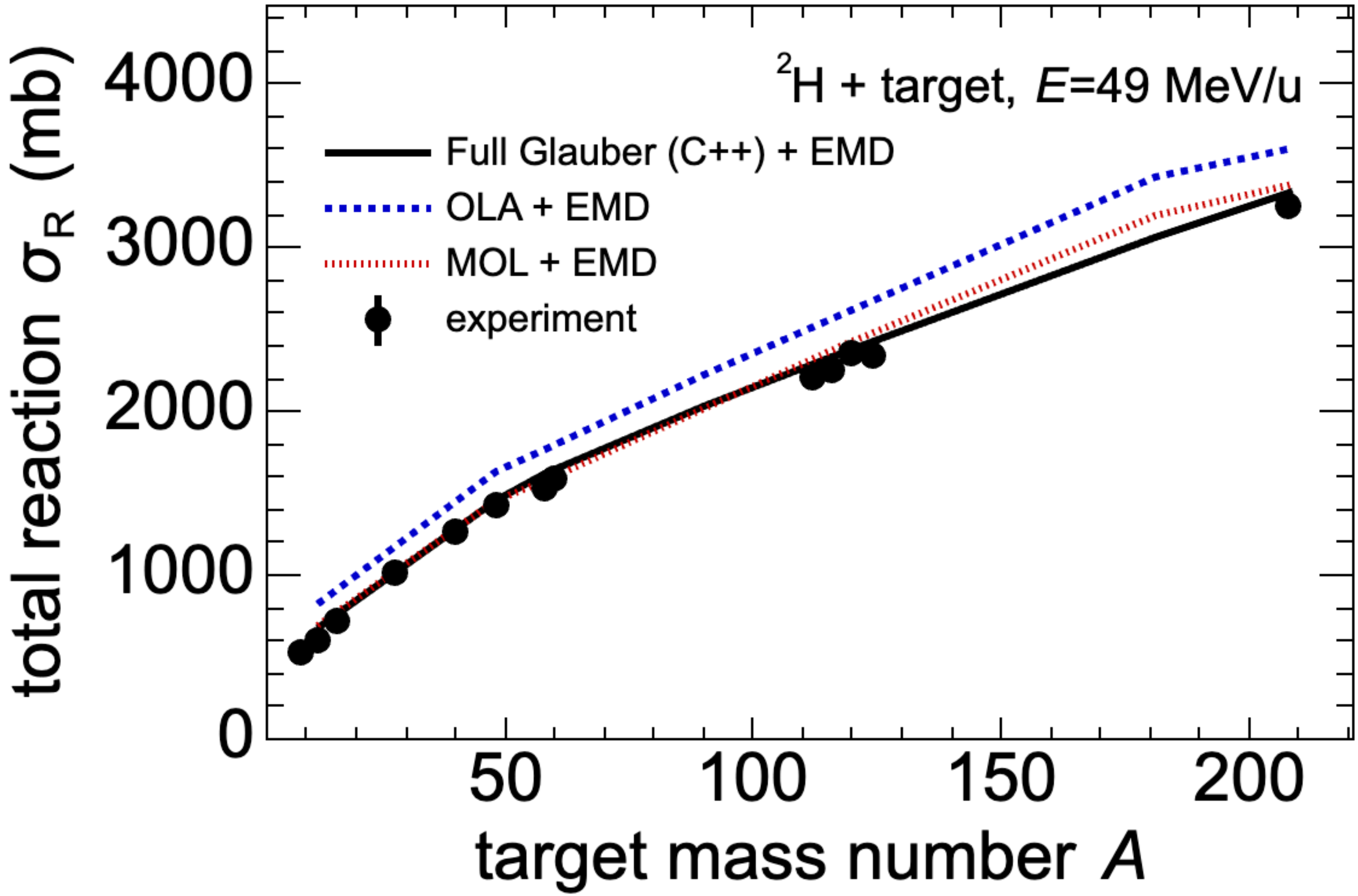


Different proton parton configurations

Deuteron dissociation



Deuteron dissociation



Deuteron dissociation

Deuteron's photo-dissociation cross section:

$$\gamma + d \to p + n$$

is acquired from JENDL/PD-2016.

Then, it is convoluted with E1 photo number:

$$\sigma_{\mathrm{EMD}}(d) = \int_{2.224\,\mathrm{MeV}}^{\infty} N_{E1}(E_\gamma)\,\sigma_{\gamma d \to pn}(E_\gamma)\,dE_\gamma$$

Deuteron dissociation

$$S_d(\mathbf{b}, \mathbf{s}) = S_p(\mathbf{b} + \mathbf{s}/2)\, S_n(\mathbf{b} - \mathbf{s}/2)$$

$$P_p = |\, S_p \,|^2, \qquad P_n = |\, S_n \,|^2$$

$$P_{\text{strip,p}} = (1 - P_p) P_n, \qquad P_{\text{strip,n}} = P_p (1 - P_n)$$

$$\sigma_x = 2\pi \int b\, db\, P_x(b)$$

$$P_{\text{diff}} = \left\langle\, |\, S_d \,|^2 \right\rangle - \left| \left\langle S_d \right\rangle \right|^2$$

$$\sigma_R^{\text{nuclear}} = \sigma_{\text{abs}} + \sigma_{\text{diff}}$$

$$\sigma_{\text{abs}} = \sigma_{\text{strip,p}} + \sigma_{\text{strip,n}} + \sigma_{\text{both abs}}$$

$$\sigma_{\text{breakup}}^{p,\text{nuclear}} = \sigma_{\text{diff}} + \sigma_{\text{strip,n}}$$

Zr inelastic scattering cross section