Collective inertia in spontaneous fission with energy density functionals

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- Collective inertia in fission by EDF
- Fission half-life in Fm isotopes
- Outlook

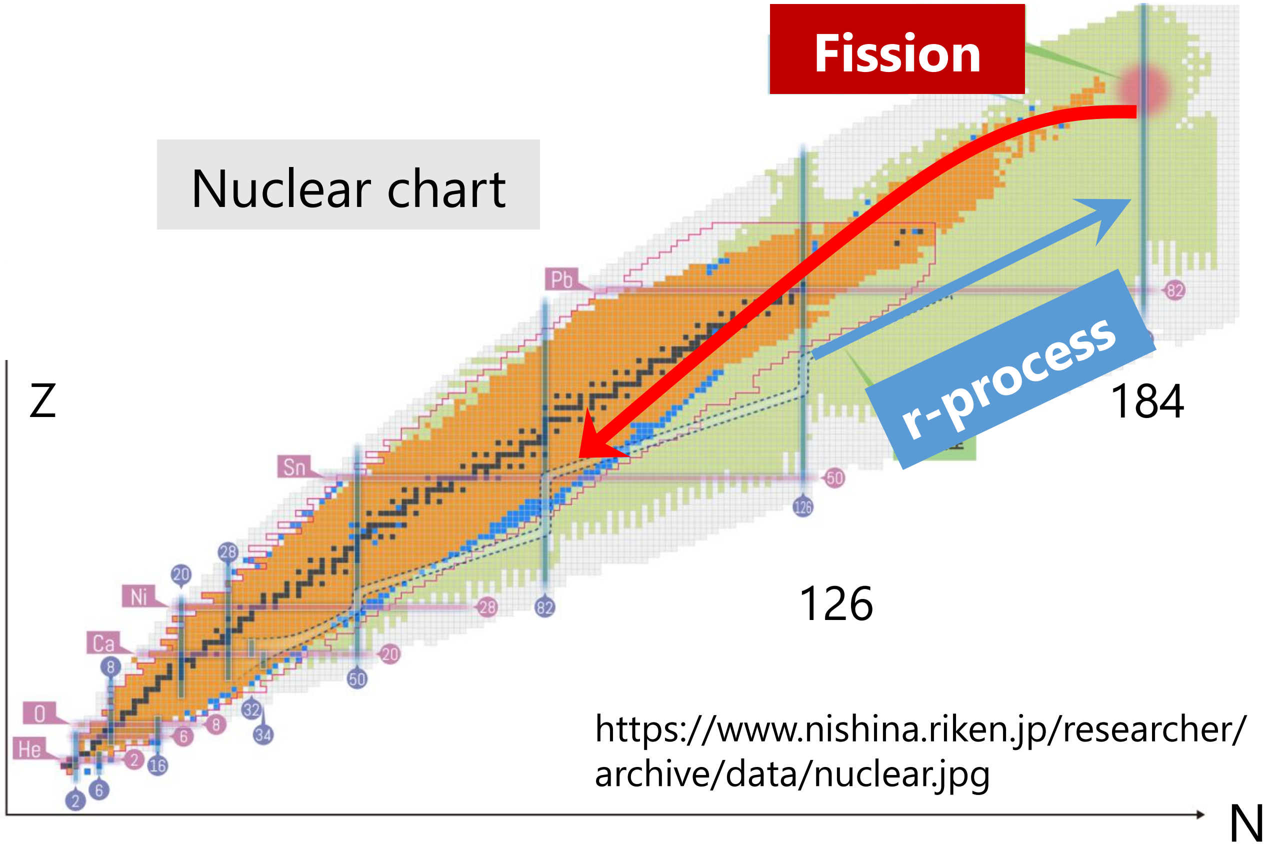
Washiyama, Hinohara, Nakatsukasa, PRC103, 014306 (2021)

Washiyama, Web of Conf. 306, 01026 (2024)

NuSRAP 2026, July 13--15, 2026@CCS, Univ. Tsukuba

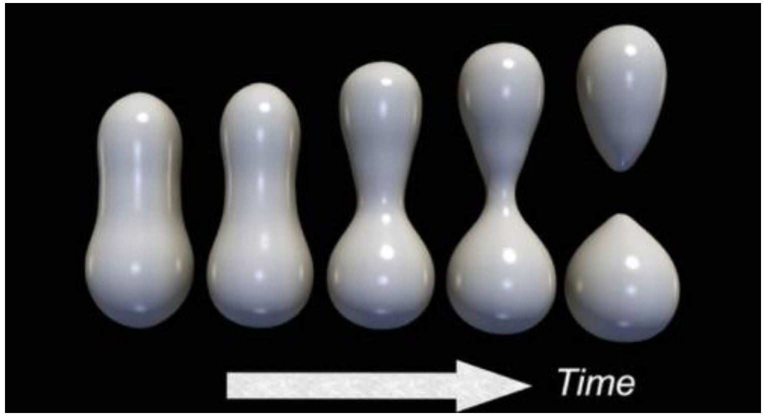
Introduction: Nuclear fission

- Decay mode in heavy nuclei
- End point of r-process



https://www.nishina.riken.jp/researcher/archive/data/nuclear.jpg

- Large amplitude collective motion
- Many-body quantum tunneling

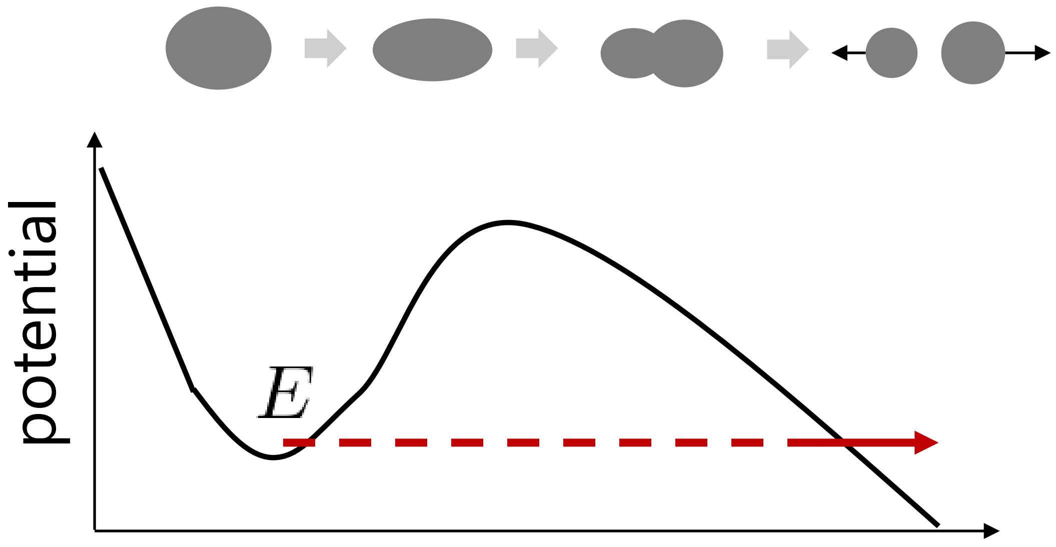


Courtesy of G. Scamps

Spontaneous fission described by DFT

J. Sadhukhan et al, PRC88, 064314 (2013), etc.

Spontaneous fission



Many-body quantum tunneling

WKB approximation

Fission half-life

$$T_{1/2} = \ln 2/(nP) \qquad P = \frac{1}{1+\exp(2S)}$$

Action

$$S = \int_{s_1}^{s_2} ds \ \sqrt{2M(s)(V(s)-E)}/\hbar$$

s Collective coordinate (Q_{20} etc.)

V Potential

***M* Collective inertia**

Aim of this talk

Fission observable

- Fission half-life
- Mass distribution

Potential energy

Collective inertia

$$S=\int_{s_1}^{s_2} ds\ \sqrt{2M(s)(V(s)-E)}/\hbar$$

To describe collective inertia M for fission half-life based on density functional theory (DFT)

Collective inertia by density functional theory

Former work: Cranking formula

Ignore residual effects

Scaling factor $D = (1+\alpha)\, D^{\mathrm{crank}}$

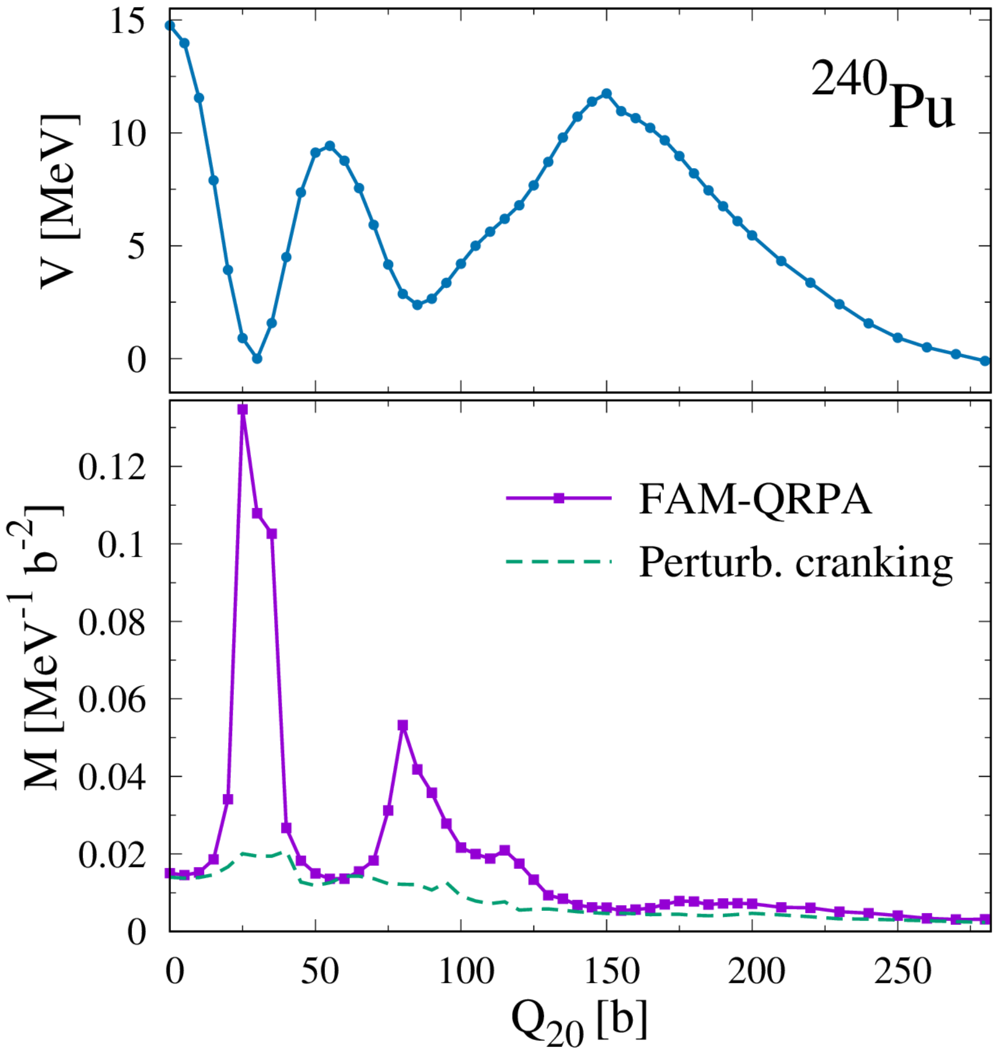
Prochniak et al., NPA730 (2004) 59
Baran et al., PRC84, 054321 (2011)
Sadhukhan et al, PRC88, 064314 (2013)
Giuliani and Robledo, PLB 787, 134 (2018)

Our work: **Local QRPA**

Include residual effects

Hinohara et al., PRC82 (2010) 064313
Sato, Hinohara, NPA849(2011)53
Yoshida, Hinohara, PRC83(2011)061302

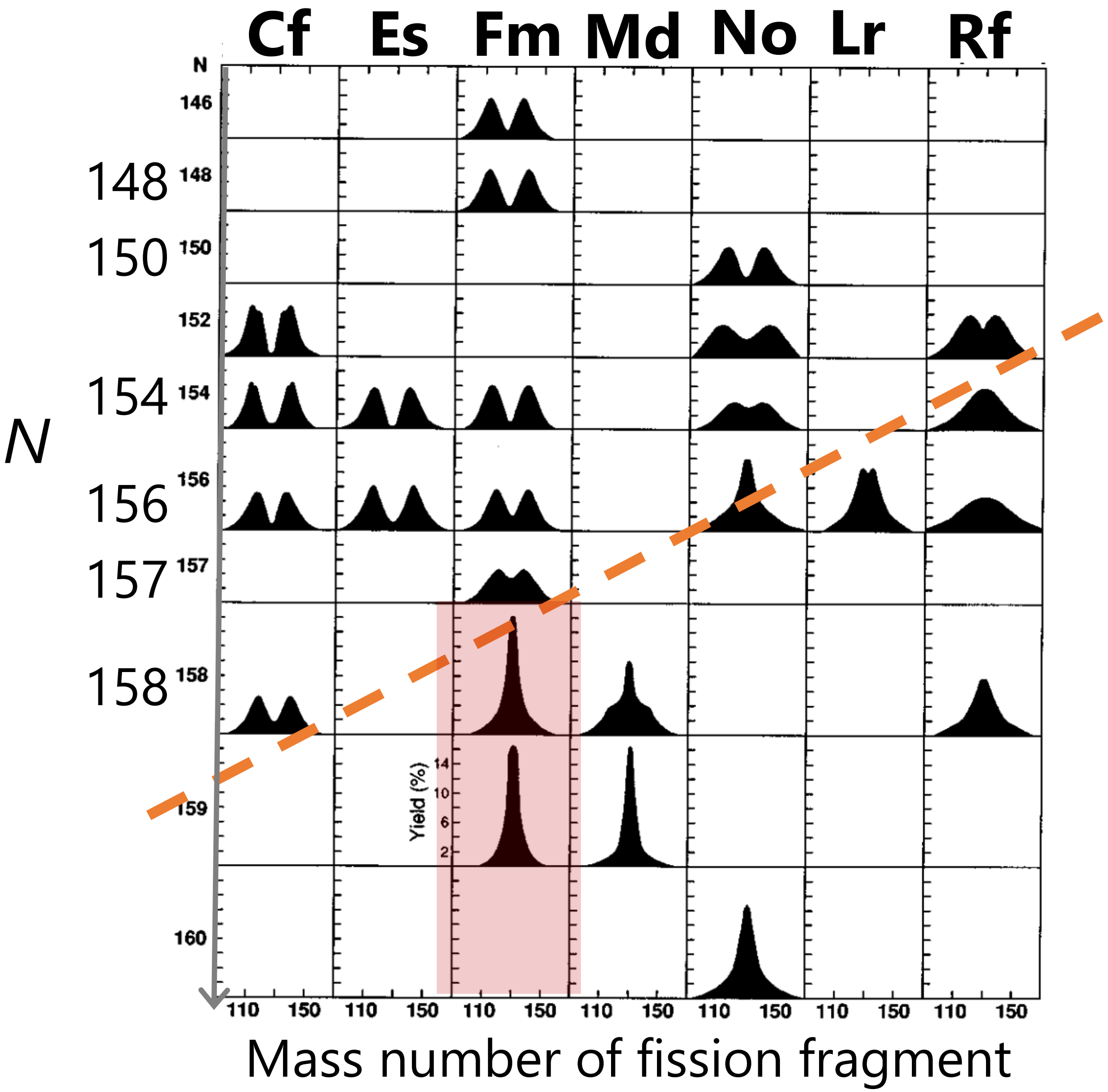
Comparison (QRPA/Cranking)



Washiyama, Hinohara, Nakatsukasa, PRC103, 014306 (2021)

Target: Fm isotopes

Mass distribution of fission fragments



Mass symmetric fission is dominant in Fm isotopes, N ≥ 158

$Q_{30} = 0$ may be a good approximation

M.R. Lane et al., PRC53, 2893 (1996)

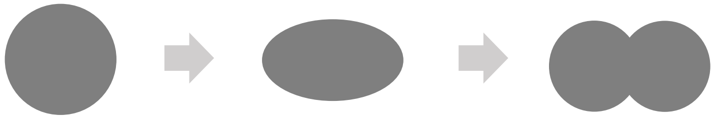
Calculation set-up

$$S = \int_{s_1}^{s_2} ds \ \sqrt{2\underline{M(s)}(\underline{V(s)} - E)}/\hbar$$

$V(s)$: Constrained HFB along the fission path

Constrained on Q_{20}

Fix Q_{30}=0 → Symmetric fission path only



$M(s)$: Local QRPA on constrained HFB states with the finite amplitude method

$$\delta\langle\phi(s)|[\hat{H}_{\mathrm{M}}(s), \hat{Q}^i(s)] - \frac{1}{i}\hat{P}^i(s)|\phi(s)\rangle = 0$$

$$\delta\langle\phi(s)|[\hat{H}_{\mathrm{M}}(s), \frac{1}{i}\hat{P}^i(s)] - C_i(s)\hat{Q}^i(s)|\phi(s)\rangle = 0$$

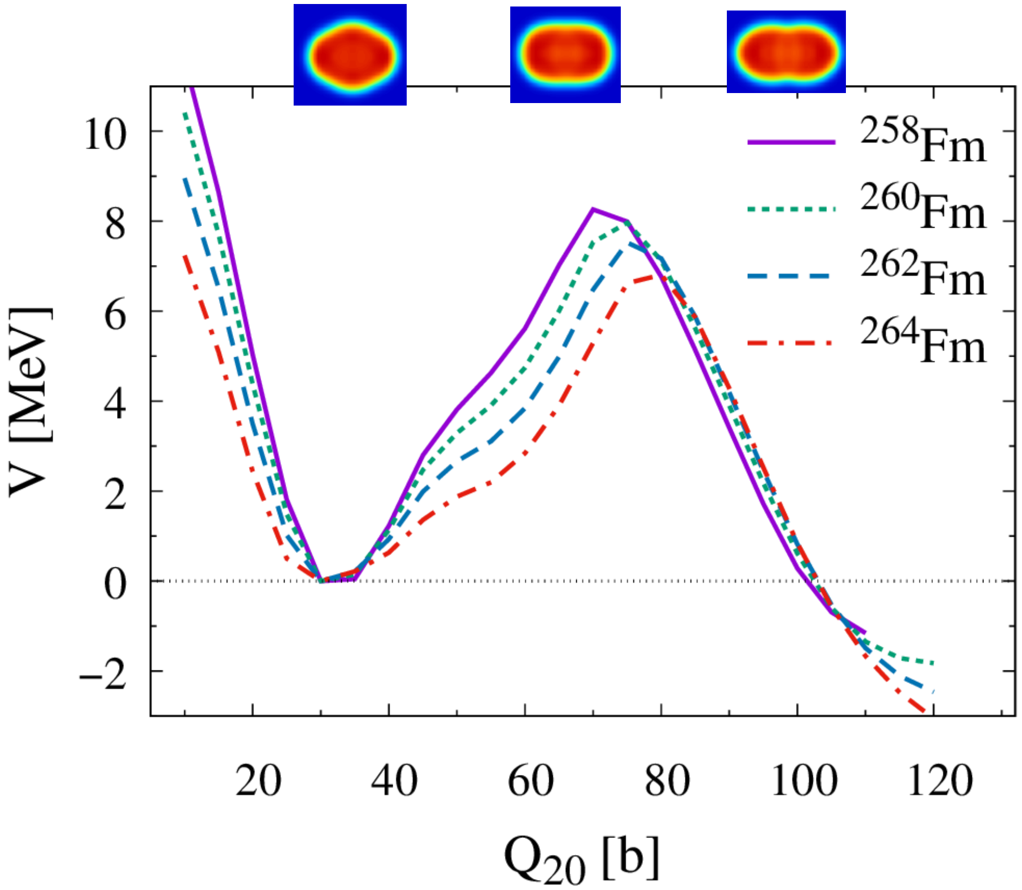
Select the most collective mode

Washiyama, Hinohara, Nakatsukasa, PRC103, 014306 (2021)

Computation time: 60000 - 70000 hours (1500 hours x 30-50 states)

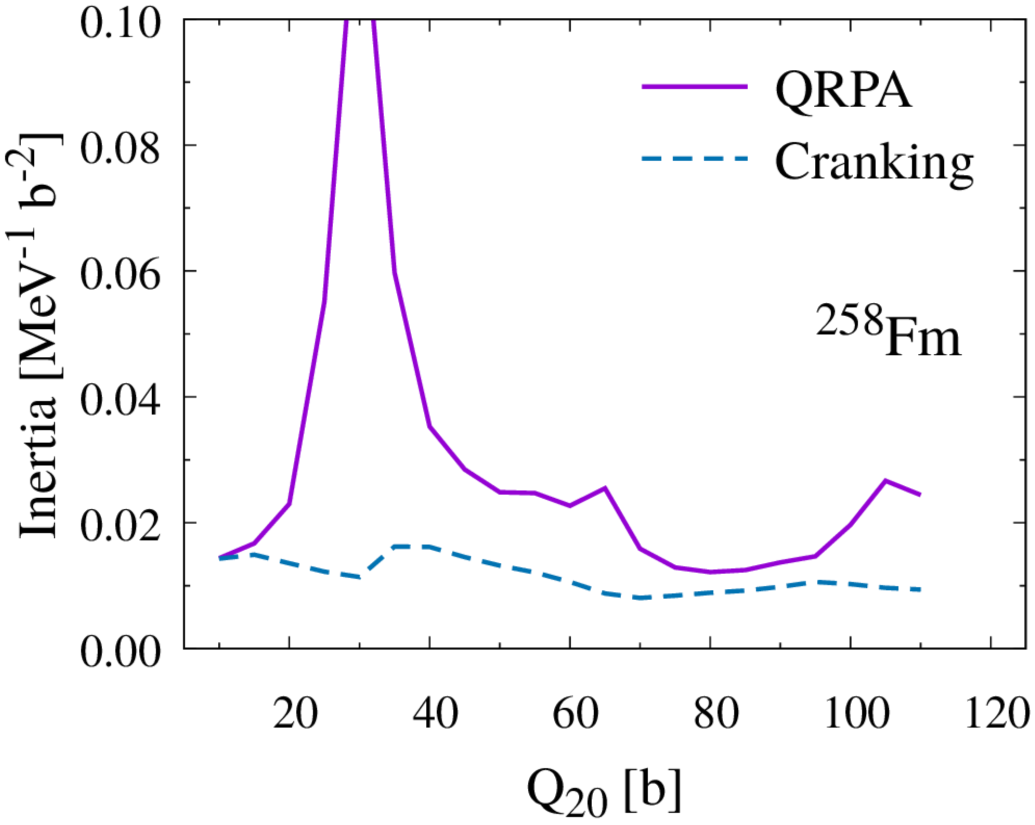
Result: fission barrier, collective inertia in Fm isotopes

Fission barrier



258Fm > 260Fm > 262Fm > 264Fm

Inertia



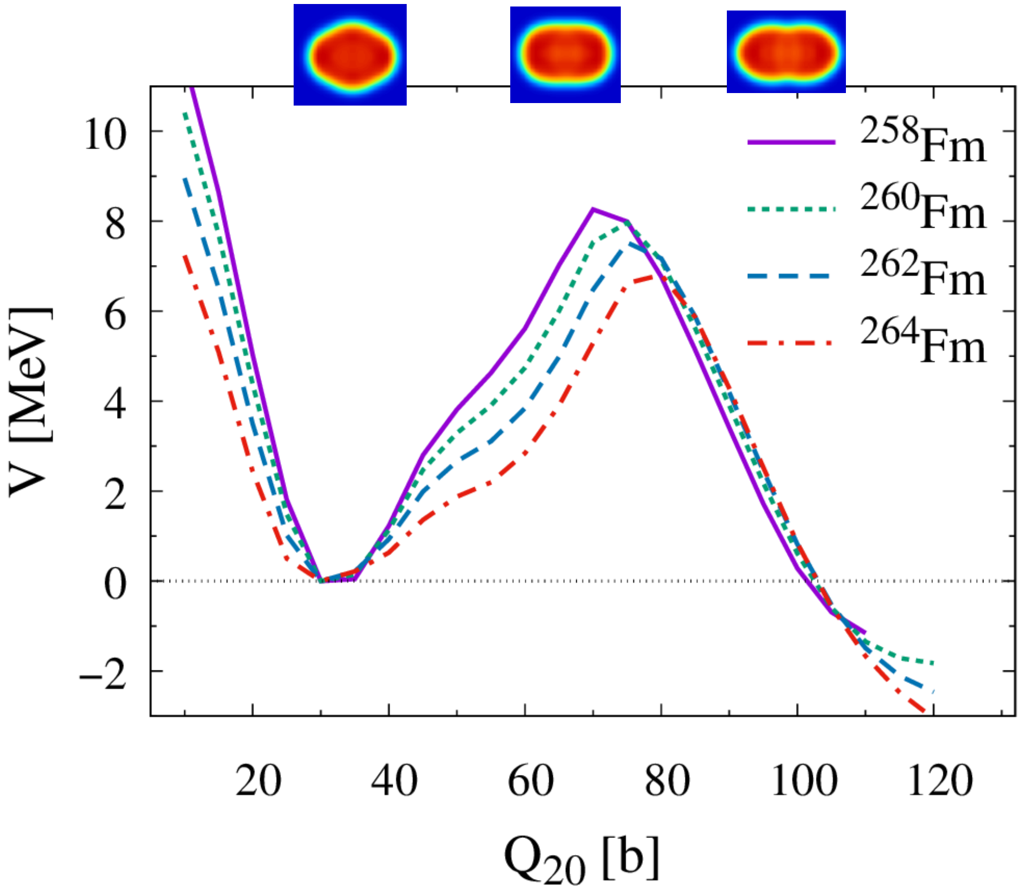
SkM* + volume pairing

25x25x35 fm^3, dx=1.0 fm

Washiyama, EPJ Web of Conf. 306, 01026 (2024)

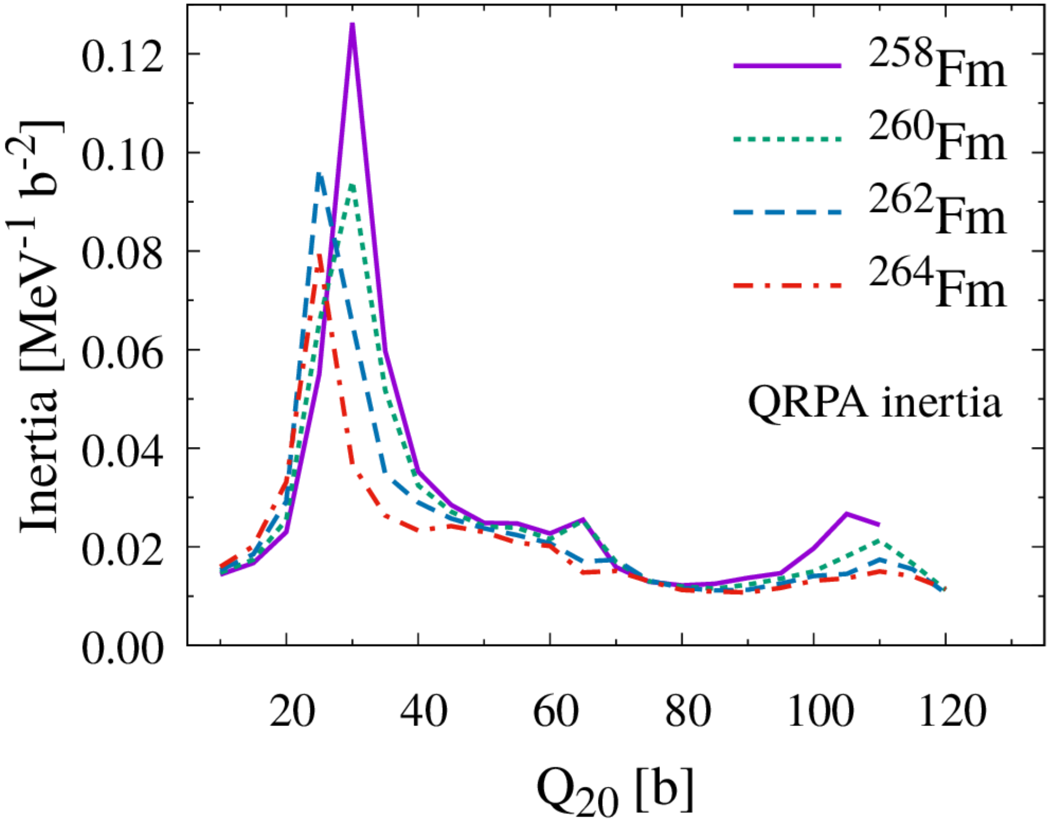
Result: fission barrier, collective inertia in Fm isotopes

Fission barrier



258Fm > 260Fm > 262Fm > 264Fm

Inertia



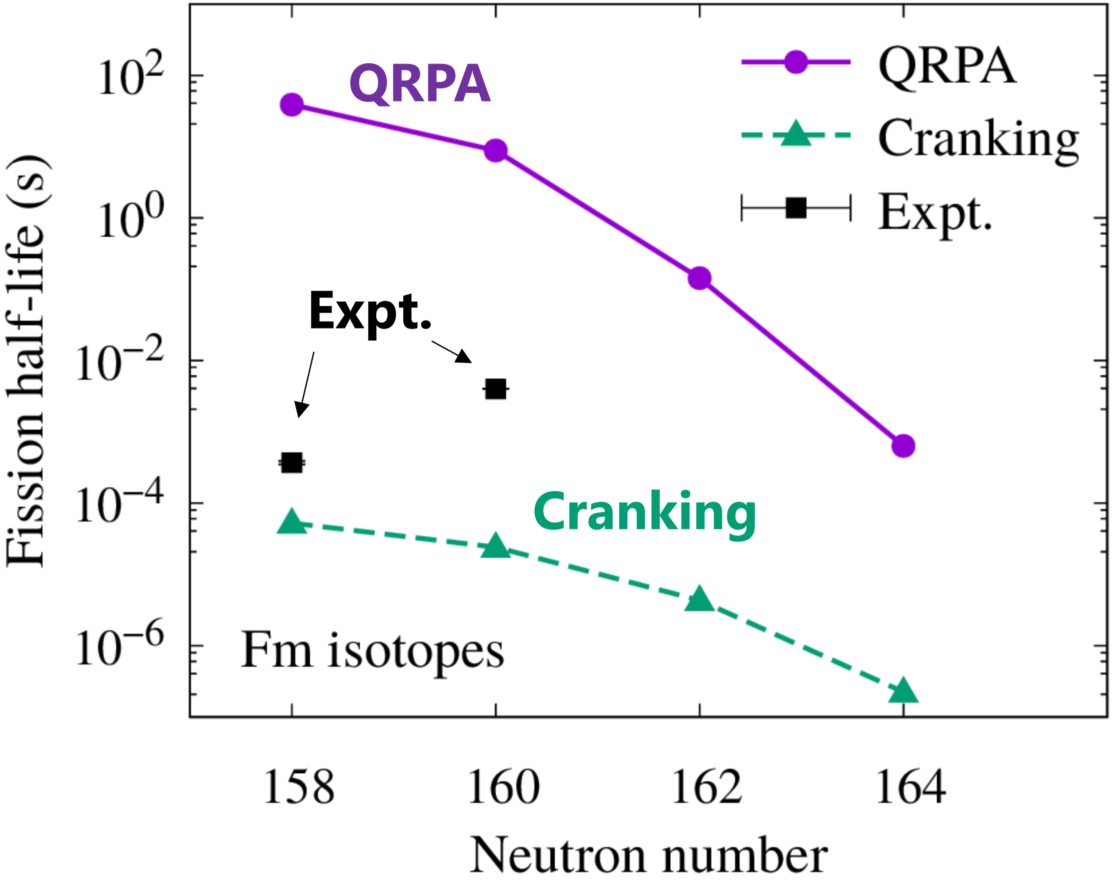
SkM* + volume pairing

25x25x35 fm^3, dx=1.0 fm

Washiyama, EPJ Web of Conf. 306, 01026 (2024)

Impact on fission half-life

Fission Half-life



$$T_{1/2} = \ln 2/(nP)$$

$$n = 10^{20.38}\ \mathrm{s}^{-1} \qquad P = \frac{1}{1+\exp(2S)}$$

A. Baran, Phys. Lett. B 76, 8 (1978)
Sadhukhan et al, PRC88, 064314 (2013), etc.

Larger *V* & *M* → longer half-life

Overestimation

Data from N. E. Holden and D. C. Hoffman,
Pure Appl. Chem. 72, 1525 (2000)

Washiyama, EPJ Web of Conf. 306, 01026 (2024)

Outlook

The overestimation of the calculated half-lives may be reduced by including additional collective coordinates.

→ Decrease the action $S = \int_{s_1}^{s_2} ds \sqrt{2M(s)(V(s) - E)}/\hbar$

Two (and more) **collective coordinates**

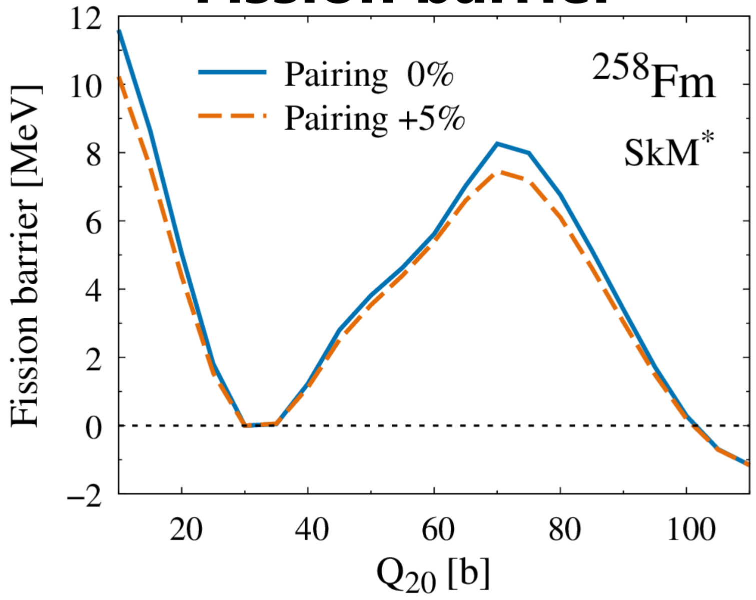
- Octupole moment, nonaxial (triaxial) quadrupole moment
- **Pairing**

  → Induce a large variation in the inertia

  "Pairing-induced speedup of nuclear spontaneous fission"
  J. Sadhukhan et al, PRC90, 061304 (2014)

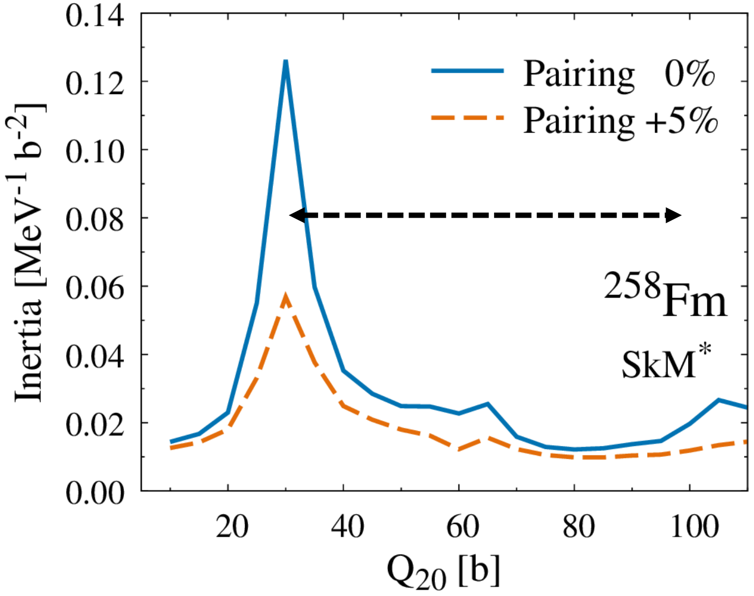
Pairing strength dependence

Fission barrier



~10% change at barrier

Inertia



20 – 50% change

Action *S*

25.6 (Pairing 0%)

20.6 (Pairing 5%)

Half-life

4.5 x $\mathbf{10^{1}}$ s **(Pairing 0%)**

2.1 x $\mathbf{10^{-3}}$ s **(Pairing 5%)**

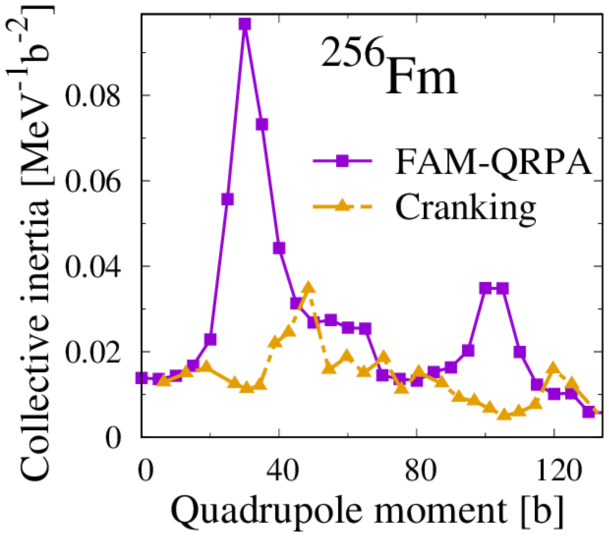
0.37(2) x 10^{-3} s (data)

+5% pairing

Summary

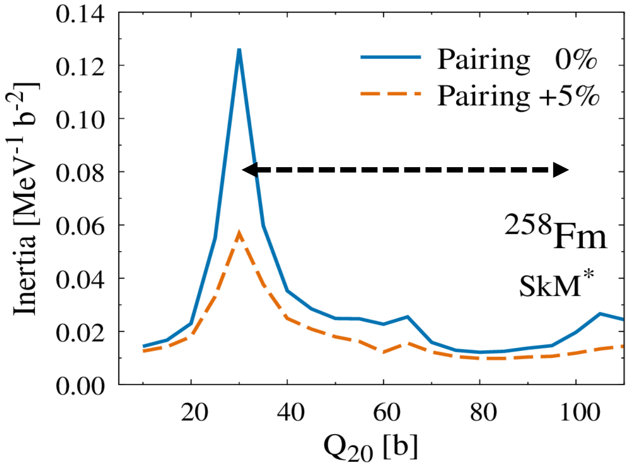
Description of collective inertia is important

- Drawbacks of the cranking formula
- Correctly including residual effects by QRPA
- Impact on fission half-life



Outlook

- Multiple collective coordinates
  - Octupole and triaxial quadrupole moment
  - Pairing



20 – 50% change

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