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Triple-α reaction by an exact three-body model and its applications

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[1] M. Katsuma, J. Phys. G 52, 085105 (2025)

1. To understand the dynamic universe

Triple α reaction, followed by the $^{12}C(\alpha,\gamma)$ reaction, is the most important nuclear reaction.

- nucleosynthesis of heavy elements
- controls stellar evolution
- supernova and concomitant nucleosynthesis
- C/O ratio

- Hoyle state (0^+_2)
- Temperature: $T_9 > 0.1$
- relatively well-understood recently

Low temperatures ($T_9 < 0.1$): the reaction rates have not been established so far.

- The direct 3α collision and direct decay measurement from ternary continuum states are experimentally impossible.
- The approximation of the low-temperature rates has been given by the pioneering works.

- Low-temperature = non-resonant

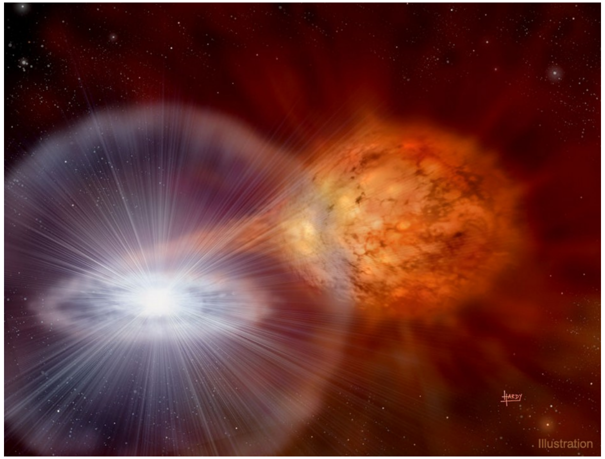
The non-resonant reaction rates may affect

a close binary system

1.) **Hot CNO cycle** and **rp-process:** nucleosynthesis in thermonuclear explosion of the accreting degenerate stars,

2.) **Type Ia supernova**, 3.) structure of degenerate stars.

To deepen our understanding of the dynamic stellar explosion in the multi-messenger era, a study of the triple α reaction is required more rigorously at the low temperatures.



(David A. Hardy & PPARC)

2. Frontier of the triple-α reaction at low temperatures

NACRE, CF88: particle approximation of ^{8}Be → the standard rates

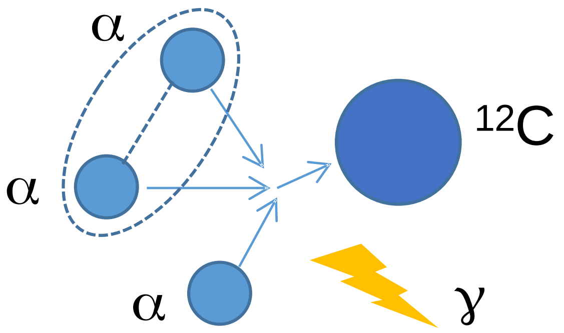
- Pioneering works of the reaction rates

  K. Nomoto, F.-K. Thielemann, S. Miyaji, Astron. Astrophys. 149, 239 (1985)
  K. Langanke, M. Wiescher, F.-K. Thielemann, Z. Phys. A 324, 147 (1986)
  (NACRE) C. Angulo, M. Arnould et al., Nucl. Phys. A 656, 3 (1999)
  (CF88) G.R. Caughlan, W.A. Fowler, At. Data Nucl. Data Tables 40, 283 (1988)

- Their formula is supported with separable Hamiltonian

  T. Akahori, Y. Funaki, K. Yabana, PRC92, 022801 (2015)

Restricted condition



Adiabatic: assuming slow internal motion of ^{8}Be, Fedorov & Jensen, PLB389, 631 (1996)

- **CMF** – Faddeev method with a correction of the long-range Coulomb interaction.

  S. Ishikawa, Phys. Rev. C 87, 055804 (2013); ibid. 90, 061604 (2014)

- **ACF** – Adiabatic channel function expansion method

  H. Suno, Y. Suzuki, P. Descouvemont, Phys. Rev. C 94, 054607 (2016)

→ Success in the rates
$T_9 > 0.1$, Adiabatic ≈ NACRE

Non-adiabatic: 3α motion varies more flexibly.

- **HHR** – Hyperspherical harmonics R-matrix expansion method – a solution of Faddeev eqs.

  N.B. Nguyen, F.M. Nunes, I.J. Thompson, E.F. Brown, Phys. Rev. Lett. 109, 141101 (2012)
  N.B. Nguyen, F.M. Nunes, I.J. Thompson, Phys. Rev. C 87, 054615 (2013)

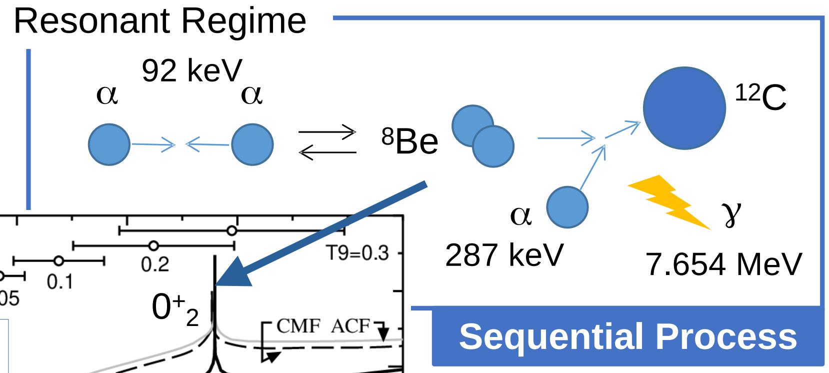
- **HHR*** improve matching procedure of w.f. in the asymptotic region
  define 3-body S-factors, give an analytic expression of the reaction rates.

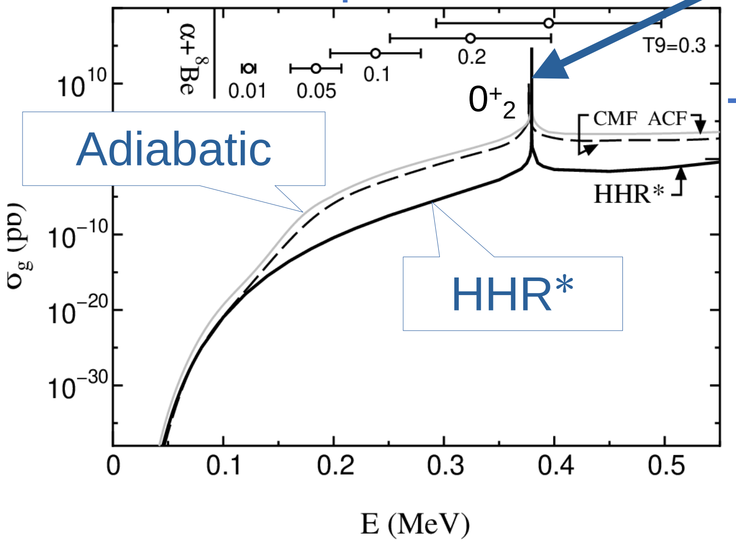
2. Frontier of the triple-α reaction at low temperatures

All models make the equivalent rates above $T_9 = 0.1$. However, they **make differences below this temperature.**

Why?

The narrow peak of 0^+_2 dominates the rates for $T_9 > 0.1$.





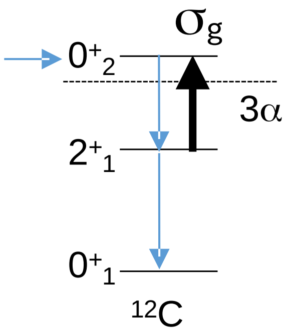
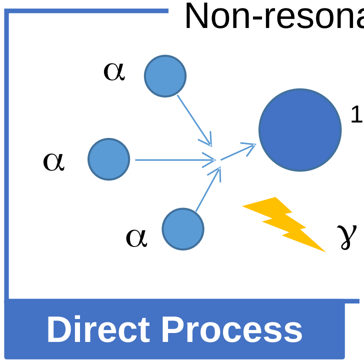
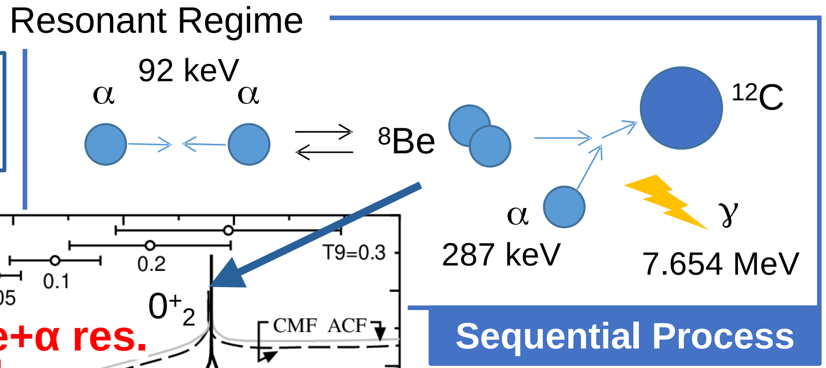
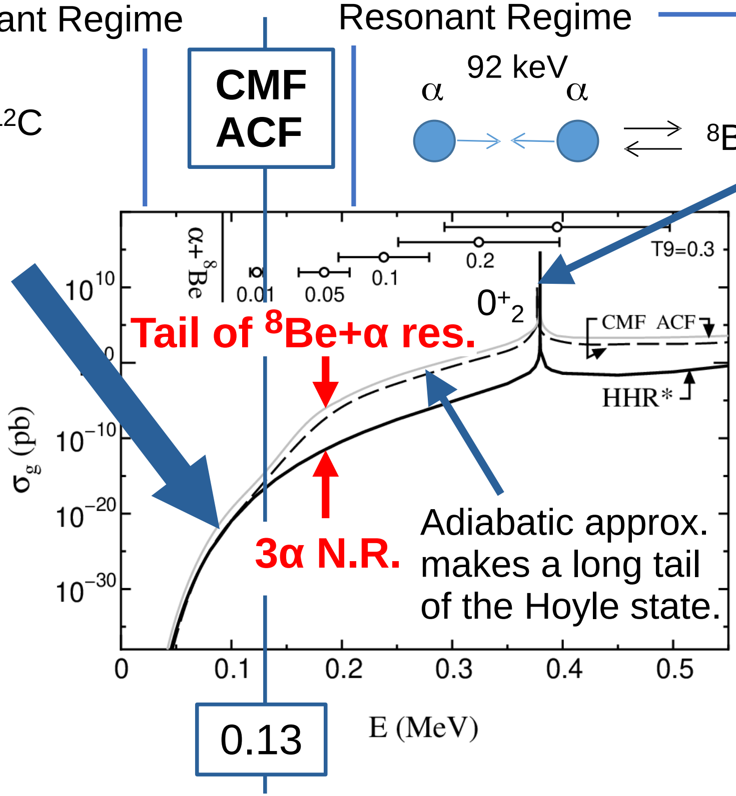


Photo-disintegration cross sections of $^{12}C(2^+_1 \rightarrow 0^+)$

2. Frontier of the triple-α reaction at low temperatures







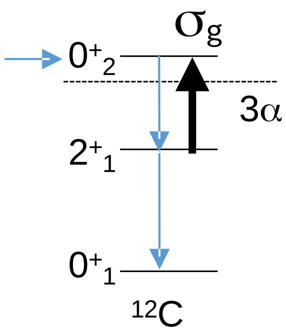
Astrophysics:

Nova, X-ray burst

Type Ia SNe

Nucleosynthesis:

HCNO, rp-process



$T_9 > 0.1$

Helium burning in degenerate stars

Low T

Helium burning in normal stars

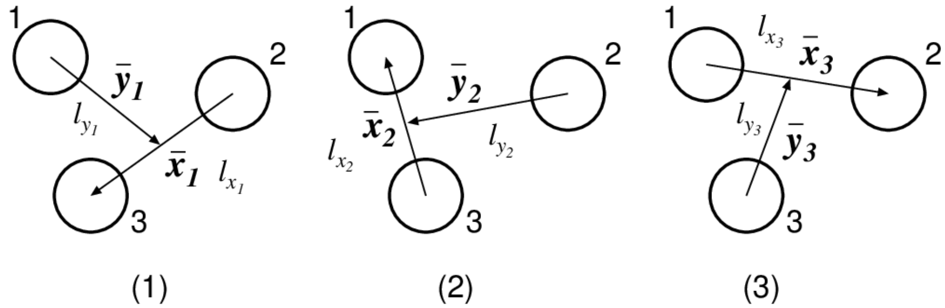
^{8}Be

The difference of theoretical approaches in the internal region makes **the deviation of the reaction rates at low temperatures.**

3. Non-adiabatic Faddeev 3-body R-matrix expansion (HHR*)

Faddeev Eqs.

$$\begin{cases} (\hat{T}_{(1)} + \hat{V}_{23} + \hat{V}_{3\alpha} - E)\,\Psi_{(1)} &= -\hat{V}_{23}\Psi_{(2)} - \hat{V}_{23}\Psi_{(3)} \\ (\hat{T}_{(2)} + \hat{V}_{31} + \hat{V}_{3\alpha} - E)\,\Psi_{(2)} &= -\hat{V}_{31}\Psi_{(3)} - \hat{V}_{31}\Psi_{(1)} \\ (\hat{T}_{(3)} + \hat{V}_{12} + \hat{V}_{3\alpha} - E)\,\Psi_{(3)} &= -\hat{V}_{12}\Psi_{(1)} - \hat{V}_{12}\Psi_{(2)} \end{cases}$$



1. Reduction by the symmetric 3α system

$$(\hat{T}_{(3)} + \hat{V} - E)\,\Psi_{(3)} = 0$$

$$\hat{V} = \sum_{i<j}^{3} \hat{V}_{ij} + \hat{V}_{3\alpha}$$

2α pot.: phase shifts, 3α potentials

2. Transform the coordinates

$$\begin{cases} \rho^2 &= x_i^2 + y_i^2, \\ \theta_i &= \arctan\left(\frac{x_i}{y_i}\right) \\ \Omega_5^{(3)} &\equiv (\theta_3, \hat{\boldsymbol{x}}_3, \hat{\boldsymbol{y}}_3) \end{cases}$$

hyper-radius

hyper-angles

3. The hyper-angular part is solved, analytically.

$$\Psi_{(3)} = \rho^{-5/2} \sum_{\gamma} \chi_\gamma^l(\rho)\, \Phi_{lm}^{\gamma}(\Omega_5^{(3)})$$

Hyperspherical harmonics

Coupled-channel (C.C.) equations

$$\left[T_\gamma + U_{\gamma\gamma}^l(\rho) - \epsilon\right] \chi_\gamma^l(\rho) = -\sum_{\gamma' \neq \gamma} U_{\gamma'\gamma}^l(\rho)\chi_{\gamma'}^l(\rho)$$

- Same as a formula for inelastic scattering with L=K+3/2. K is the hyper-angular momentum.

3. Non-adiabatic Faddeev 3-body R-matrix expansion (HHR*)

4. CC eqs.: Hamiltonian is diagonalized in a box with an R-matrix radius a_c.

- Eigen-states

$$\begin{cases} \chi^l_{\gamma i}(\rho) = \sum_{n=0} c^n_{\gamma i}\varphi^K_n(\rho) \\ \tilde{E}(l^\pi_i) = -\frac{\hbar^2}{2m_N}\epsilon_i \end{cases}$$

- Reduced width amplitudes

$$\tilde{\gamma}_{\alpha i} = \sqrt{\frac{\hbar^2}{2m_N a_c}}\chi^l_{\alpha i}(a_c)$$

- 3-body R-matrix

$$R^l_{\alpha\beta}(E, a_c) = \sum_i \frac{\tilde{\gamma}_{\alpha i}\tilde{\gamma}_{\beta i}}{\tilde{E}(l^\pi_i) - E}$$

R-matrix expansion method

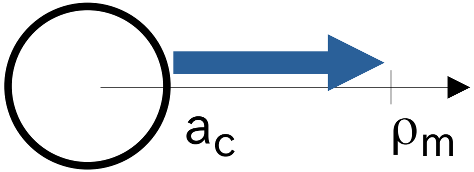
- Continuum states with the scattering boundary condition are expanded by a set of the eigen-states of Hamiltonian.

$$\chi^{l\ in}_{\alpha\beta}(k, \rho) = \sum_i A_{i\beta}(k)\chi^l_{\alpha i}(\rho)$$

5. To include the effect of the long-range Coulomb force, I solve numerically the external w.f. from a_c.

$$\tilde{\chi}^l_{\gamma\gamma_0}(k, \rho) = \sum_{\gamma'} C_{\gamma'\gamma_0}(k)\chi^{l\ ext}_{\gamma\gamma'}(k, \rho)$$

$$\to \frac{i}{2}\left[I^{(\gamma_0)}_{\gamma, K+3/2}(\eta_\gamma; k\rho_m) - \sum_{\gamma'} S^l_{\gamma'\gamma_0}(E) O^{(\gamma')}_{\gamma, K+3/2}(\eta_\gamma; k\rho_m)\right]$$



Matching to the Coulomb w.f.

6. Calculate cross sections by using the continuum states and bound state.
7. Convert them into the reaction rates.

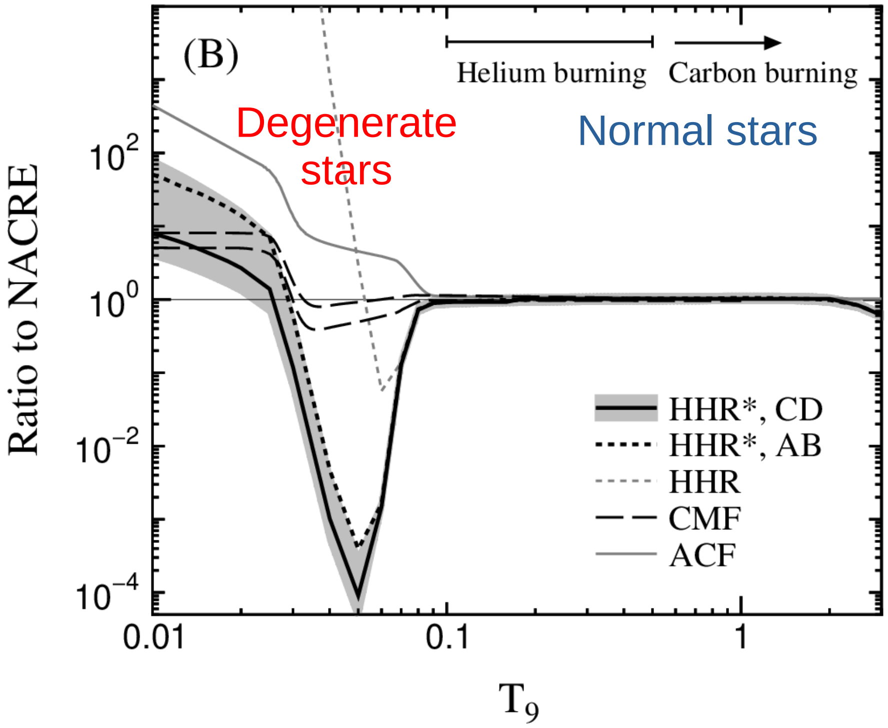
4. Triple α reaction rates – Comparison between the adiabatic models and HHR*

NACRE ratio

- At $T_9 = 0.05$, the reaction rates are found to be reduced by a factor of 10^{-4}.
  - The cross sections at E = 0.18 MeV are reduced from the adiabatic ones.
  - The accurate description of 3-body motion, including ^{8}Be break-up.

- This difference may be expected to affect nucleosynthesis and surface structure of accreting degenerate stars in some degree.

- On the other hand, all results are consistent with NACRE for $0.08 < T_9 < 3$, so that stellar evolution of normal stars remains intact.



NACRE, CF88, Reaclib (FY05)

——— Res. of ^{8}Be + α

- - - - - - - - N.R. of ^{8}Be + α

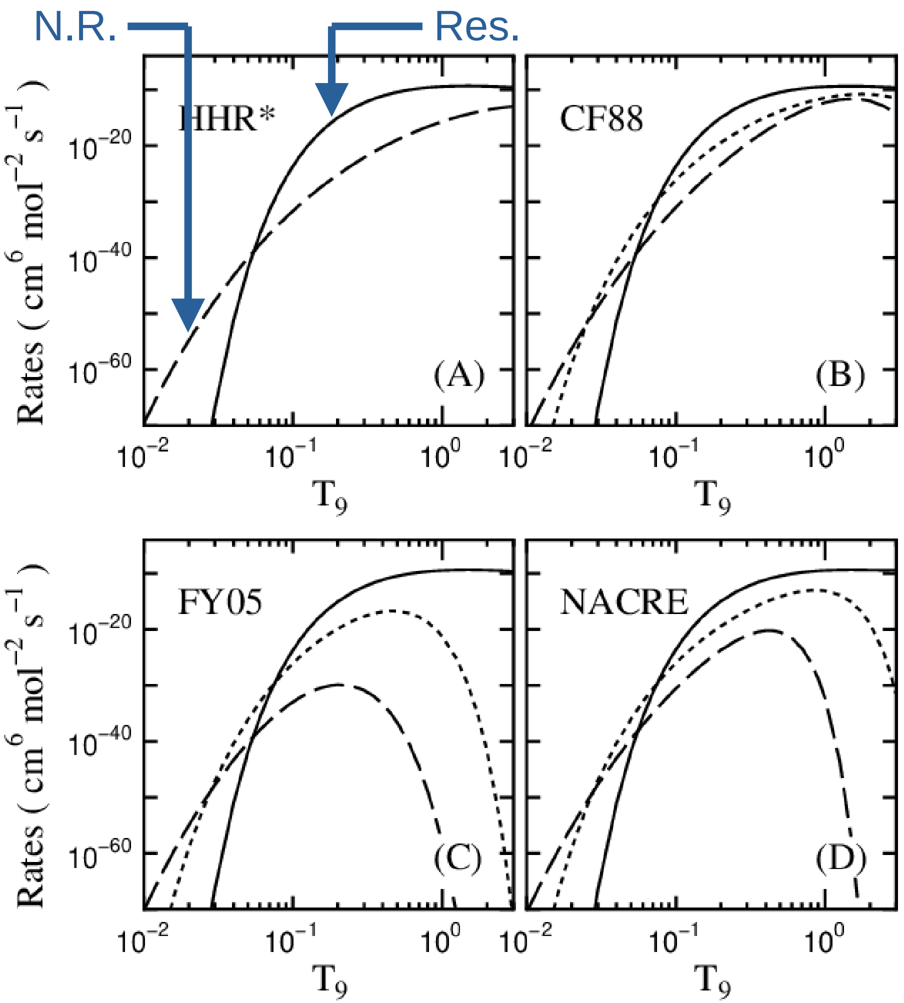
— — — N.R. of ((2α N.R.) + α)

HHR*

No N.R. of ^{8}Be+α in HHR*.

This means that a statistically generated ^{8}Be is destroyed by the third α-particle at $T_9 \approx 0.05$ before its lifetime.

(FY05) H.O.U. Fynbo, et al., Nature 433, 136 (2005), the experimental upgrade of NACRE and CF88 formula, adopted in JINA Reaclib, https://reaclib.jinaweb.org/he4(aa,g)c12

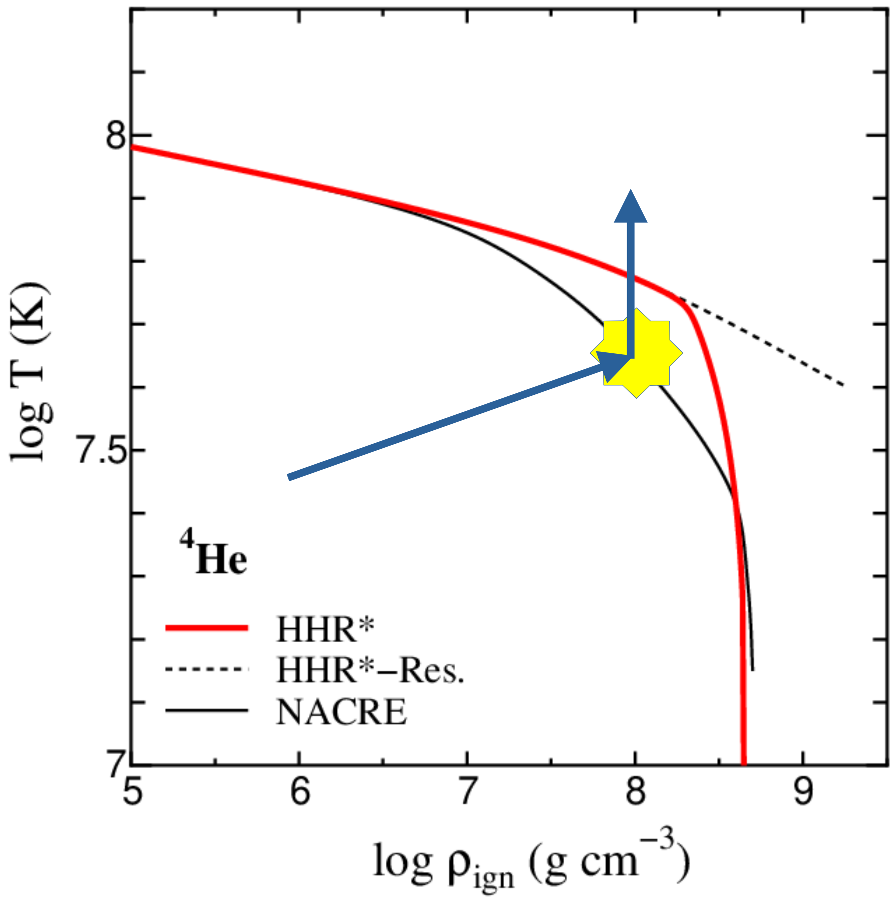


5. Ignition curve of helium burning in accreting white dwarfs

N.R. component plays an important role in helium burning at (log T) = 7.*)

- Due to the reduction of rates at $T_9 = 0.05$, the ignition density becomes insensitive to temperatures in $0.01 < T_9 < 0.05$.

The accretion grows from low temperature and low density, surrounding white dwarfs.



* K. Nomoto, F.-K. Thielemann, S. Miyaji, Astron. Astrophys. 149, 239 (1985).
K. Langanke, M. Wiescher, F.-K. Thielemann, Z. Phys. A 324, 147 (1986).

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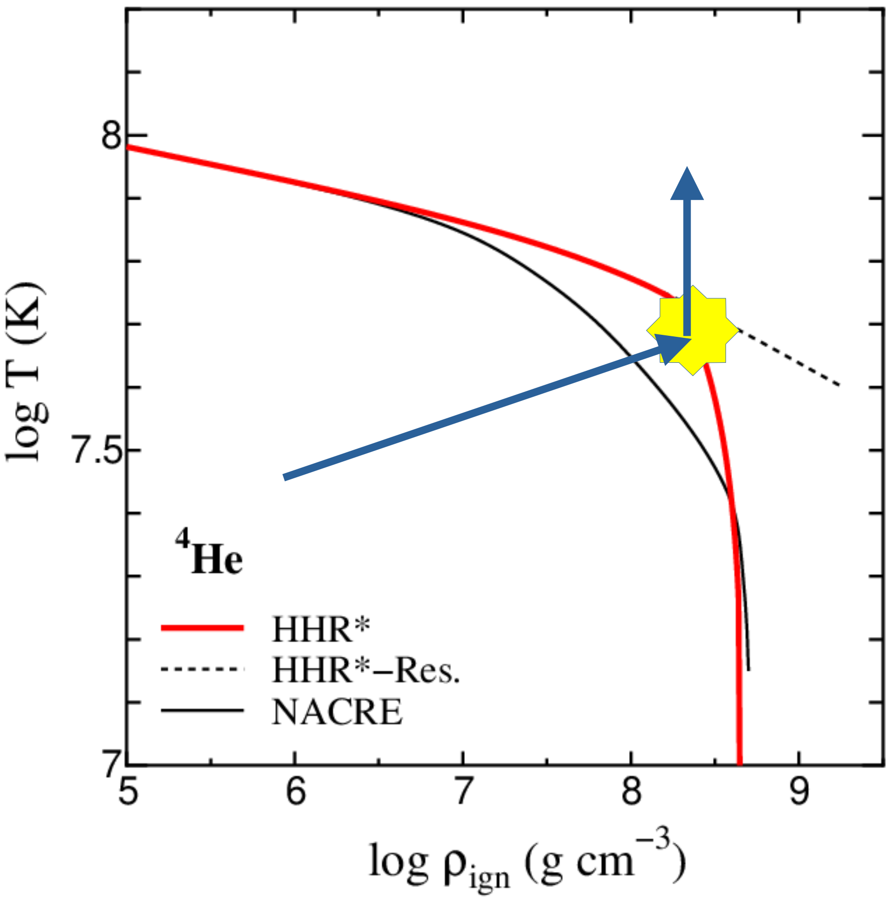
When the new rates are used, the accretion grows denser (heavier).

After that, helium burning starts to ignite.



* K. Nomoto, F.-K. Thielemann, S. Miyaji, Astron. Astrophys. 149, 239 (1985).
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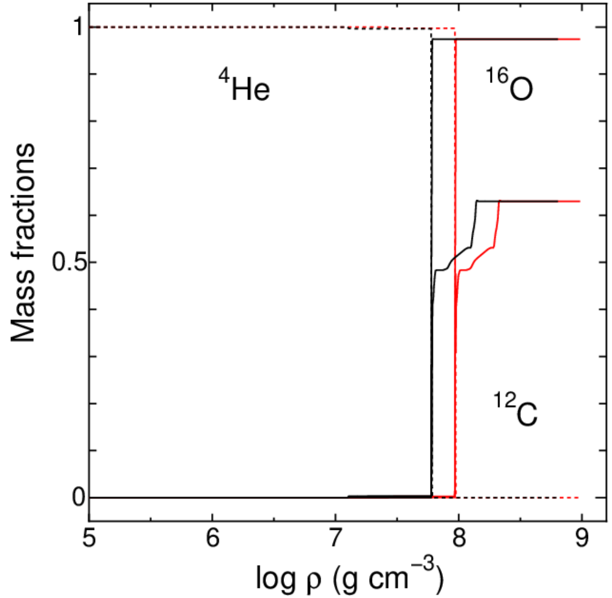
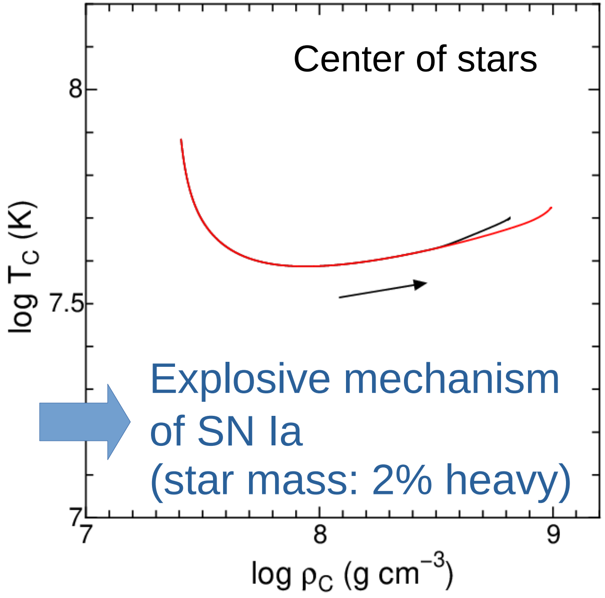
5. Test calculation of astrophysical applications

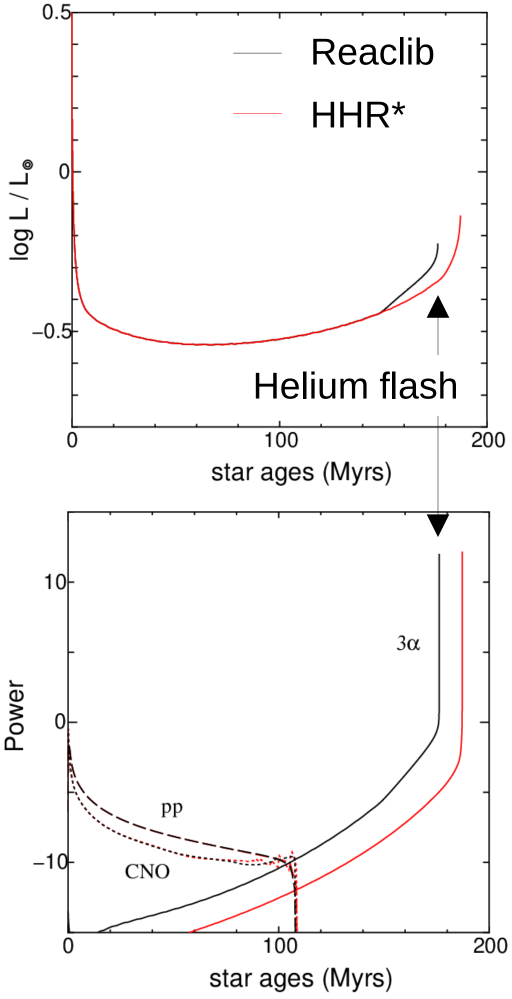
Helium nova (white dwarf helium shell ignition)

- CO white dwarfs of 0.96 $M_\odot$,
  with ^{4}He accretion (X_{He}=1) of $dM/dt = 2 \times 10^{-9}$ $M_\odot$/yr.
- **MESA** [B Paxton et al., ApJS 192, 3 (2011)], 1D stellar evolution code
  - Time before helium flash becomes longer.
  - Make a denser CO core because of the heavy accretion.





Explosive mechanism of SN Ia
(star mass: 2% heavy)



5. ρT phase diagram of Helium ion plasma

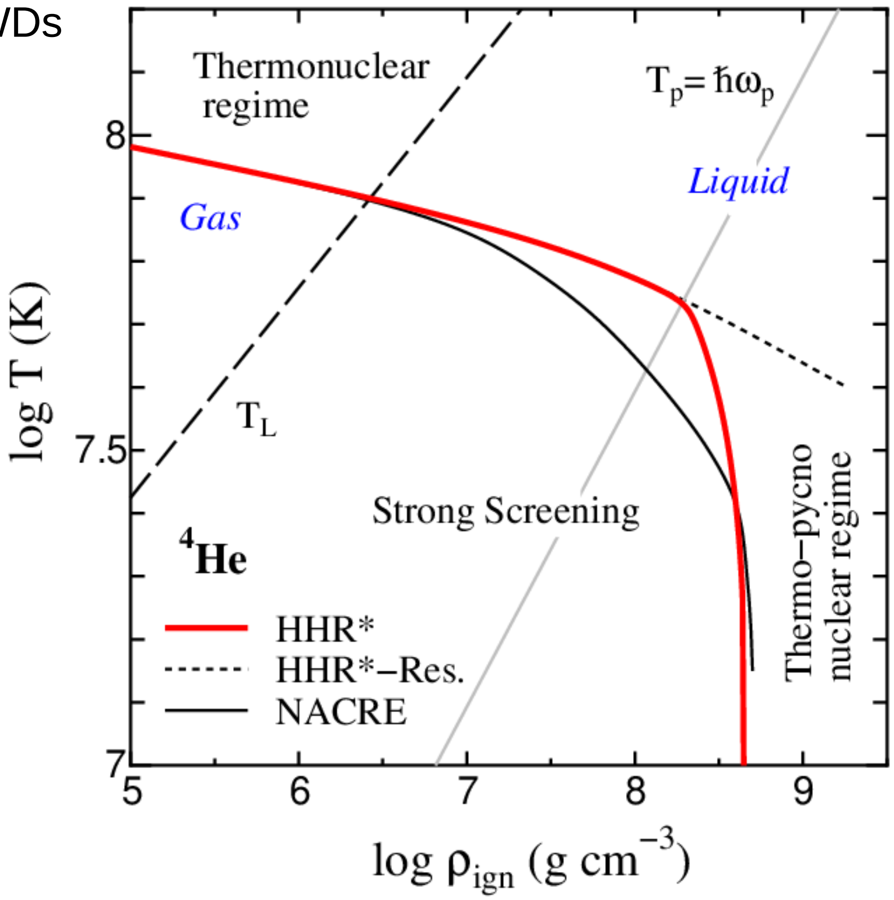
Ignition curve of helium burning in accreting WDs

- appears to bend at (log ρ) ≈ 8.3 g cm^{-3}, (log T) ≈ 7.7.

Helium ion plasma‡ : Gas, Coulomb liquid

- Below plasma temperatures T_p, ions seem to start to form a sort of lattice structure with regular tetrahedrons, *i.e.* fcc, hcp.
- The 3-body N.R. component appears only in the intermediate region between ”thermo” and ”pycno”.

The direct triple α process is a reaction mechanism characterizing the thermo-pycno nuclear regime.



‡ E E Salpeter, H M Van Horn, ApJ 155, 183 (1969); L R Gasques et al., PRC72, 025806 (2005);
S. Schramm, S E Koonin, ApJ 365, 296 (1990); N. Itoh et al., ApJ 362, 620 (1990); ibid. 234, 1079 (1979)

Summary

1. Purpose of this project: Precise estimate of the triple α reaction rates at low temperatures

2. Implementation: Performed an exact calculation without any restriction on the internal motion of ^{8}Be

3. Theory: Non-adabatic Faddeev 3-body R-matrix expansion method
(Hyperspherical harmonics R-matrix (HHR) expansion method)

4. Results: The reaction rates are found to be reduced by a factor of 10^{-4} at $T_9 = 0.05$. This means that the non-resonant ^{8}Be+α component is removed from the standard rates.

5. Applications:

a) Due to this reduction, the ignition density is found to become insensitive to temperatures in $0.01 < T_9 < 0.05$, which leads to the late helium flash in accreting WD of 0.96 $M_\odot$ with pure ^{4}He accretion (X_{He}=1) of dM/dt = 2 x 10^{-9} $M_\odot$/yr. The test calculation has been done with MESA.

b) The direct triple α process at low temperatures is also found to be a reaction mechanism characterizing the thermo-pycno nuclear regime below plasma temperatures.