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Quasiparticle finite amplitude method on the deformed relativistic Hartree-Bogoliubov theory in continuum: beta-decay half-life

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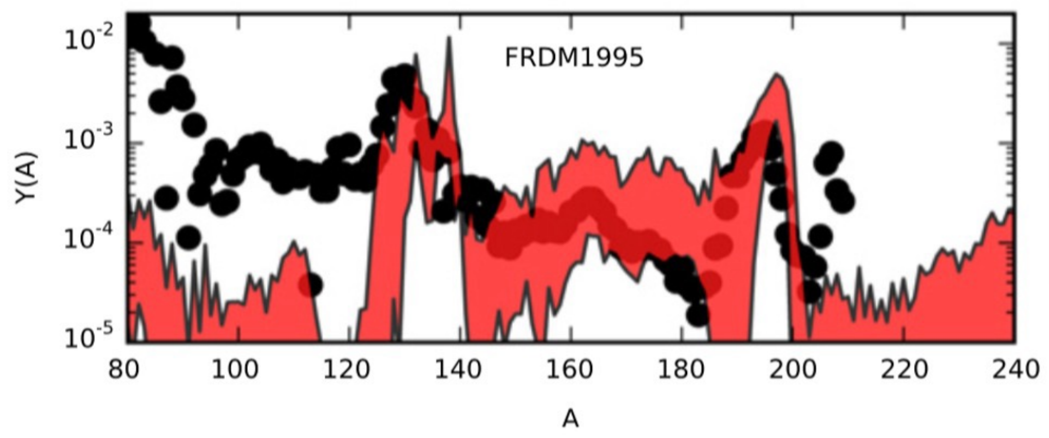
Beta-decay half-lives

- Nuclear β-decay half-lives are important for understanding the spin-isospin properties of the nuclear interaction and exploring the origin of heavy elements in the universe.

  Burbidge, Burbidge, Fowler, Hoyle, RMP 29, 547 (1957);
  Cowan, et al. RMP 93, 015002 (2021).

- Accurate β-decay half-lives of nuclei on the neutron-rich side are required to simulate the r-process and constrain its astrophysical sites.

  Mumpower, et al., PPNP 86, 86 (2016)



Multiplicative factors ranging from 10^{-1} to 10

- So far, the β-decay half-lives for more than a thousand neutron-rich nuclei have been measured, most of which lie relatively close to the β-stability line.

  Kondev, M. Wang, W. Huang, Naimi, Audi, Chinese Phys. C 45, 030001 (2021)

- For nuclei far from the β-stability line, the data on β-decay half-lives rely on theoretical predictions.

QRPA for beta-decay half-lives

- pnQRPA is one of the widely used models for beta-decay half-lives; it describes nuclear excitations as coherent superpositions of 2qp excitations and can be applied across the nuclear chart.

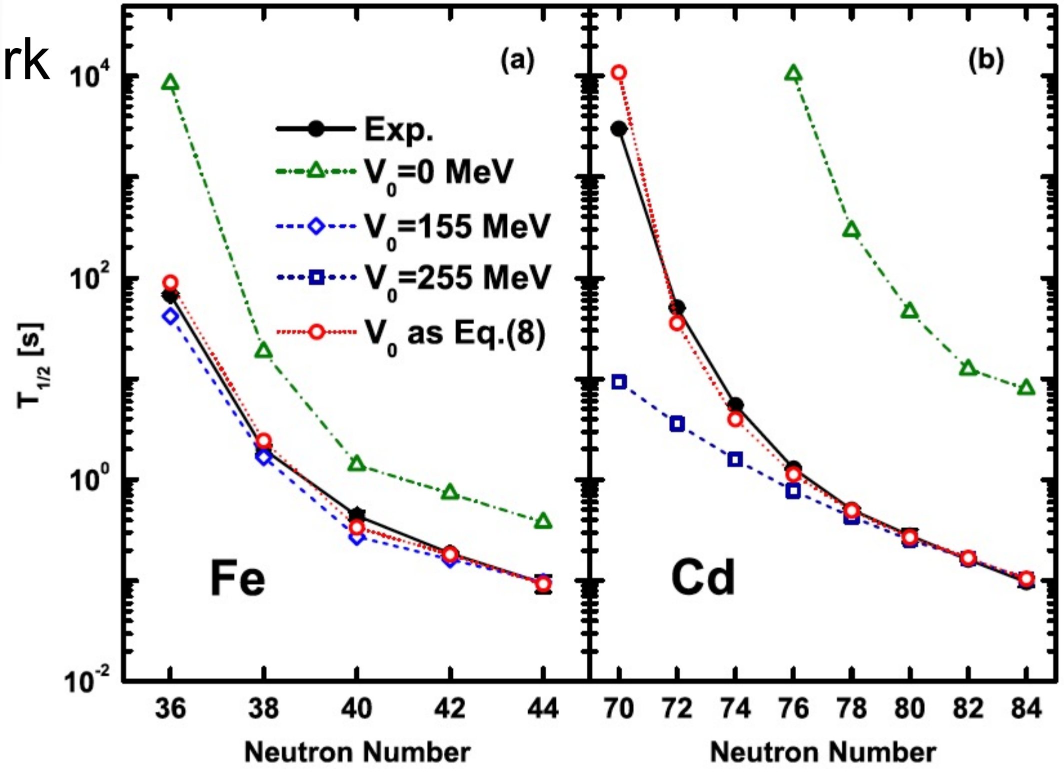
  Based on FRDM 1992: *Moller, Nix, Kratz, ADNDT 66, 131 (1997).*
  Based on FRDM 2012: *Moller, Mumpower, Kawano, Myers, ADNDT 125, 1 (2019).*

- pnQRPA has been built based on the DFT framework
  - Open-shell nuclei: proton-neutron pairing improves the descriptions

    J. Engel, et al., PRC 60, 014302 (1999);
    Z.M. Niu, et al., PLB 723, 172 (2013).

  - Closed-shell nuclei: tensor effects, beyond (Q)RPA effects

    F. Minato, C.L. Bai, PRL 110, 122501 (2013);
    Y.F. Niu, et al., PRL 114, 142501 (2015).



Z.M. Niu, et al., PLB 723, 172 (2013).

For the deformed nucleus

- Traditional pnQRPA has been developed based on Skyrme and Gogny EDFs
  - Beta-decay half-lives based on ground states and local minima in Ge, Se, Kr, Ru, and Pd isotopes *Sarriguren, PRC 91, 044304 (2015).*
  - By analyzing the β-decay half-lives in Zr isotopes: β-decay half-lives can serve as indicators of shape-phase transitions *Yoshida, Y.F. Niu, Minato, PRC 108, 034305 (2023).*

- pnQFAM theory has been implemented based on Skyrme EDFs:

  Mustonen, Shafer, Zenginerler, Engel, PRC 90, 024308 (2014).
  - By implementing the equal filling approximation for odd nuclei, global calculations for beta-decay half-lives have been carried out in nuclei on the neutron-rich side.

    Mustonen, Engel, PRC 93, 014304 (2016); Shafer et al, PRC 94, 055802 (2016).
  - Beyond QRPA correlations are studied.

    Q.Q. Liu, Engel, Hinohara, Kortelainen, PRC 109, 044308 (2024).

For the deformed nucleus

- The relativistic DFT takes Lorentz symmetry into account and has been proven to be a powerful theory in nuclear physics.

J. Meng, Relativistic density functional for nuclear structure (World Scientific, 2016).

- Recently, the pnQFAM approach has been developed based on relativistic DFT with a harmonic oscillator basis.

*C.Chen, **Z.Z. Li**, Y.N. Zhang, Y.F. Niu, PRC 112, 064332 (2025).*

Due to the use of harmonic oscillator basis, this model is not suitable for describing the β-decay half-lives around the drip line.

DRHBc + pnQFAM

- DRHBc theory includes pairing correlations and continuum effects in a microscopic and self-consistent way, and has been successfully applied to the studies of nuclear masses, the halo structures, and exotic excitations.

P. Guo, et al. (DRHBc Mass Table Collaboration), ADNDT 158, 101661 (2024).
K.Y. Zhang, et al., PLB 844, 138112 (2023).
*X.F. Jiang, **Z.Z. Li**, X.W. Sun, J. Meng, PLB 874, 140264 (2026).*

- Therefore, the combination of DRHBc and pnQFAM approach offers a powerful tool for investigating β-decay half-lives in exotic nuclei.

- In this work:
  - We will develop the pnQFAM based on DRHBc, adopting PC-PK1 functional.
  - In the allowed GT approximation, the deformation and pn pairing effects on β-decay half-lives will be studied based on the relativistic DFT framework.

pnQFAM approach

- We consider the system within the one-body external field $F(t)$

$$\hat{F}(t) = \hat{F}(\omega)e^{-i\omega t} + \hat{F}^{\dagger}(\omega)e^{i\omega t} \qquad \hat{F}(\omega) = \sum_{l_1 l_2} f_{l_1 l_2}(\omega) c_{l_1}^{\dagger} c_{l_2}$$

- In the RHB approximation, the state of the nuclear system is characterized by the generalized density

$$\mathbb{R}(t) = \begin{pmatrix} \rho(t) & \kappa(t) \\ -\kappa^*(t) & \hat{1} - \rho^*(t) \end{pmatrix}$$

⇒ Observables

$$\rho_{ll'}(t) = \langle \Phi(t) | c_{l'}^{\dagger} c_l | \Phi(t) \rangle \qquad \kappa_{ll'}(t) = \langle \Phi(t) | c_{l'} c_l | \Phi(t) \rangle$$

- Time evolution of $\mathbb{R}(t)$ is derived from Ehrenfest's theorem, which gives TDRHB equation:

$$i\hbar \frac{\partial}{\partial t} \mathbb{R}(t) = [\mathbb{H}(t) + \mathbb{F}(t), \mathbb{R}(t)]$$

$$\mathbb{H}(t) = \begin{pmatrix} h(t) & \Delta(t) \\ -\Delta^*(t) & -h^*(t) \end{pmatrix}$$

$$\mathbb{F}(t) = \mathbb{F}(\omega)e^{-i\omega t} + \mathbb{F}^{\dagger}(\omega)e^{i\omega t}$$

$$\mathbb{F}(\omega) = \begin{pmatrix} f(\omega) & \\ & -f^T(\omega) \end{pmatrix}$$

Decided by external field

pnQFAM approach

- If the external field is weak, it introduces small-amplitude oscillations around the stationary density and Hamiltonian,

$$\mathbb{R}(t) = \mathbb{R}_0 + \delta\mathbb{R}(\omega)e^{-i\omega t} + \delta\mathbb{R}^\dagger(\omega)e^{i\omega t},$$

$$\mathbb{H}(t) = \mathbb{H}_0 + \delta\mathbb{H}(\omega)e^{-i\omega t} + \delta\mathbb{H}^\dagger(\omega)e^{i\omega t}.$$

$\mathbb{R}(t) \to \delta\mathbb{R}(\omega)$

- By linearizing the TDRHB equation with respect to $\delta\mathbb{R}$, one obtains the linear response equation

$$\omega\delta\mathbb{R}(\omega) = \left[\mathbb{H}_0, \delta\mathbb{R}(\omega)\right] + \left[\delta\mathbb{H}(\omega) + \mathbb{F}(\omega), \mathbb{R}_0\right]$$

$\mathbb{H}_0, \mathbb{R}_0$: Stationary RHB theory

$\mathbb{F}$: Decided by external field only

$\delta\mathbb{H}$: functional of $\delta\mathbb{R}$

One way to solve the LR equation is QRPA

pnQFAM approach

- To obtain the pnQFAM equations, the matrices are written in terms of proton and neutron components as

$$\mathbb{R} = \begin{pmatrix} \mathbb{R}_{nn'} & \mathbb{R}_{np'} \\ \mathbb{R}_{pn'} & \mathbb{R}_{pp'} \end{pmatrix}, \ \mathbb{H} = \begin{pmatrix} \mathbb{H}_{nn'} & \mathbb{H}_{np'} \\ \mathbb{H}_{pn'} & \mathbb{H}_{pp'} \end{pmatrix}, \ \mathbb{F} = \begin{pmatrix} \mathbb{F}_{nn'} & \mathbb{F}_{np'} \\ \mathbb{F}_{pn'} & \mathbb{F}_{pp'} \end{pmatrix}$$

- For charge-exchange transitions, one has $\mathbb{F}_{nn'} = \mathbb{F}_{pp'} = 0$, which gives

$$\omega\delta\mathbb{R}_{pn}(\omega) = \left[\mathbb{H}_0, \delta\mathbb{R}(\omega)\right]_{pn} + \left[\delta\mathbb{H}(\omega) + \mathbb{F}(\omega), \mathbb{R}_0\right]_{pn}$$

- In the Bogoliubov basis (π for protons and ν for neutrons), $\mathbb{W}_{lk} = \begin{pmatrix} W_{n\nu} & 0 \\ 0 & W_{p\pi} \end{pmatrix}$ $\mathcal{W}_{lk} = \begin{pmatrix} U & V^* \\ V & U^* \end{pmatrix}_{lk}$

$$\mathbb{F}_{\pi\nu}(\omega) = \begin{pmatrix} F^{11}_{\pi\nu}(\omega) & F^{20}_{\pi\nu}(\omega) \\ -F^{02}_{\pi\nu}(\omega) & -F'^{11}_{\pi\nu}(\omega) \end{pmatrix} \quad \delta\mathbb{H}_{\pi\nu}(\omega) = \begin{pmatrix} \delta H^{11}_{\pi\nu}(\omega) & \delta H^{20}_{\pi\nu}(\omega) \\ -\delta H^{02}_{\pi\nu}(\omega) & -\delta H'^{11}_{\pi\nu}(\omega) \end{pmatrix} \quad \delta\mathbb{R}_{\pi\nu}(\omega) = \begin{pmatrix} & X_{\pi\nu}(\omega) \\ -Y_{\pi\nu}(\omega) & \end{pmatrix}$$

pnQFAM approach

- One can obtains the pnQFAM equations

$$(E_\pi + E_\nu - \hbar\omega)X_{\pi\nu}(\omega) + \delta H^{20}_{\pi\nu}(\omega) = -F^{20}_{\pi\nu}(\omega)$$

$$(E_\pi + E_\nu + \hbar\omega)Y_{\pi\nu}(\omega) + \delta H^{02}_{\pi\nu}(\omega) = -F^{02}_{\pi\nu}(\omega)$$

$\delta H^{20}_{\pi\nu}$ ($\delta H^{02}_{\pi\nu}$) are functionals of $X_{\pi\nu}$ and $Y_{\pi\nu}$: the equations can be solved iteratively.

- For a given ω, the strength function with respect to F_μ is calculated by

$$S(F_\mu;\omega) = -\frac{1}{\pi}\mathrm{Im}R_F(F_\mu;\omega_\gamma)$$

$$R_F(F_\mu;\omega_\gamma) = \sum_{\pi\nu} F^{20*}_{\pi\nu}X_{\pi\nu}(\omega_\gamma) + F^{02*}_{\pi\nu}Y_{\pi\nu}(\omega_\gamma)$$

$\omega_i = \omega + i\gamma$: the imaginary part is used to avoid the infinity in the strength function, which corresponds to a smoothing width of 2γ.

Beta-decay half-lives

- In the allowed GT approximation, the β-decay half-life of an even-even nucleus reads

$$\frac{1}{T_{1/2}} = D^{-1} g_A^2 \sum_{\mu} \sum_{m} B_m^{(\mu)} f(Z,\omega)$$

  - $D = \frac{2\pi^3 \hbar^7 \ln 2}{m_e^5 c^4 G_F^2} = 6163.4\ \mathrm{s}$ *and g_A is set as 1.0*
  - $\mu = 0, \pm 1$: all the components of GT transition operator.
  - m runs over all final states with $E < Q_\beta$.
  - $f(Z,\omega)$ is the phase-space factor.

$$B_m^{(\mu)} = \left| \langle m | F_\mu | 0 \rangle \right|^2 = \frac{1}{2\pi i} \oint_{C_m} R_F(F_\mu, \omega) d\omega$$

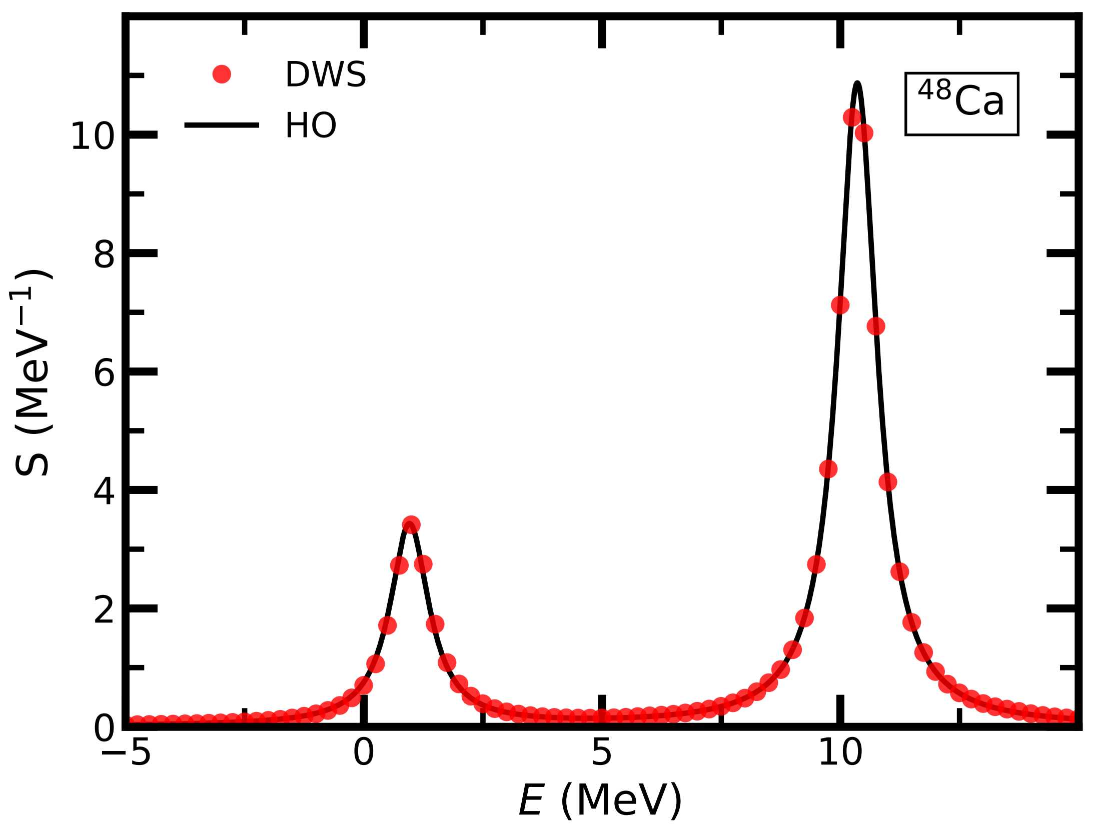
- In the pnQFAM case, the half-life can be calculated by a single contour integration:

$$\frac{1}{T_{1/2}} = D^{-1} g_A^2 \frac{1}{2\pi i} \sum_{\mu} \oint_C R_F(F_\mu; \omega) f_{\mathrm{poly}}(Z,\omega) d\omega$$

C encloses all the eigenstates with $E < Q_\beta$.

Numerical checks

- In the doubly-magic nucleus ^{48}Ca: pnQFAM gives the same strength distribution as the one from spherical pnQRPA code based on HO basis.



Spherical pnQRPA (HO) code from: Z.Y.Wang, et al., JPG (2016).

- Ikeda sum rules by pnQFAM

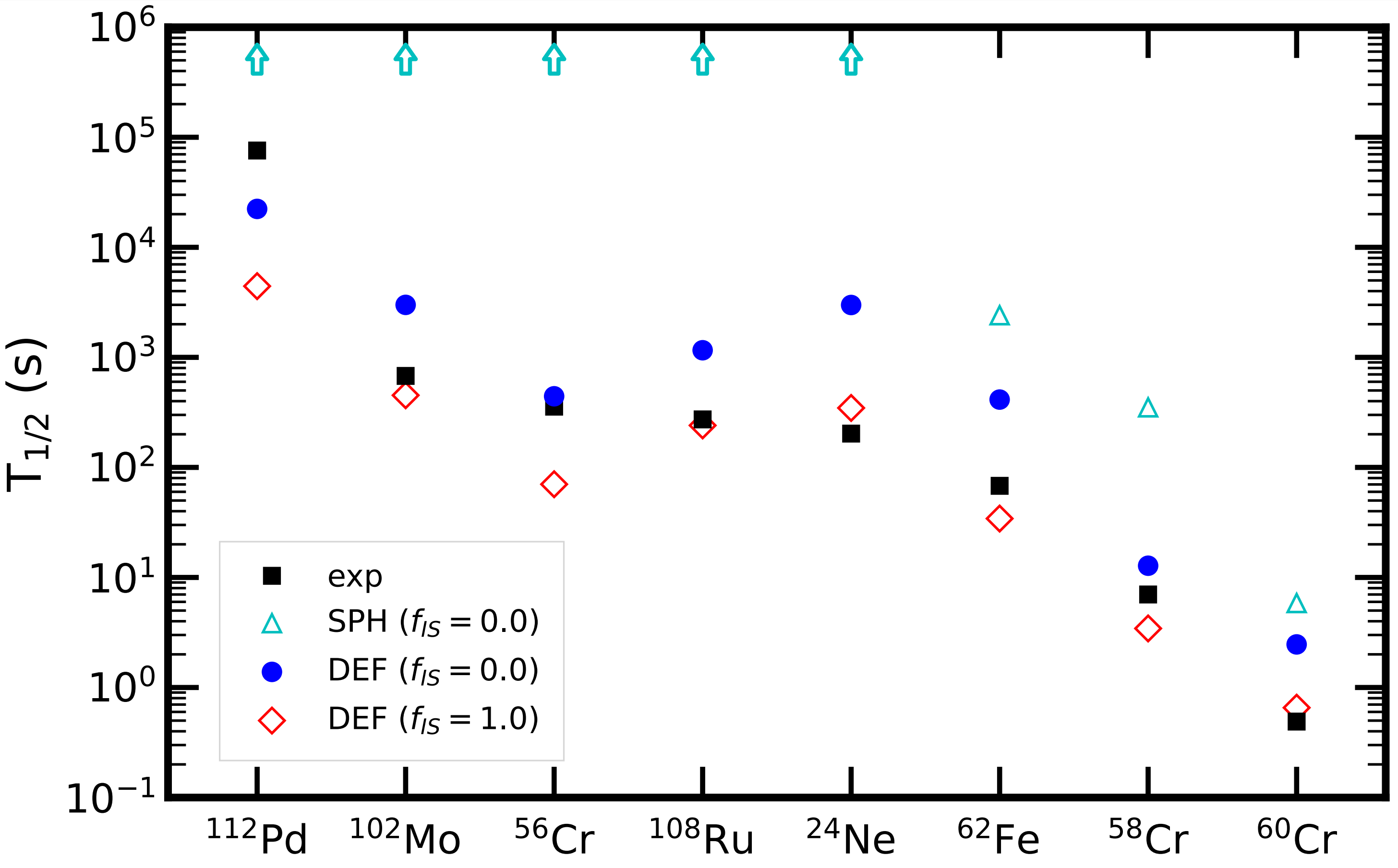
$$\sum_n B_n^{(\mathrm{GT}-)} - B_n^{(\mathrm{GT}+)} = 3(N-Z)$$

✓ ^{48}Ca exhausted 99.9779% of the Ikeda sum rule.

✓ ^{24}Ne exhausted 100.0516% of the Ikeda sum rule.

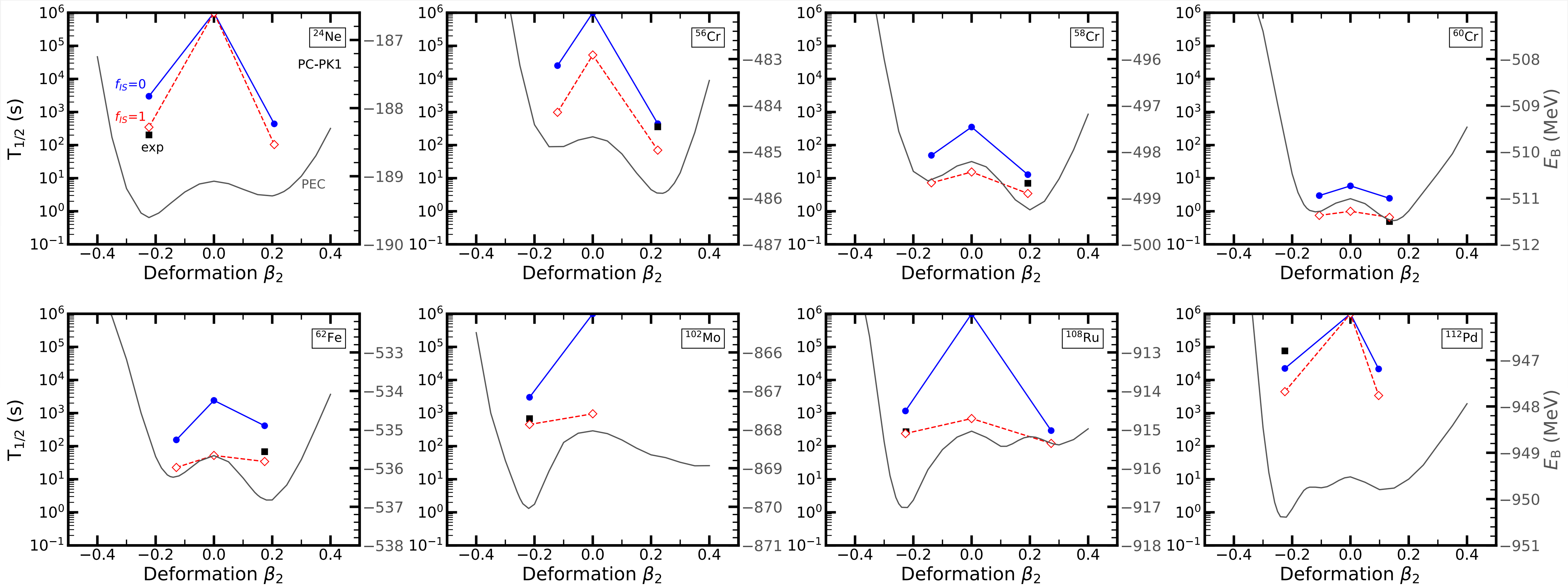
Deformation effects on beta-decay half-lives

- We firstly select some nuclei to show the deformation effects:
  - Deformed nuclei : from *P. Guo, et al. (DRHBc Mass Table Collaboration), ADNDT 158, 101661 (2024).*
  - dominated by allowed transitions: from *Marketin, Huther, Martinez-Pinedo, PRC 93, 025805 (2016).*



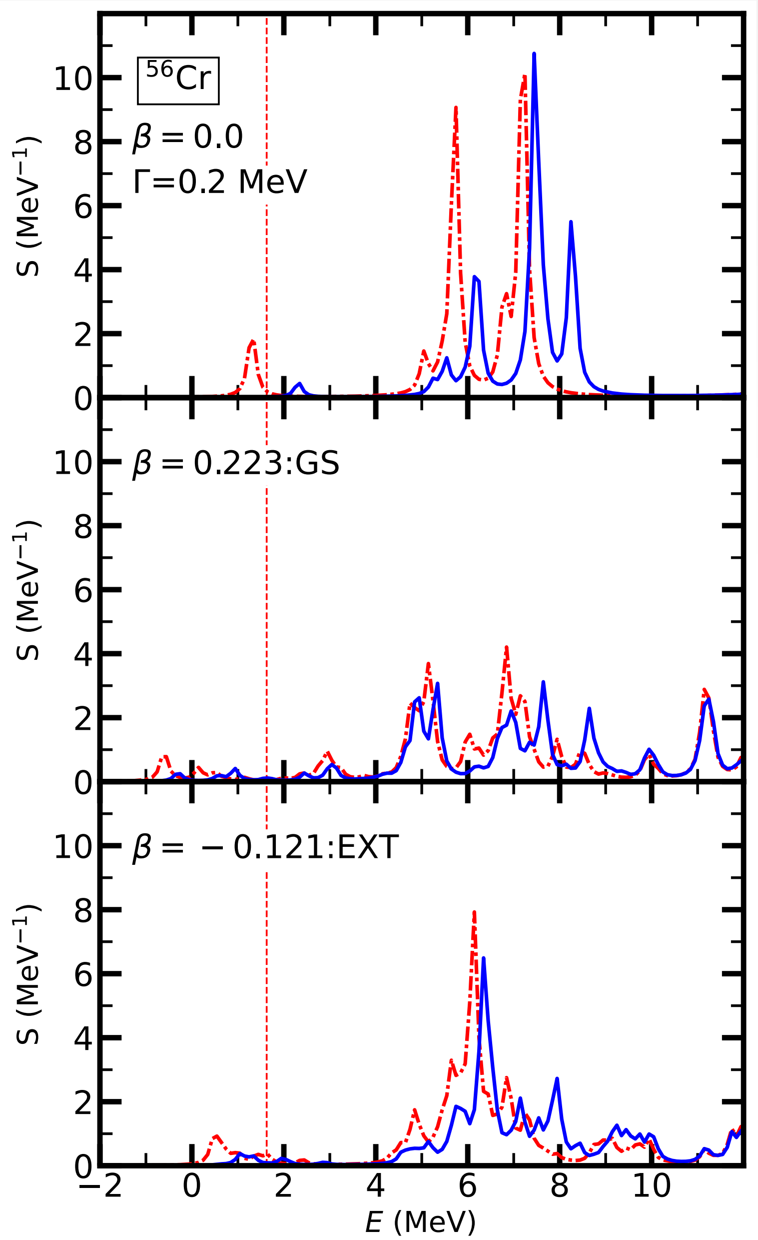
- Deformation significantly improves the descriptions of β-decay half-lives systematically.
- IS pairing further improves the descriptions in most cases.

Deformation effects on beta-decay half-lives



- Deformation reduces the half-lives in all the cases, and significantly improves the descriptions of $T_{1/2}$ overall: which is particularly important in ^{24}Ne, ^{56}Cr, and ^{112}Pd.
- The IS pairing effects further reduces $T_{1/2}$.
- IS pairing effect in spherical cases is stronger than in deformed case.

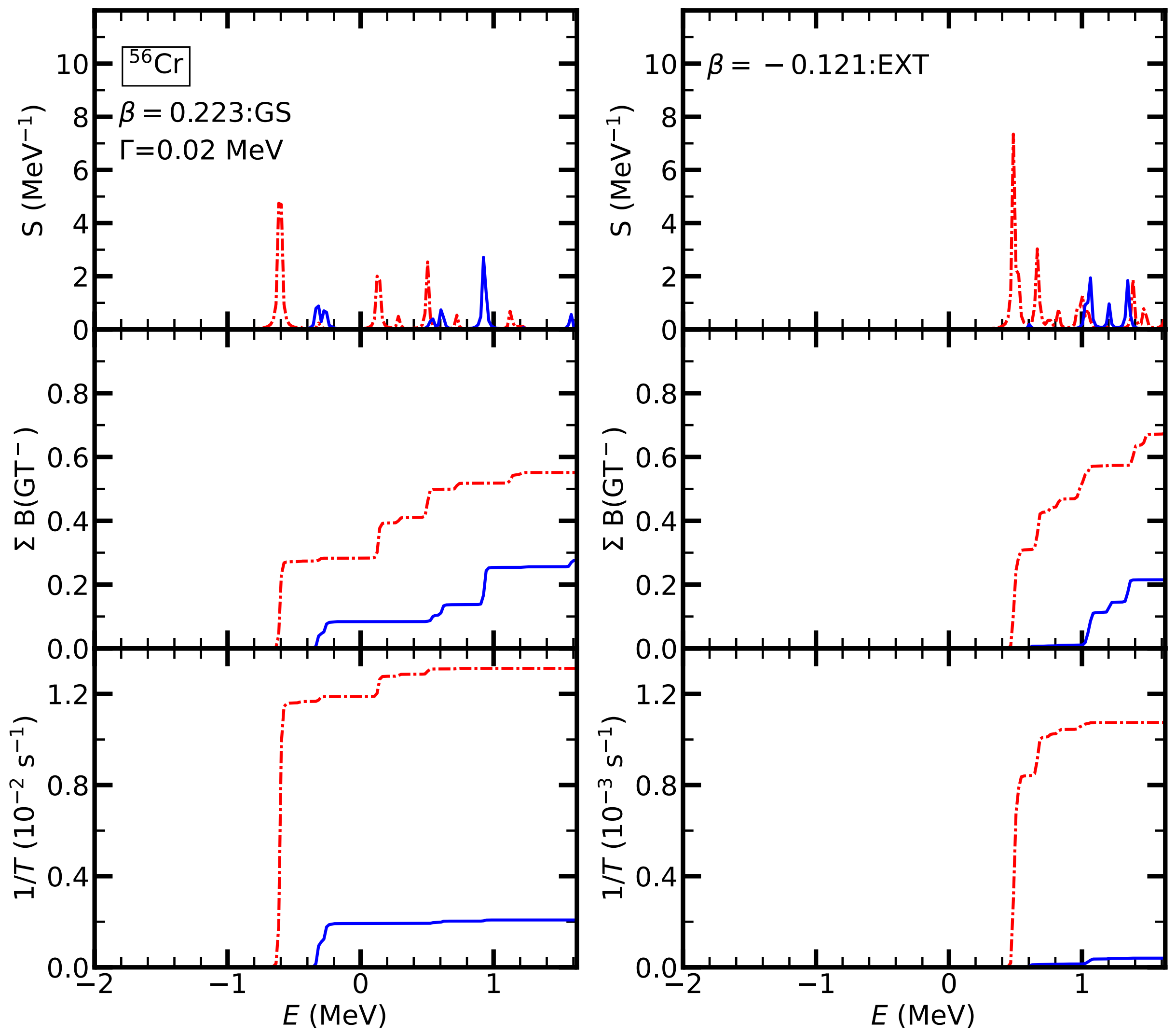
Gamow-Teller strength in ^{56}Cr



- Deformation significantly splits the GT strength distributions, and considerable low-lying strengths appear at $E < Q_\beta$.
- IS pairing further shifts the strength to lower energy, and enhances the strengths.
- In GS, the IS pairing effects are weaker, which can be attributed to the smaller pairing gaps:

56Cr	Δ_n (MeV)	Δ_p (MeV)
SPH	1.432356	1.620640
GS	1.224937	1.005486
EXT	1.374023	1.461058

The contributions to $T_{1/2}$ from each peak in ^{56}Cr



- Many peaks are found blow Q_β with a smaller widths.
- The IS pairing enhanced the strength below Q_β about 2 times.
- The contribution to T1/2 mainly comes from the lowest states due to the large phase-space factors.

Summary

- DRHBc+pnQFAM is developed, adopting the PC-PK1 functional and a density-dependent zero-range pairing force.

- Deformation effects on β-decay half-lives are investigated within the relativistic DFT framework.
  - ✓ Deformation effects systematically improve the agreement between calculated and experimental β-decay half-lives.
  - ✓ IS pairing effects further improves the descriptions in most cases.
  - ✓ IS pairing effect in spherical cases is stronger than in deformed case.

- Taking ^{56}Cr as an example, the contributions to $T_{1/2}$ from the low-lying GT strength distributions are analyzed.

Thank you for your attention !