

# Constraining theoretical corrections to Gamow-Teller transition rates (Toward Precision Tests of the Standard Model)

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- Lee-Yang model of  $\beta$  decay

$$\begin{aligned}
 \mathcal{H}_{\text{LY}} = & +\bar{p}n(C_S \bar{e}v - C'_S \bar{e}\gamma_5 v) && \text{scalar} \\
 & +\bar{p}\gamma_\mu n(C_V \bar{e}\gamma^\mu v - C'_V \bar{e}\gamma^\mu \gamma_5 v) && \text{vector} \\
 & +\bar{p}\sigma_{\mu\nu} n(C_T \bar{e}\sigma^{\mu\nu} v - C'_T \bar{e}\sigma^{\mu\nu} \gamma_5 v) / 2 && \text{tensor} \\
 & -\bar{p}\gamma_\mu \gamma_5 n(C_A \bar{e}\gamma^\mu \gamma_5 v - C'_A \bar{e}\gamma^\mu v) && \text{axial-vector} \\
 & +\bar{p}\gamma_5 n(C_P \bar{e}\gamma_5 v - C'_P \bar{e}v) && \text{pseudoscalar} \\
 & +\text{H.c.}
 \end{aligned}$$

- Non-relativistic limit

$$\begin{aligned}
 \text{vector} + \text{scalar} & \propto \tau^\pm, && (\text{Fermi}) \\
 \text{axial-vector} + \text{tensor} & \propto \sigma\tau^\pm, && (\text{Gamow-Teller}) \\
 \text{pseudoscalar} & \propto \mathcal{O}(|q|/m)
 \end{aligned}$$

- Signatures of physics beyond the  $V-A$  interaction

- 1) Scalar currents lead to violation of CVC in F transitions (**nuclear structure calculations**)
- 2) Tensor currents induce mirror asymmetries in GT transitions (**nuclear structure calculations**)
- 3) Both also modify the  $\beta$ -spectrum shape (**angular correlation measurements**).

# Probing the vector (and scalar) sectors

- Quark flavor mixing

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Current data indicate  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 < 1$  ( $\sim 95\%$  C.L.)  
**New degrees of freedom?**

$|V_{ud}|$  (nuclear  $\beta$  decays),  $|V_{us}|$  (kaon decays),  $|V_{ub}|$  negligible

- Extraction of  $|V_{ud}|$

- Superaligned  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decay (pure Fermi process)

$$\mathcal{F}t = ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS}) = \frac{K}{2G_F^2 |V_{ud}|^2 (1 + \Delta_R^V)}$$

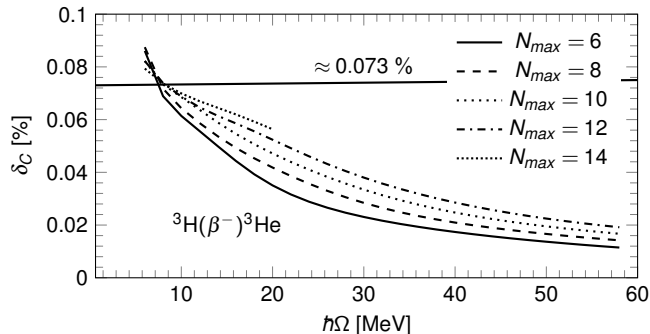
Experimental input:  $ft$  values measured with  $\sim 0.1\%$  precision for 15 cases. Radiative corrections include the nucleus-independent term  $\Delta_R^V$ , the outer correction  $\delta'_R$  (dependent on  $Z$  and  $Q_\beta$ ), and **the structure-dependent correction  $\delta_{NS}$ . Whereas,  $\delta_C$  is the isospin-symmetry breaking correction.**

- Complementary data from mirror  $\beta$  transitions (admixture of Fermi and Gamow-Teller)

*Towner and Hardy, PRC102, 045501 (2020); Severijns et al., PRC107, 015502 (2023); and Xayavong et al. (2016-present)*

# Probing the vector (and scalar) sectors

- A long-standing challenge in first-principles treatments of  $\delta_C$



Similar convergence issues also arise in IMSRG calculations.

*Stroberg, Particles 4, 521-535 (2021)*

**The Coulomb interaction induces strong coupling across oscillator shells**

*Stetcu et al., PRC71, 044325 (2005)*

## NCSM w/o 3-body forces!

TBMEs: N3LO-EMN500, SRG  
evolved with  $\lambda = 2 \text{ fm}^{-1}$ .

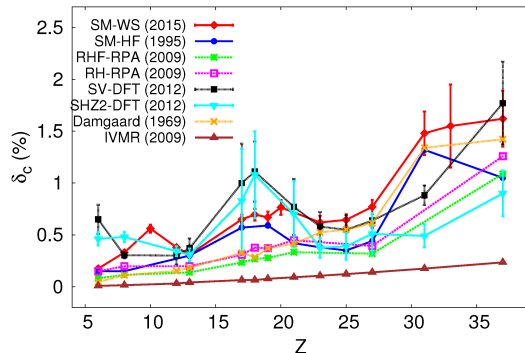
*Xayavong et al., PRC113, 034317 (2026)*

*Miyagi, EPJA59, 150 (2023)*

*Johnson et al., arXiv:1801.08432*

# Probing the vector (scalar) sectors

- While computationally tractable, phenomenological approaches yield diverse predictions



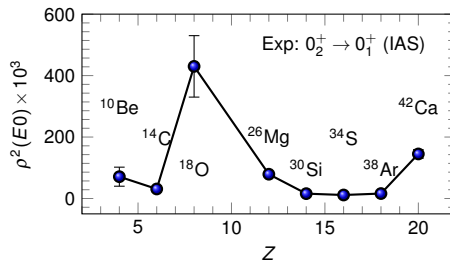
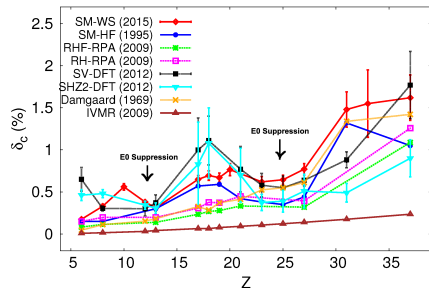
*Towner and Hardy, PRC102, 045501 (2020); Ormand and Brown, PRC52, 2455 (1995); Liang et al., PRC79, 064316 (2009); Satula et al., PRC86, 054316 (2012); Damgaard, NPA130, 233 (1969), Auerbach, PRC79, 035502 (2009); Xayavong, Smirnova, Lim et al., PRC112, 055503 (2025); PRC113, 034317 (2026); PRC109, 014317 (2024); PRC105, 044308 (2022); PRC97, 024324 (2018)*

Several constraints have been proposed. For example, low-energy contribution to  $\delta_C$  can be extracted from isospin-forbidden Fermi transitions.

Further independent constraints are still needed.

# Probing the vector (scalar) sectors

- Possible anti-correlation between  $\delta_C$  and  $E0$ -transition strengths



- For further insight, we are analyzing  $-2\hat{T}^{(1)}(E0) = [F_{\mp}, [\hat{T}(E0), F_{\pm}]]$ . The expectation value relation is not immediately transparent, but is highly isospin protected:

$$\begin{aligned}
 -2 \langle a | \hat{T}^{(1)}(E0) | a \rangle &= \sum_{bb'} \langle a | F_{\mp} | b \rangle \langle b | \hat{T}(E0) | b' \rangle \langle b' | F_{\pm} | a \rangle - \sum_{ba'} \langle a | F_{\mp} | b \rangle \langle b | F_{\pm} | a' \rangle \langle a' | \hat{T}(E0) | a \rangle \\
 &\quad - \sum_{ca'} \langle a | \hat{T}(E0) | a' \rangle \langle a' | F_{\pm} | c \rangle \langle c | F_{\mp} | a \rangle + \sum_{cc'} \langle a | F_{\pm} | c' \rangle \langle c' | \hat{T}(E0) | c \rangle \langle c | F_{\mp} | a \rangle,
 \end{aligned}$$

In the two-state mixing limit, we indeed recover  $\delta_C \propto 1/\rho^2(E0)$ .

A NCSM study of this behavior using natural (transition) orbital basis is in progress

# Probing the axial-vector (and tensor) sectors

- Gamow-Teller transitions

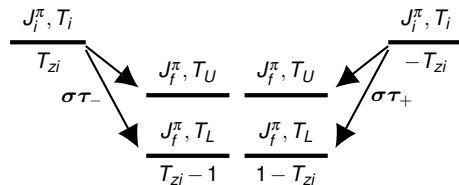
$$(ft)^{x\pm}(1+\delta_R'^{x\pm})(1+\delta_{NS}^{x\pm}-\delta_C^{x\pm}) = \frac{K}{G_F^2 |V_{ud}|^2 (g_A^{eff})^2 |\mathcal{M}^x|^2 (1+\Delta_R^A)}$$

where  $x = L$  or  $U$ . Here,  $\mathcal{M}^x$  and  $q_A^{eff}$  are isoscalar quantities, and can be eliminated through the following ways:

## 1) combining the mirror $\beta^\pm$ transitions

$$\Delta_I^x = \left[ \frac{(ft)^{x+}}{(ft)^{x-}} - 1 \right] - (\delta_C^{x+} - \delta_C^{x-}) \approx 0$$

applicable separately for the  $L$  and  $U$  branches



## 2) combining the $L$ and $U$ branches

$$\Delta_{II}^\pm = \left( \frac{\mathcal{M}^U}{\mathcal{M}^L} \right)^2 - \frac{(ft)^{L\pm}}{(ft)^{U\pm}} \left[ 1 - (\delta_C^{L\pm} - \delta_C^{U\pm}) \right] \approx 0$$

applicable separately for the  $\beta^+$  and  $\beta^-$  decays

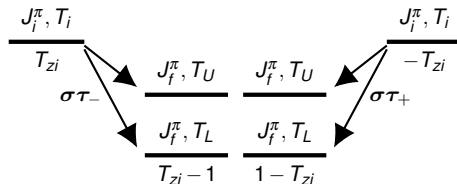
## 3) combining all the 4 transitions

$$\Delta_{III} = \left[ (ft)^{L-} / (ft)^{L+} \times (ft)^{U+} / (ft)^{U-} - 1 \right] + (\delta_C^{L+} - \delta_C^{L-}) + (\delta_C^{U+} - \delta_C^{U-}) \approx 0$$

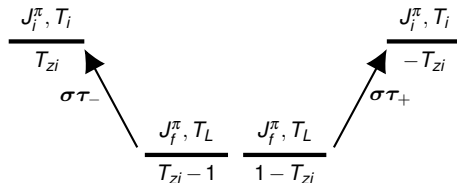
The radiative corrections are mostly isoscalar. The finiteness of  $\Delta_I^x$ ,  $\Delta_{II}^\pm$ , and  $\Delta_{III}$  may be attributed to tensor currents

# Probing the axial-vector (and tensor) sectors

- **Weak**  $\beta^\pm$  decays



- **Strong** charge-exchange reactions in reverse directions



In many cases, daughter nuclei are stable or long-life isotopes. Therefore, the mirror asymmetry can also be studied using nuclear reactions. This could be an alternative way to calibrate GT unit cross sections.

# Shell-model calculations

- Solution to  $H|\Psi_i\rangle = E_i|\Psi_i\rangle$  using the shell model

- 1) HO basis  
Orthogonality, integrability, symmetries, ...
- 2) Valence space
  - ▶  $0\hbar\omega$ :  $p$ ,  $sd$ ,  $pf$ , ...
  - ▶  $N\hbar\omega$ : ZBM,  $psd$ , ZBM2,  $sdpf$ , ...
- 3) Effective interactions

## Configuration-interaction method

Inputs

$$\Psi_i = \sum_n C_{in} \Phi_n$$

$$H_{ij} = \langle \Phi_j | H | \Phi_i \rangle$$

$\Rightarrow$

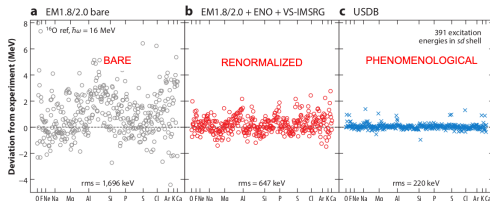
Efficient Code

$$\begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1N} \\ H_{21} & H_{22} & \cdots & H_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1} & H_{N2} & \cdots & H_{NN} \end{pmatrix}$$

$\Rightarrow$

Outputs

$$E_i, \Psi_i$$



| Masses | Spaces     | Interactions                       |
|--------|------------|------------------------------------|
| 8-14   | $p$ shell  | CKPOT, CKI, CKII<br>PJP, PJT, PMOM |
| 15-24  | ZBM        | REWIL                              |
| 25-35  | $sd$ shell | USD, USDA, USDB                    |

Stroberg, Annu. Rev. Nucl. Part. Sci. 2019. 69:307-62

## Conventional INC interaction model

The full Hamiltonian can be decomposed as

$$H = H_0 + V_{INC}, \quad [V_{INC}, T] \neq 0$$

where  $V_{INC}$  is parametrized as

$$\begin{aligned} V_{INC} &= \sum_{k=0}^2 V_{INC}^{(k)} \\ &= \sum_{k=0}^2 \left[ C^{(k)} V_C(\mathbf{r}) + P^{(k)} V_\pi(\mathbf{r}) + R^{(k)} V_\rho(\mathbf{r}) \right] I^{(k)} \end{aligned}$$

with  $V_C(\mathbf{r})$  is the Coulomb, and  $V_\pi(\mathbf{r})/V_\rho(\mathbf{r})$  are Yukawa pots. Whereas  $C^k$ ,  $P^k$  and  $R^k$  are adjustable parameters.

*Lam et al., PRC87, 054304 (2013)*

*Ormand and Brown, NPA440 (1985) 174; NPA491, 1 (1989)*

## Conventional constraints

Isobaric multiplet mass equation (IMME)

$$M(\alpha, T, T_z) = a(\alpha, T) + b(\alpha, T) T_z + c(\alpha, T) T_z^2$$

where

$$a(\alpha, T) = \frac{\langle \Psi || V_{INC}^{(0)} || \Psi \rangle}{\sqrt{2T+1}} - \frac{T(T+1) \langle \Psi || V_{INC}^{(2)} || \Psi \rangle}{(2T+1) \sqrt{T(T+1)(2T-1)}}$$

$$b(\alpha, T) = \frac{\langle \Psi || V_{INC}^{(1)} || \Psi \rangle}{\sqrt{T(2T+1)(T+1)}}$$

$$c(\alpha, T) = \frac{3 \langle \Psi || V_{INC}^{(2)} || \Psi \rangle}{(2T+1) \sqrt{T(T+1)(2T-1)}}$$

where  $b(\alpha, T)$  is important, because the isovector interaction is dominant

**Possible issue: off-diagonal elements remain unconstrained!**

**For this study, we simply adopt the INC interactions from**

*Ormand and Brown, NPA491, 1 (1989)*

# Evaluation of the isospin corrections

- With a spherical basis, GT matrix element can be written as

$$M_{GT} = \sum_{k_\alpha k_\beta} \langle k_\alpha || \sigma \tau_\pm || k_\beta \rangle \times \text{OBTD}(k_\alpha k_\beta \text{ if } \lambda)$$

where  $k \equiv nj\tau$  (without  $m$ ),  $\lambda = 1$  (the rank of  $\sigma \tau_\pm$ ). The transition densities are defined as

$$\text{OBTD}(k_\alpha k_\beta \text{ if } \lambda) = \frac{\langle f || [c_{k_\beta}^\dagger \otimes \tilde{c}_{k_\alpha}]^{(\lambda)} || i \rangle}{\sqrt{2\lambda + 1}}$$

where  $\tilde{c}_{j,m} = (-1)^{j-m} c_{j,-m}$ . The SPME can be broken into

$$\langle k_\alpha || \sigma \tau_\pm || k_\beta \rangle = \xi_{\alpha\beta} \theta_{\alpha\beta} \Omega_{\alpha\beta}$$

with

$$\xi_{\alpha\beta} = \langle \tau_\alpha | \tau_\pm | \tau_\beta \rangle, \quad \text{charge changing}$$

$$\theta_{\alpha\beta} = (-1)^{l_\alpha + j_\alpha + \frac{3}{2}} \sqrt{6} \hat{j}_\alpha \hat{j}_\beta \left\{ \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & 1 \\ j_\beta & j_\alpha & l_\alpha \end{array} \right\} \delta_{l_\alpha l_\beta}, \quad l \text{ is conserved}$$

$$\Omega_{\alpha\beta} = \int_0^\infty R_{k_\alpha}(r) R_{k_\beta}(r) r^2 dr, \quad \text{radial mismatch}$$

This formalism allows one to mimic shell-model transition densities with realistic radial wave functions

- WS potential (flexible and realistic choice for shell-model basis)

$$U(r) = -V_0 f(r, a_0, r_0) - \frac{V_0 \lambda \hbar^2}{4\mu^2 c^2} \frac{1}{r} \frac{d}{dr} f(r, a_s, r_s) \langle \mathbf{l} \cdot \boldsymbol{\sigma} \rangle \\ + V_g \frac{1}{r} \frac{d}{dr} f(r, a_s, r_s) + V_{coul}(r)$$

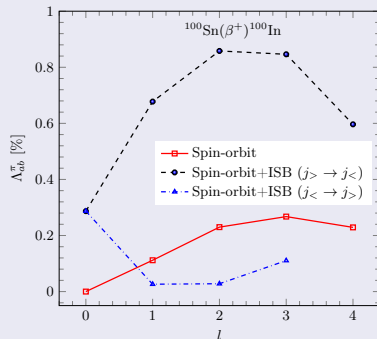
where  $V_g$  and  $r_0$  are constrained with  $S_p/S_n$  and  $\langle r_{ch}^2 \rangle$ , respectively

- Problem with HO basis. According to Thomas precession, one has

$$U_{SO}(r) \propto \frac{1}{r} \frac{d}{dr} U_{HO}(r) \langle \boldsymbol{\sigma} \cdot \mathbf{l} \rangle = m\omega^2 \langle \boldsymbol{\sigma} \cdot \mathbf{l} \rangle$$

Therefore, WS eigenfunctions corrects for both ISB and SO effects (GT can link spin-up to spin-down and vice-versa).

## Role of the spin-orbit potential



**HF eigenfunctions are generally more realistic, but they lack flexibility, and hence unsuitable for precision applications**

Xayavong and Smirnova, PRC105, 044308 (2002)

# Evaluation of the isospin corrections

- From the definition  $M_{GT}^2 = \mathcal{M}_{GT}^2(1 - \delta_C)$ , one can write  $\delta_C = \sum_{i=1}^6 \delta_{Ci}$  where

$$\delta_{C1} = \frac{2}{\mathcal{M}_{GT}} \sum_{k_\alpha k_\beta} \theta_{\alpha\beta} \xi_{\alpha\beta} D(k_\alpha k_\beta \text{ if } \lambda), \quad \text{LO}$$

$$\delta_{C2} = \frac{2}{\mathcal{M}_{GT}} \sum_{k_\alpha k_\beta} \theta_{\alpha\beta} \Lambda_{\alpha\beta} \xi_{\alpha\beta} \text{OBTD}_0(k_\alpha k_\beta \text{ if } \lambda), \quad \text{LO}$$

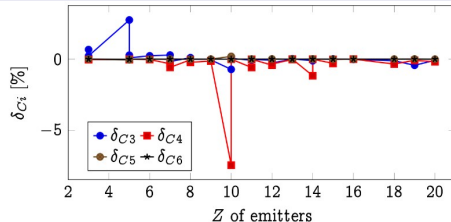
$$\delta_{C3} = -\frac{2}{\mathcal{M}_{GT}} \sum_{k_\alpha k_\beta} \theta_{\alpha\beta} \Lambda_{\alpha\beta} \xi_{\alpha\beta} D(k_\alpha k_\beta \text{ if } \lambda), \quad \text{NLO}$$

$$\delta_{C4} = -\frac{1}{4} (\delta_{C1} + \delta_{C2})^2, \quad \text{NLO}$$

$$\delta_{C5} = -\delta_{C3} \sqrt{|\delta_{C4}|}, \quad \text{N}^2\text{LO}$$

$$\delta_{C6} = -\frac{1}{4} (\delta_{C3})^2, \quad \text{N}^3\text{LO}$$

## Higher-order contributions



Xayavong and Lim, PRC111, 015501 (2025)

PRC112, 014313 (2025)

Here, we define  $D = \text{OBTD}_0 - \text{OBTD}$  and  $\Lambda = 1 - \Omega$  as isospin-perturbation parameters. Unlike Fermi,  $\mathcal{M}_{GT}$  is not model-independent. Furthermore,  $\delta_C$  could be constructive. At LO, the isospin-mixing ( $\delta_{C1}$ ) and radial mismatch ( $\delta_{C2}$ ) contributions are decoupled.

**Contributions from radial excitations are not included.**

Xayavong et al., PRC113, 034317 (2026)

# Radial mismatch corrections

- As an essence of the shell-model approach, one can expand OBTDs in terms of intermediate states

$$\text{OBTD}(k_\alpha k_\beta if \lambda) = \sum_{\pi} \Theta_{\alpha\beta if}^{\pi\lambda} \langle i || c_{k_\beta}^\dagger || \pi \rangle \langle f || c_{k_\alpha}^\dagger || \pi \rangle,$$

$$\text{where } \Theta_{\alpha\beta if}^{\pi\lambda} = (-1)^{J_f + J_\pi + J_\alpha + \lambda} \begin{Bmatrix} J_i & J_f & \lambda \\ J_\beta & J_\alpha & J_\pi \end{Bmatrix}$$

Subsequently,  $\delta_{C2}$  and  $\delta_{C3}$  can be evaluated as

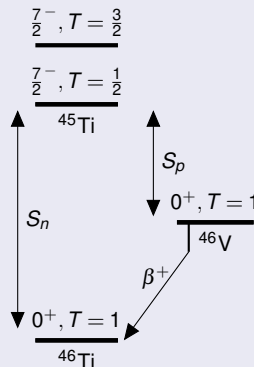
$$\delta_{C2} = \frac{2}{\mathcal{M}_{GT}} \sum_{k_\alpha k_\beta \pi} \theta_{\alpha\beta} \Lambda_{\alpha\beta}^\pi \xi_{\alpha\beta} \Theta_{\alpha\beta if}^{\pi\lambda} \mathcal{A}(f; \pi k_\alpha) \mathcal{A}(i; \pi k_\beta),$$

$$\tilde{\delta}_{C3} = \frac{2}{\mathcal{M}_{GT}} \sum_{k_\alpha k_\beta \pi} \theta_{\alpha\beta} \Lambda_{\alpha\beta}^\pi \xi_{\alpha\beta} \Theta_{\alpha\beta if}^{\pi\lambda} A(f; \pi k_\alpha) A(i; \pi k_\beta)$$

where  $\tilde{\delta}_{C3} = \delta_{C3} + \delta_{C2}$ . Whereas  $A(i; \pi k_\beta)$ ,  $A(f; \pi k_\alpha)$  are spectroscopic amplitudes and  $\mathcal{A}(i; \pi k_\beta)$ ,  $\mathcal{A}(f; \pi k_\alpha)$  the respective isospin-symmetry limit.

This expansion indicates that the  $g_A$  quenching may be related to the quenching of spectroscopic factors

## Schematic rep.



**This approach is non-standard, but provides an efficient means of capturing contributions outside the model space through radial mismatch**

*Towner and Hardy, PRC77, 025501 (2008)*

# Radial mismatch corrections

- Isospin structure of  $\delta_{C2}$  when  $T_i = T_f$

$$\delta_{C2} = \frac{2}{\mathcal{M}_{GT}} \sum_{k_\alpha k_\beta} \theta_{\alpha\beta}^\lambda \xi_{\alpha\beta} \left[ \sum_{\pi'} |C_{\pi'}| \Theta_{\alpha\beta fi}^{\pi'\lambda} \Lambda_{\alpha\beta}^{\pi'} \frac{(f|||c_{k_\alpha}^\dagger|||\pi')}{\hat{J}_f \hat{T}_f} \frac{(i|||c_{k_\beta}^\dagger|||\pi')}{\hat{J}_i \hat{T}_i} \right. \\ \left. - \sum_{\pi''} |C_{\pi''}| \Theta_{\alpha\beta fi}^{\pi''\lambda} \Lambda_{\alpha\beta}^{\pi''} \frac{(f|||c_{k_\alpha}^\dagger|||\pi'')}{\hat{J}_f \hat{T}_f} \frac{(i|||c_{k_\beta}^\dagger|||\pi'')}{\hat{J}_i \hat{T}_i} \right],$$

where  $C_\pi$  is an isospin coefficient, and  $\pi'$  ( $\pi''$ ) denotes states with  $T_\pi = T_i - \frac{1}{2}$  ( $T_\pi = T_i + \frac{1}{2}$ ).

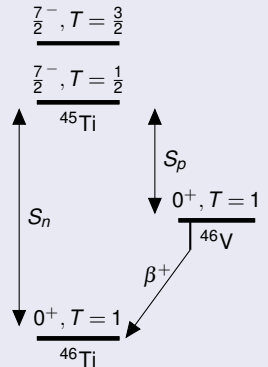
- Isospin structure of  $\delta_{C2}$  when  $T_i \neq T_f$

$$\delta_{C2} = \frac{2}{\mathcal{M}_{GT}} \sum_{k_\alpha k_\beta \tilde{\pi}} C_{\tilde{\pi}} \theta_{\alpha\beta}^\lambda \Theta_{\alpha\beta fi}^{\tilde{\pi}\lambda} \Lambda_{\alpha\beta}^{\tilde{\pi}} \xi_{\alpha\beta} \frac{(f|||c_{k_\alpha}^\dagger|||\tilde{\pi})}{\hat{J}_f \hat{T}_f} \frac{(i|||c_{k_\beta}^\dagger|||\tilde{\pi})}{\hat{J}_i \hat{T}_i},$$

where  $\tilde{\pi}$  denotes states with  $T_\pi = \min(T_i, T_f) + \frac{1}{2}$ .

According to these structures, one expects larger  $\delta_{C2}$  values for GT transitions with  $T_i \neq T_f$ , which are typical.

## Schematic rep.



Xayavong and Lim, PRC112, 014313 (2025)

# Radial mismatch corrections

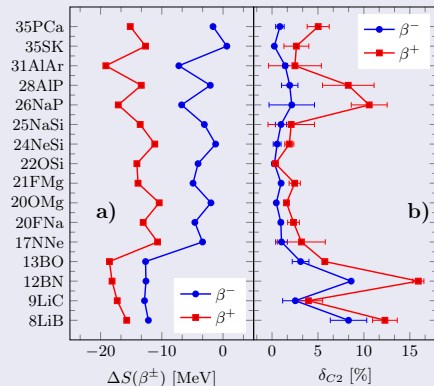
- To maintain correct asymptotics, WS parameters are adjusted to reproduce  $S_p$  and  $S_n$

$$\phi(r) \sim \exp\left(-r \frac{\sqrt{2m|E|}}{\hbar}\right)$$

Subsequently, in addition to the enhancement obtained in weakly bound nuclei ( $S_p \approx 0$ ),  $\delta_{C2}$  strongly correlates with separation-energy difference. In general,  $\delta_{C2}$  is stronger in  $\beta^+$  than in  $\beta^-$  counterpart (because Coulomb effect dominated)

- Charge-radius constraint is also essential, particularly for strongly deformed nuclei. However, charge-radius data are unavailable for most cases.

## Another features of $\delta_{C2}$



This property is essential in reproducing experimental data. It completely absent if one uses harmonic oscillator basis!

# Isospin-mixing corrections

- In the scenario when the isospin mixing occurs on the following way

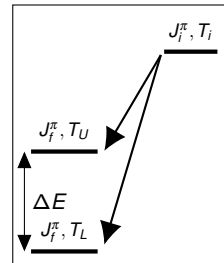
$$\begin{aligned} |i\rangle &= |J_i^\pi, T_i\rangle, \\ |f\rangle &= \sqrt{1-\alpha^2} |J_f^\pi, T_L\rangle + \alpha |J_f^\pi, T_U\rangle, \end{aligned}$$

one can easily derive

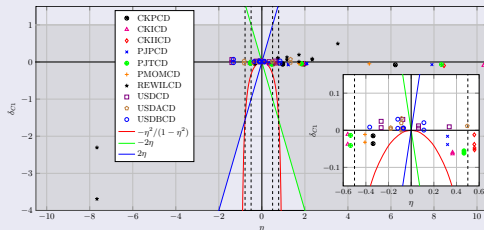
$$\delta_{C1} = -2\eta\alpha + (1-\eta^2)\alpha^2 + \mathcal{O}(\alpha^3)$$

where  $\alpha = \langle J_f^\pi T_L | V_{INC} | J_f^\pi T_U \rangle / \Delta E$  and  $\eta^2 = |\mathcal{M}_{GT}^U|^2 / |\mathcal{M}_{GT}^L|^2$ .

Note that  $\mathcal{M}_{GT}^L$  is generally not isospin forbidden!



## Some regions are not allowed



- $\delta_{C1}$  contains a LO term in  $\alpha$  (hence  $\Im\{\alpha\} = 0$ !)
- $\delta_{C1}$  can be enhanced by  $\eta$  (isospin symmetric qty!)
- These behaviors are in good agreement with shell-model calculations

Xayavong and Lim, PRC111, 015501 (2025)

## Selected shell-model results for $\delta_{C1}$

| Mirror Pair | Interaction | $\mathcal{M}_{GT}^L$ | $\mathcal{M}_{GT}^U$ | $\eta$ | $\Delta E$ [MeV] | $\delta_{C1}$ [%] |           |
|-------------|-------------|----------------------|----------------------|--------|------------------|-------------------|-----------|
|             |             |                      |                      |        |                  | $\beta^+$         | $\beta^-$ |
| 8LiB        | CKPOT/CD    | 0.58                 | 3.599                | 6.205  | 9.051            | -7.597            | -10.775   |
| 9LiC        | CKPOT/CD    | 0.604                | 0.586                | 0.970  | 4.665            | -6.082            | -4.785    |
| 12BN        | CKPOT/CD    | 0.96                 | 1.893                | 1.972  | 13.467           | -4.926            | -5.602    |
| 13BO        | CKPOT/CD    | 1.376                | 0.462                | 0.336  | 8.782            | -1.489            | -3.589    |
| 17NNe       | REWIL/CD    | 0.553                | 1.952                | 3.530  | 1.189            | 49.436            | 13.602    |
| 20FNa       | REWIL/CD    | 0.882                | 1.029                | 1.167  | 6.242            | 12.516            | 6.156     |
| 20OMg       | REWIL/CD    | 1.37                 | 1.017                | 0.742  | 2.1              | 11.32             | 9.079     |
| 21FMg       | REWIL/CD    | 0.452                | 1.058                | 2.341  | 4.657            | 8.775             | 5.967     |
| 22OSi       | REWIL/CD    | 0.245                | 1.878                | 7.665  | 0.673            | 127.78            | 164.147   |
| 24NeSi      | REWIL/CD    | 0.877                | 1.513                | 1.725  | 1.804            | 19.496            | 0.694     |
| 25NaSi      | USD/CD      | 0.423                | 0.142                | 0.336  | 1.995            | 0.937             | 1.973     |
| 26NaP       | USD/CD      | 0.923                | 0.068                | 0.074  | 1.224            | 2.934             | 0.332     |
| 28AlP       | USD/CD      | 0.81                 | 0.053                | 0.065  | 5.592            | 1.188             | 0.566     |
| 31AlAr      | USD/CD      | 0.894                | 1.246                | 1.394  | 2.252            | 6.657             | -1.66     |
| 35SK        | USD/CD      | 0.471                | 0.125                | 0.265  | 2.625            | 2.414             | 0.71      |
| 35PCa       | USD/CD      | 1.059                | 0.661                | 0.624  | 2.89             | 2.441             | -2.902    |

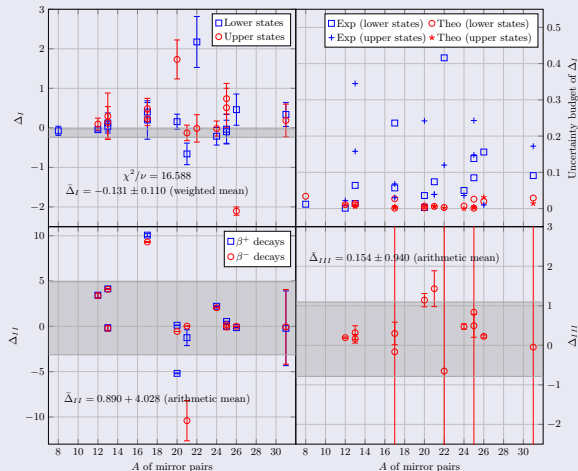
- Strong dependence on  $\eta$  is clearly observed. The sensitivity to  $\alpha$  is relatively weaker. This behavior is consistent with the results of the two-state model
- $\delta_{C1}$  is sometime constructive (carrying negative sign)
- Its magnitude spans over a wide range (from 0 to 164 % because of  $\eta$  enhancement)

*Xayavong and Lim, PRC111, 015501 (2025)*

- We obtain a remarkable agreement for  $\Delta_I$  and  $\Delta_{III}$ . Significant discrepancies persist only in a few cases
- The test based on  $\Delta_{II}$  is less precise due to the presence of  $\eta^2$  (isoscalar quantity!).
- In cases where the corrections are small,  $\Delta_{II}$  may instead serve as an experimental means to extract  $\eta^2$  (test of isoscalar interaction)
- Experimental uncertainties dominate

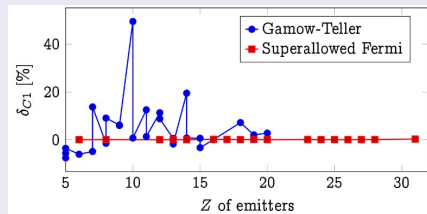
Xayavong and Lim, PRC111, 015501 (2025)

## Experimental tests of shell-model calculations

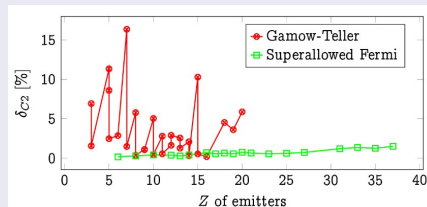


- Comprehensive study of GT mirror asymmetry within the shell-model approach.
- GT matrix elements are very sensitive to isospin-symmetry breaking (ISB) effects.
- This present study supports the reliability of the shell-model predictions employed in superallowed  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decay, where ISB effects are comparatively less pronounced.
- GT data can serve as constraints for the future development of isospin nonconserving (INC) interactions
- Further investigation using charge-exchange reactions may be possible

## GT vs Fermi



## GT vs Fermi



Thank You