Intertwined Orders and the Physics of High Temperature Superconductors

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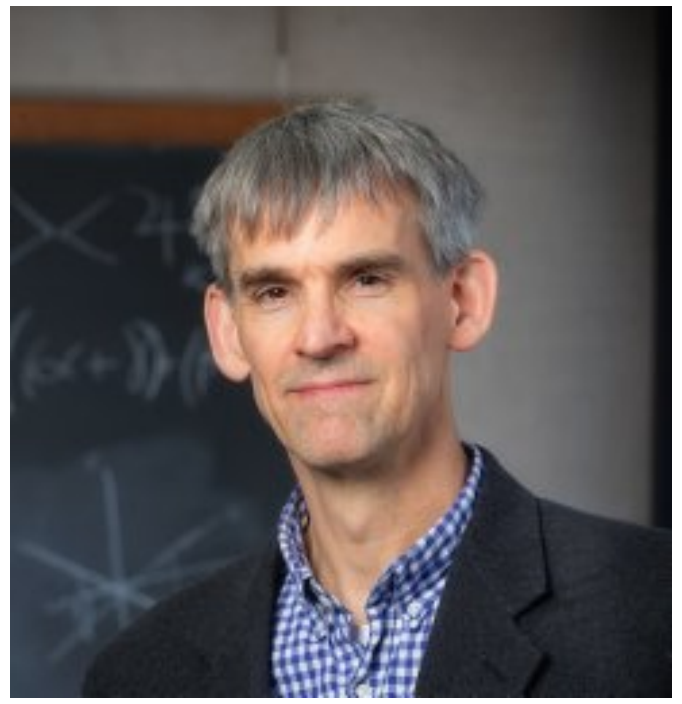
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Outline

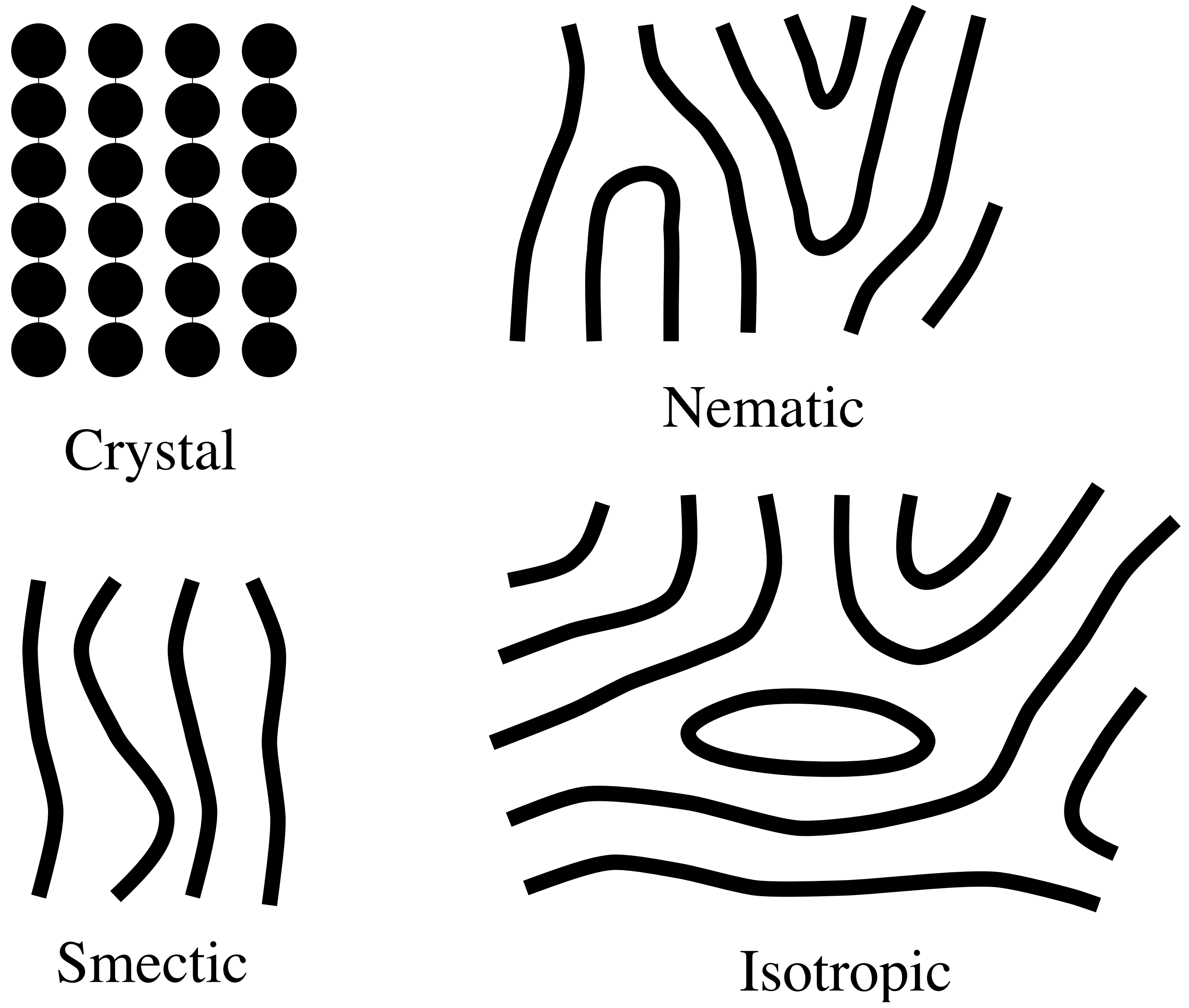
- The Problem of High Temperature Superconductors
- The Competing Orders Scenario
- Intertwined Orders
- Electronic Liquid Crystal Phases and Frustrated Phase Separation
- The Role of Quasi-two-dimensionality
- Pair-Density-Wave Order: an Intertwined Order
- Outlook

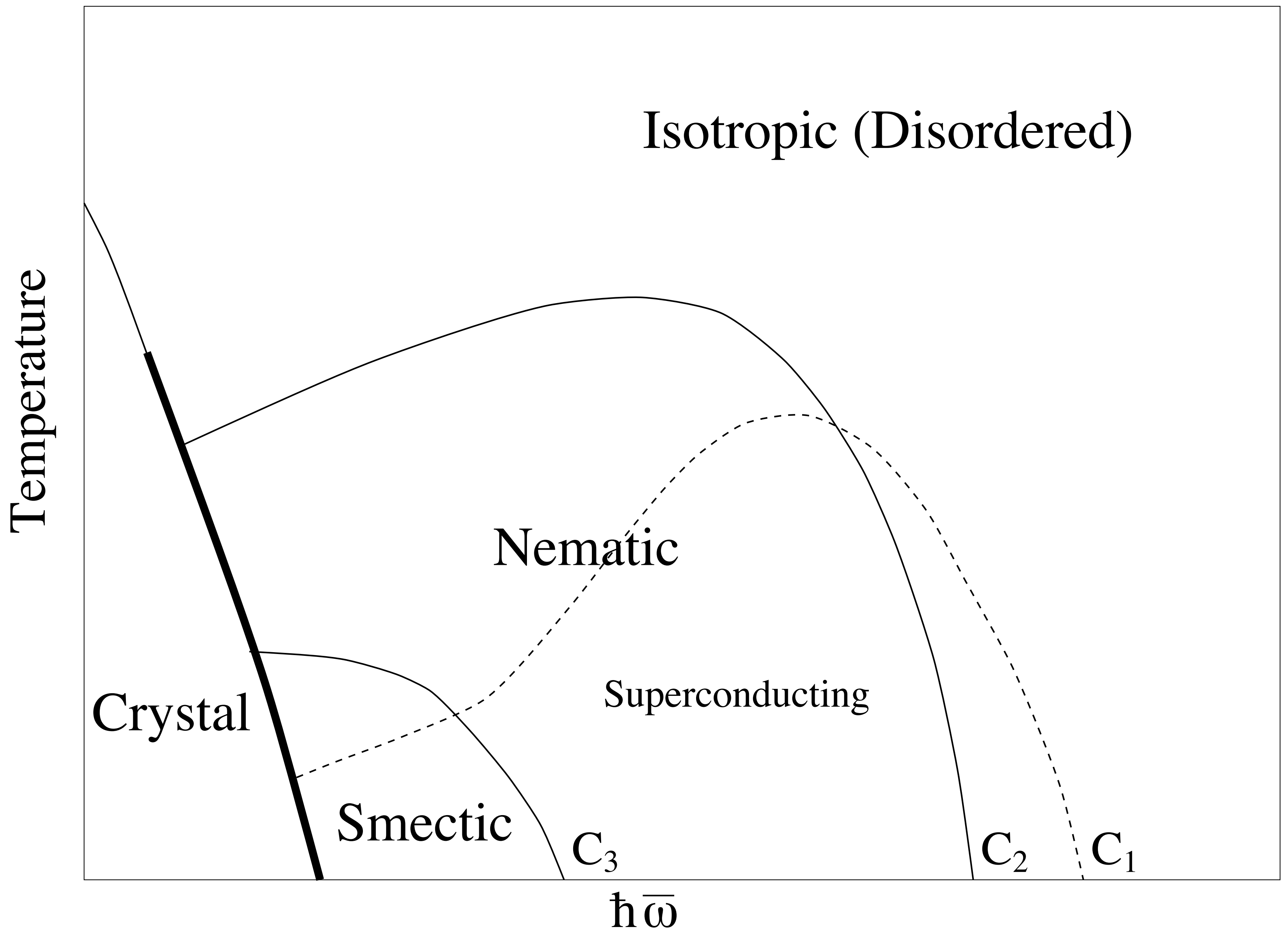
Why is High Temperature Superconductivity different

- Most high T_c superconductors are quasi-two dimensional materials
- The "normal" phase is not a good metal: the electronic quasiparticles are not well defined, $R \propto T$, etc.
- The dominant interactions are strong and repulsive
- The SC state arises from doping a strongly correlated Mott insulator
- Several other ordered (or almost ordered) states are also exist

Electronic Liquid Crystal Phases of Doped Mott Insulators

S. A. Kivelson, E. Fradkin and V. J. Emery (1998)





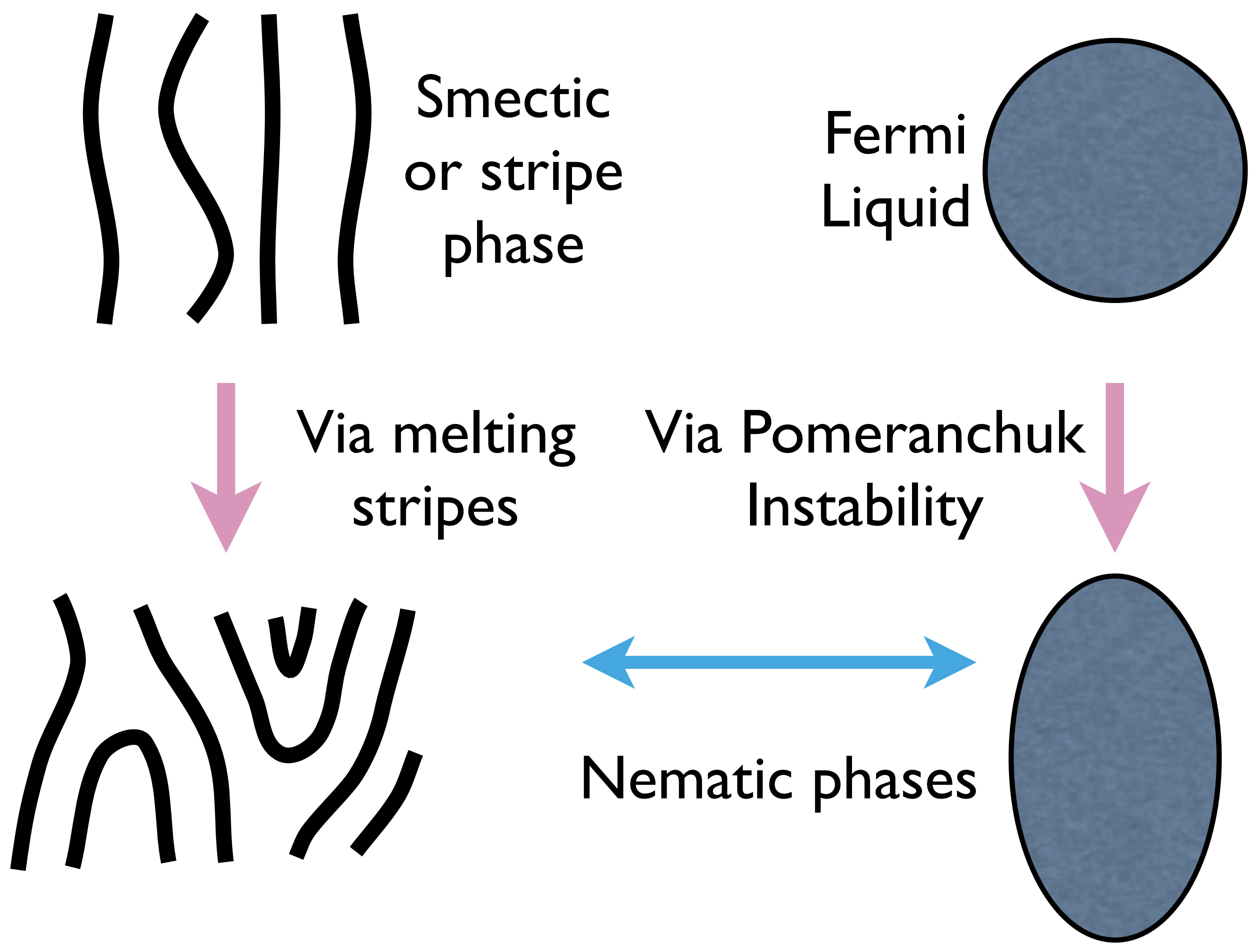
- Break translation and rotational invariance to varying degrees
- Smectic (stripe): breaks translational symmetry in one direction
- Nematic: uniform and anisotropic metallic or SC phase; breaks the point group symmetry
- How are they related to SC?

Electronic Nematic Fluids

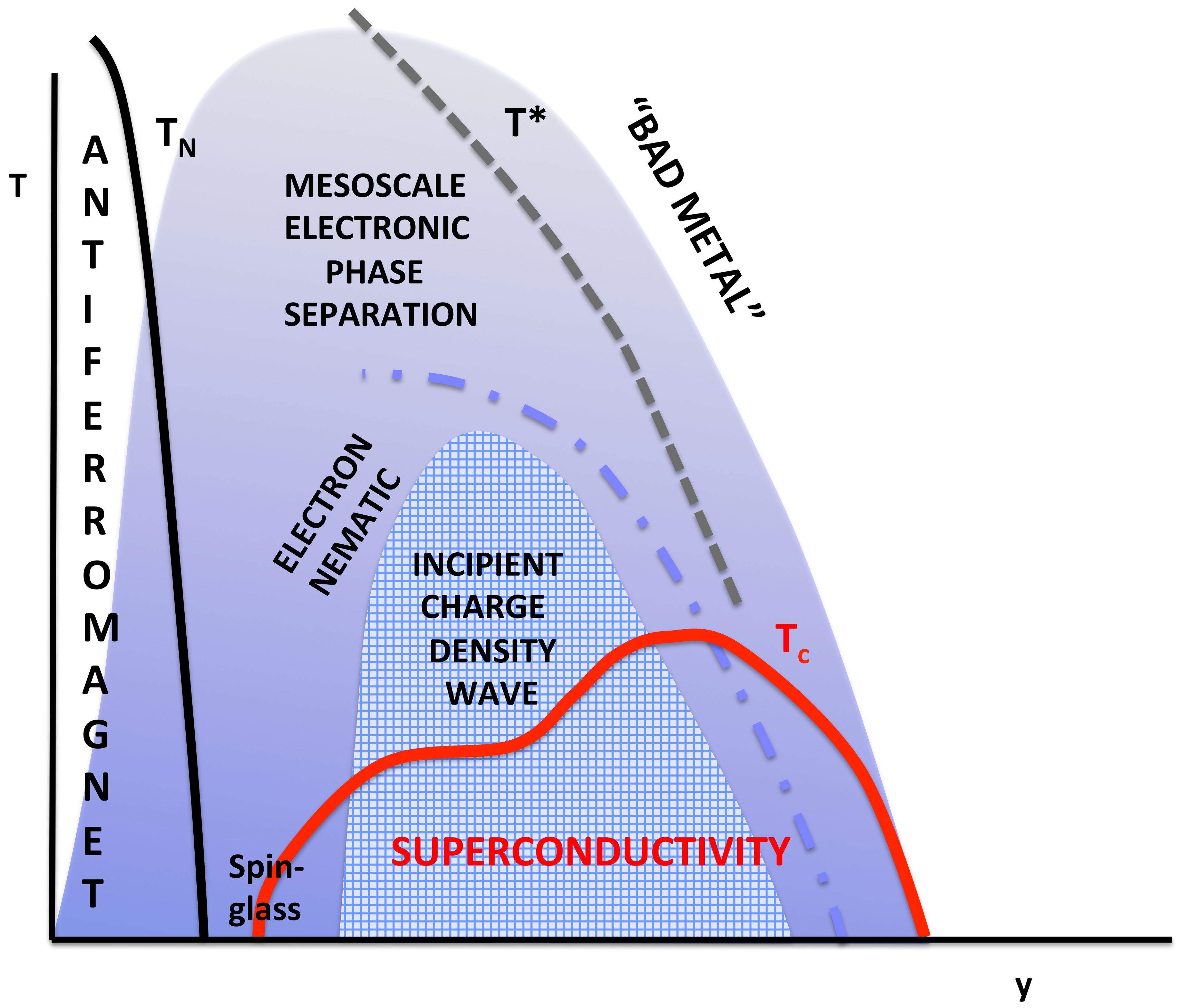
Kivelson, Fradkin, Emery, 1998; Fradkin and Kivelson 1998; Oganesyan, Kivelson and Fradkin (2001)

- Uniform phase of a fluid that breaks spontaneously rotational invariance
- Order parameter: traceless symmetric tensor (in 2D this is equivalent to a director)
- In an electronic system a nematic state is a phase with a large, sharp temperature-dependent transport anisotropy (2DEG near ν=9/2; $Sr_3Ru_2O_7$ for H~7-8T; YBCO in the pseudogap regime; $BaFe_2As_2$,FeSe)
- Two pathways to an electronic nematic state: a) by melting a charge stripe phase, and b) by a Pomeranchuk instability of a Fermi liquid: particle-hole condensate in the quadrupolar, ℓ=2, channel)

$$\mathcal{Q}(\boldsymbol{x}) = \frac{1}{k_F^2}\begin{pmatrix} \psi^\dagger(\boldsymbol{x})(\partial_x^2-\partial_y^2)\psi(\boldsymbol{x}) & \psi^\dagger(\boldsymbol{x})2\partial_x\partial_y\psi(\boldsymbol{x}) \\ \psi^\dagger(\boldsymbol{x})2\partial_x\partial_y\psi(\boldsymbol{x}) & \psi^\dagger(\boldsymbol{x})(\partial_y^2-\partial_x^2)\psi(\boldsymbol{x}) \end{pmatrix}$$



The YBCO Phase Diagram



It applies generally to all the cuprates, LBCO, YBCO, LSCO… Fe SCs, Kagome SCs, UTe_2

- T_c vs x has a plateau with an anomaly at "1/8" (Bonn & Hardy)
- INS finds nematic order for y~6.45 with T_{cN}~150 K
- NMR measurements at high fields finds a charge stripe signature in agreement with quantum oscillations (dHvA) with a T_{cdw}~60 K
- Transport and Peltier experiments find nematic order in the pseudogap regime, with T_V~150 K
- RIXS, X ray diffraction, and ultrasound anomalies find static stripe charge order near x=1/8 with a T_{cdw}~150 K

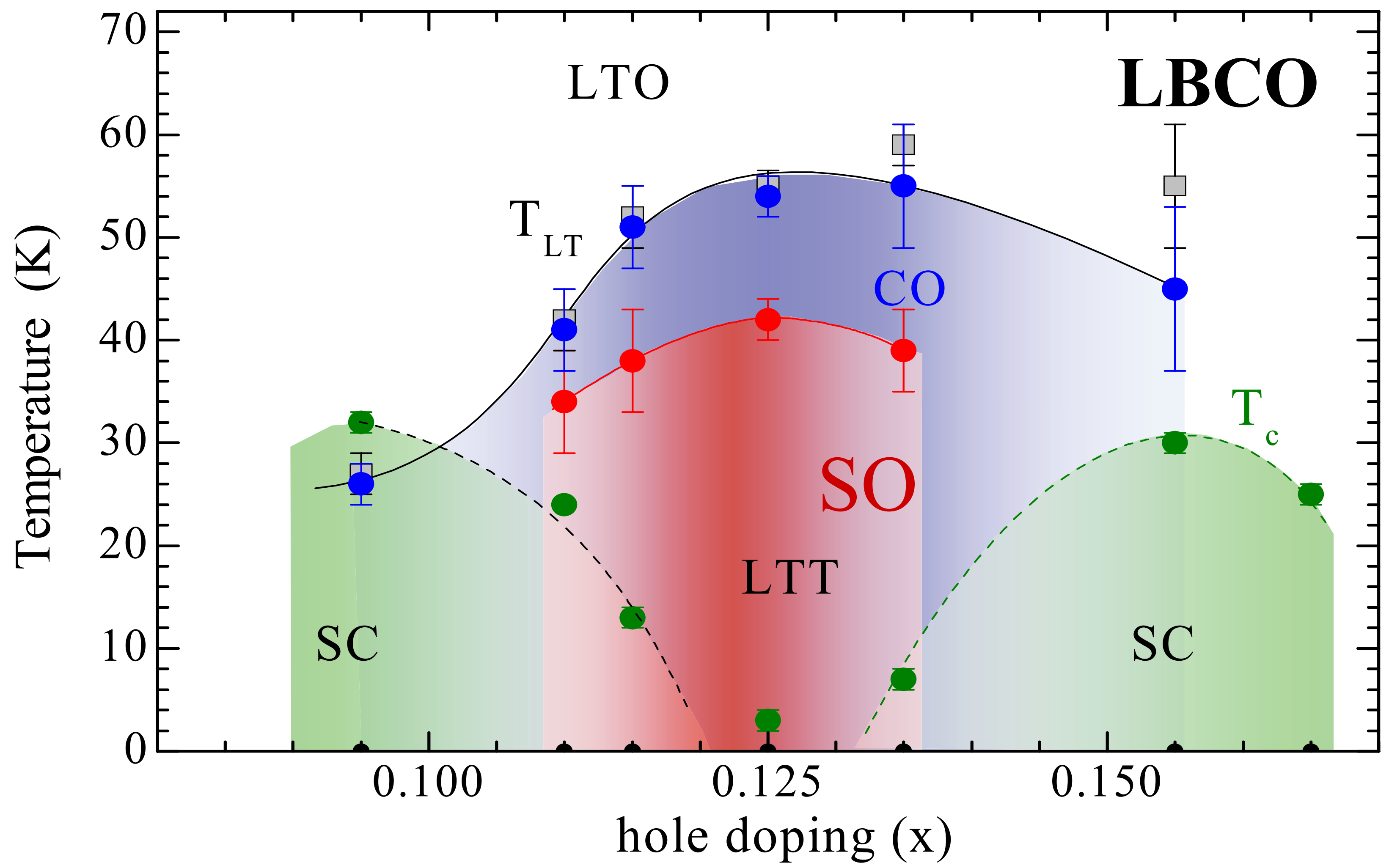
The Competing Orders Scenario

- Landau Theory of Phase Transitions with several order parameters: one order is strongest and the others are suppressed
- T_c's for the different phases are quite different
- Regimes in which orders have similar T_c's are exceptional and require fine tuning (multicritical point)
- Why are the T_cs of different orders are comparable without fine tuning?

Why are the orders of comparable strength?

- This is difficult to understand in terms of the competing order scenario
- This fact suggests that all the observed orders may have a common physical origin and are **intertwined**
- Strong hint: electronic inhomogeneity. STM sees stripe and nematic local order in exquisite detail in BSCCO on a broad range of temperatures, voltage and field
- This phenomenology is natural in doped Mott insulators: frustrated phase separation
- Electronic liquid crystal phases have also been seen many unconventional superconductors

PDW SC in $La_{2-x}Ba_xCuO_4$ @ x~1/8



- LBCO: low energy stripe fluctuations

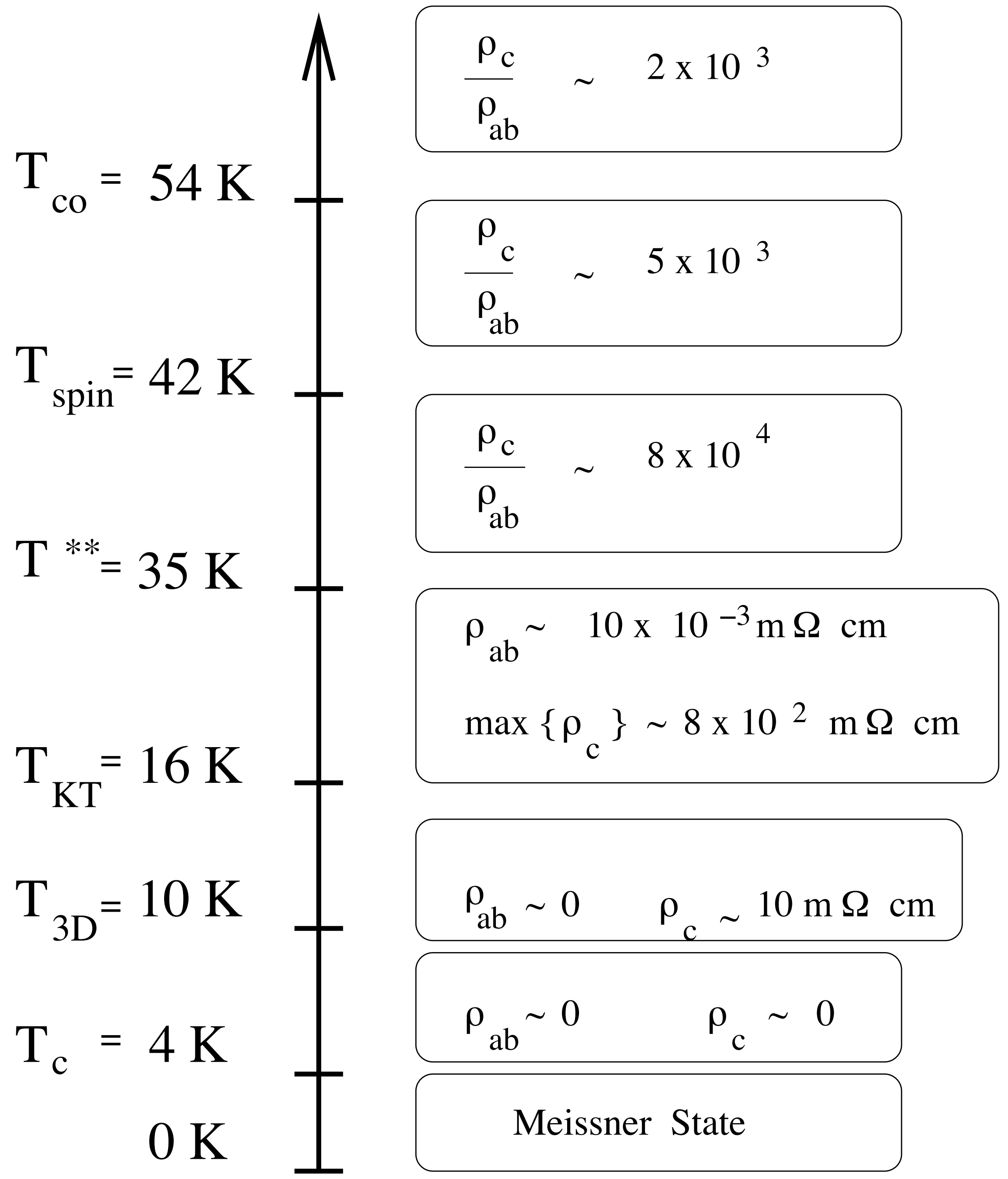
- Very low T_c @ x=1/8 with static stripe order

- SC layer decoupling at x=1/8 (Li et al, 2007)

- Also seen in underdoped LSCO in magnetic fields where INS saw a field induced SDW (Basov 2010)

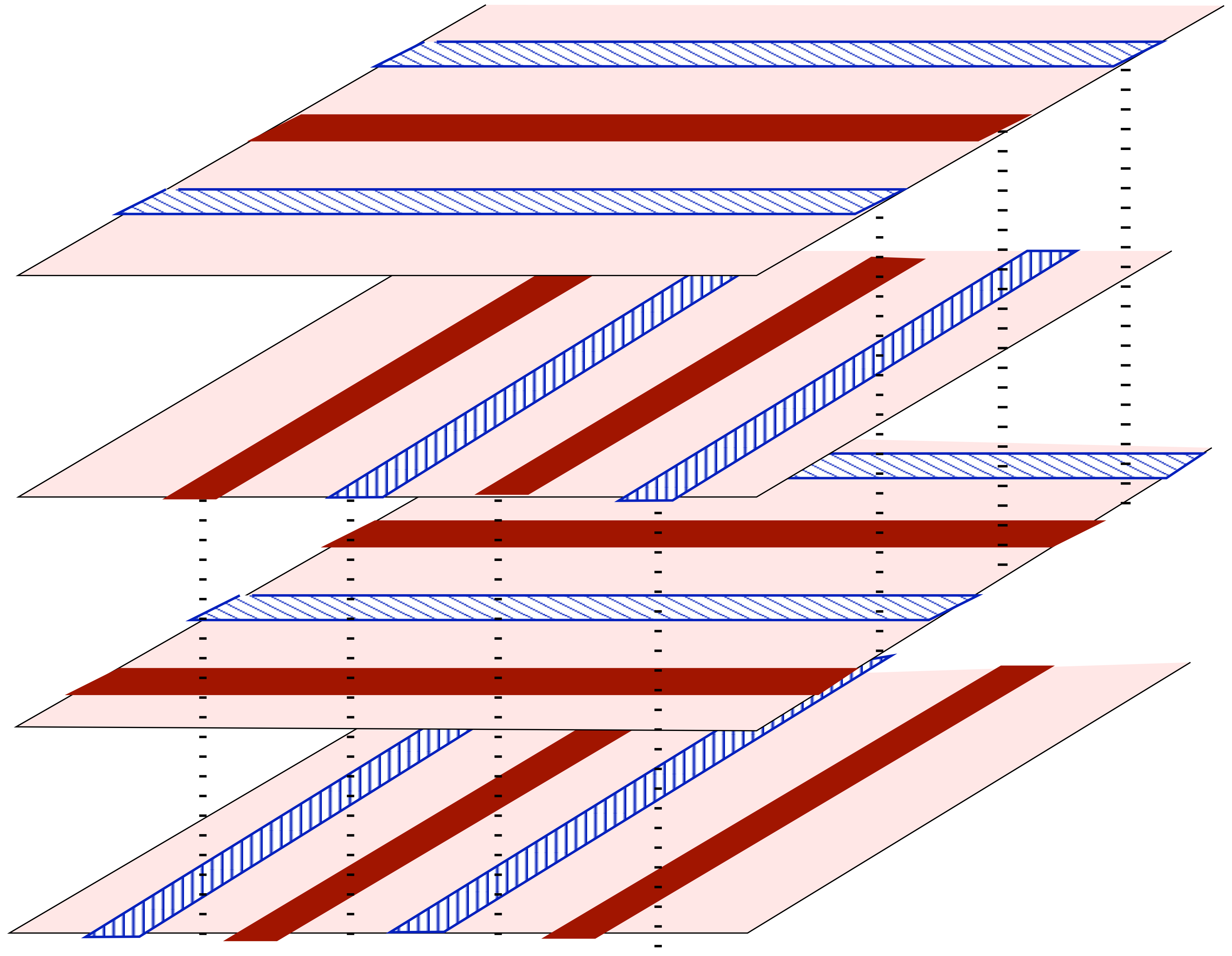
- Can only happen if a special symmetry of the SC in the striped state frustrates the c-axis Josephson coupling.

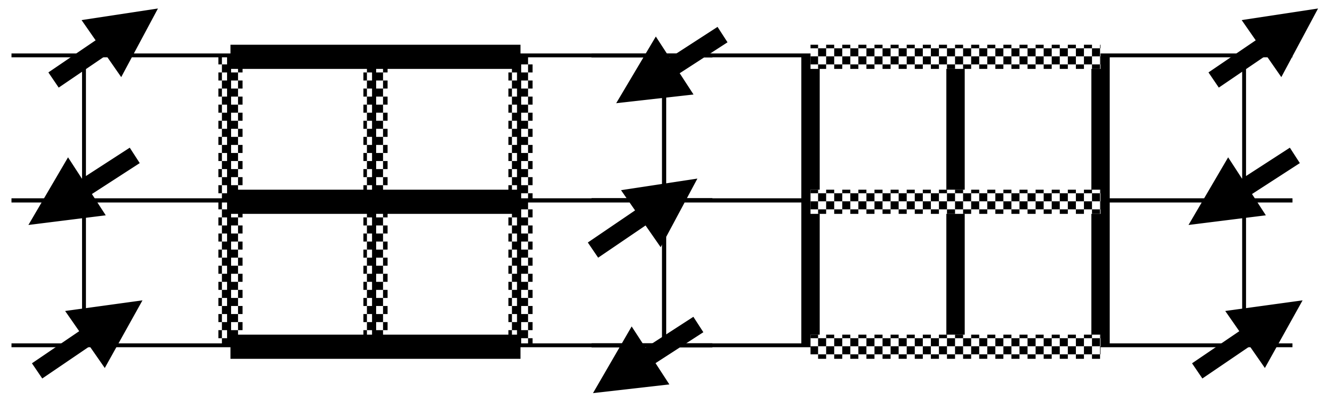
- We conjectured a novel striped superconducting state, a Pair Density Wave, in which CDW, SDW, and d-wave SC orders are intertwined! (Berg et al, 2007, 2008)

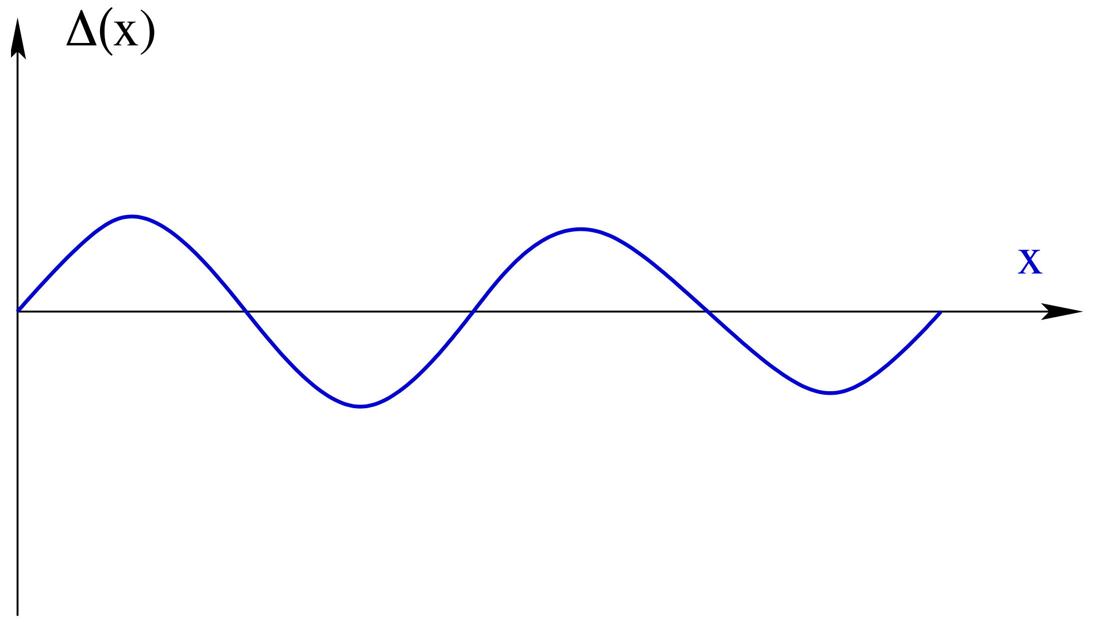


Prototype of Intertwined Orders: the PDW State

Berg, EF, Kim, Kivelson, Oganesyan, Tranquada, Zhang, PRL 2007







- PDW locally is a d-wave SC
- π phase shift

- Intertwined striped charge, spin and superconducting orders.
- Found in variational Monte Carlo (Himeda *et al* 2004), MFT (Poilblanc *et al* 2007), and iPEPS (Corboz *et al*, PRL 2014)
- Has not yet been found in DMRG on 4 leg Hubbard and t-J ladders

Order Parameters of the PDW

- Local pairing amplitude: $\Delta(\mathbf{r})=\Delta_0(\mathbf{r})+\Delta_{\mathbf{Q}}(\mathbf{r})\, e^{i\,\mathbf{Q}.\mathbf{r}}+\Delta_{-\mathbf{Q}}(\mathbf{r})\, e^{-i\mathbf{Q}.\mathbf{r}}$
- Unidirectional PDW: $\Delta_0=0$, $|\Delta_{\mathbf{Q}}|=|\Delta_{-\mathbf{Q}}|$ (LO SC without a B field)
- Complex charge 2e singlet pair condensate with wave vector $\mathbf{Q}$
- Two complex SC order parameters: $\Delta_{\mathbf{Q}}(\mathbf{r})$ and $\Delta_{-\mathbf{Q}}(\mathbf{r})$
- Two amplitude (Higgs) modes and two phase fields
- Charge stripe: $\rho_{\mathbf{K}}$, unidirectional charge stripe with wavevector $\mathbf{K}$
- Spin stripe order parameter: $\mathbf{S}_{\mathbf{Q}}$, charge neutral complex spin vector order parameter
- PDW has a “Fermi surface” of charge-neutral Bogoliubov qp’s

Ginzburg-Landau Theory
(Berg, EF, Kivelson, Tranquada, 2009)

- The quadratic and quartic terms are standard
- Interesting cubic terms allowed if **K**=2**Q** ("2Q" CDW)

$$F_3 = \gamma_\Delta \rho^*_{\mathbf{K}} \Delta^*_{-\mathbf{Q}} \Delta_{\mathbf{Q}} + \mathrm{c.c.}$$

- However, in the Landau theory orders do not have comparable strength **unless** all the parameters are fine-tuned to a multicritical point with **very large** symmetry!
- Composite order parameters

$$\rho_{\mathbf{K}} \sim \Delta^*_{-\mathbf{Q}} \Delta_{\mathbf{Q}} \qquad \Delta_{4e} \sim \Delta_{-\mathbf{Q}} \Delta_{\mathbf{Q}} \qquad \rho_{\mathbf{Q}} \sim \Delta^*_0 \Delta_{\mathbf{Q}}$$

If PDW and uniform SC coexist: a "1Q" CDW order must appear

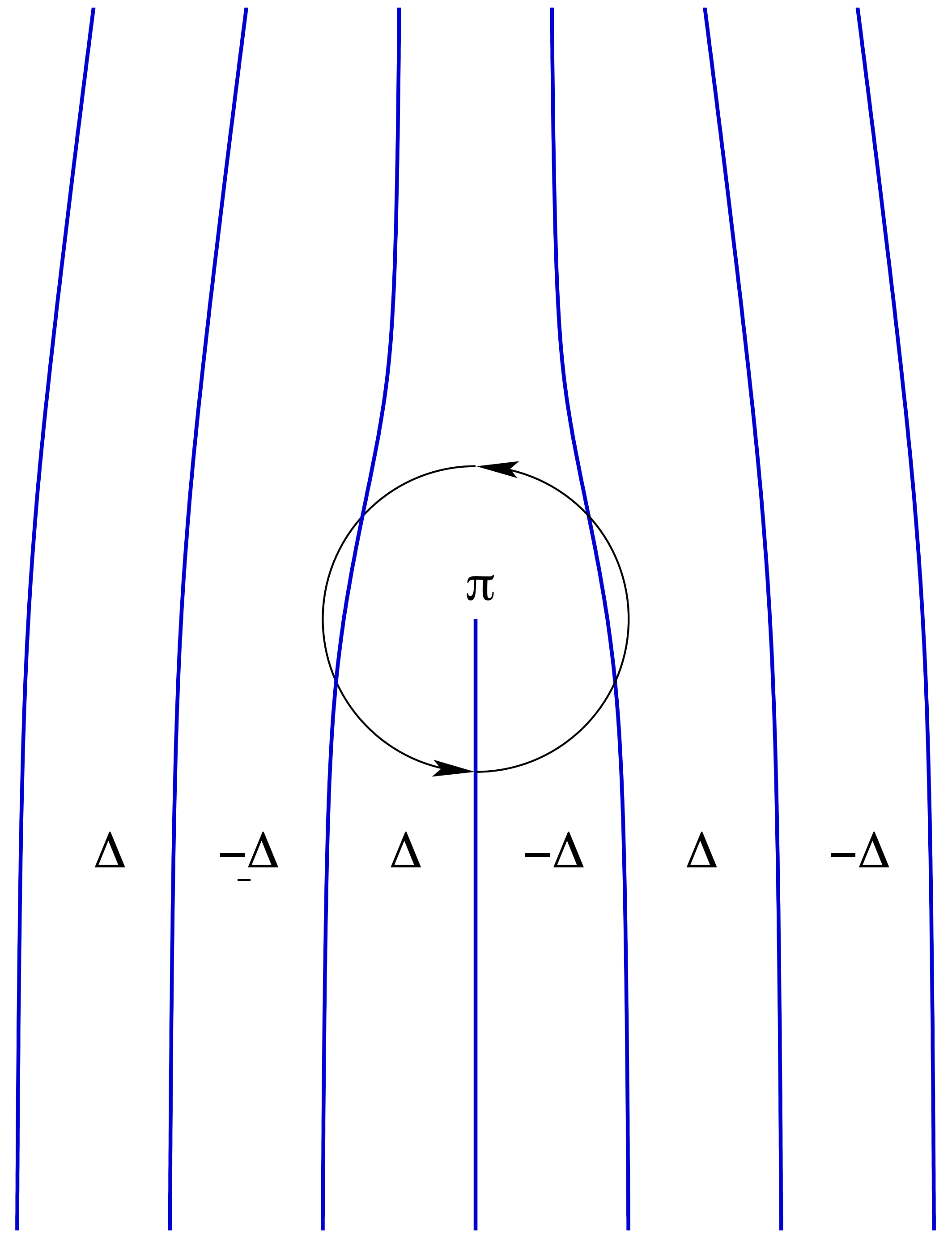
Topological Excitations of the PDW SC

E. Berg. E. Fradkin and S.A. Kivelson (2009)

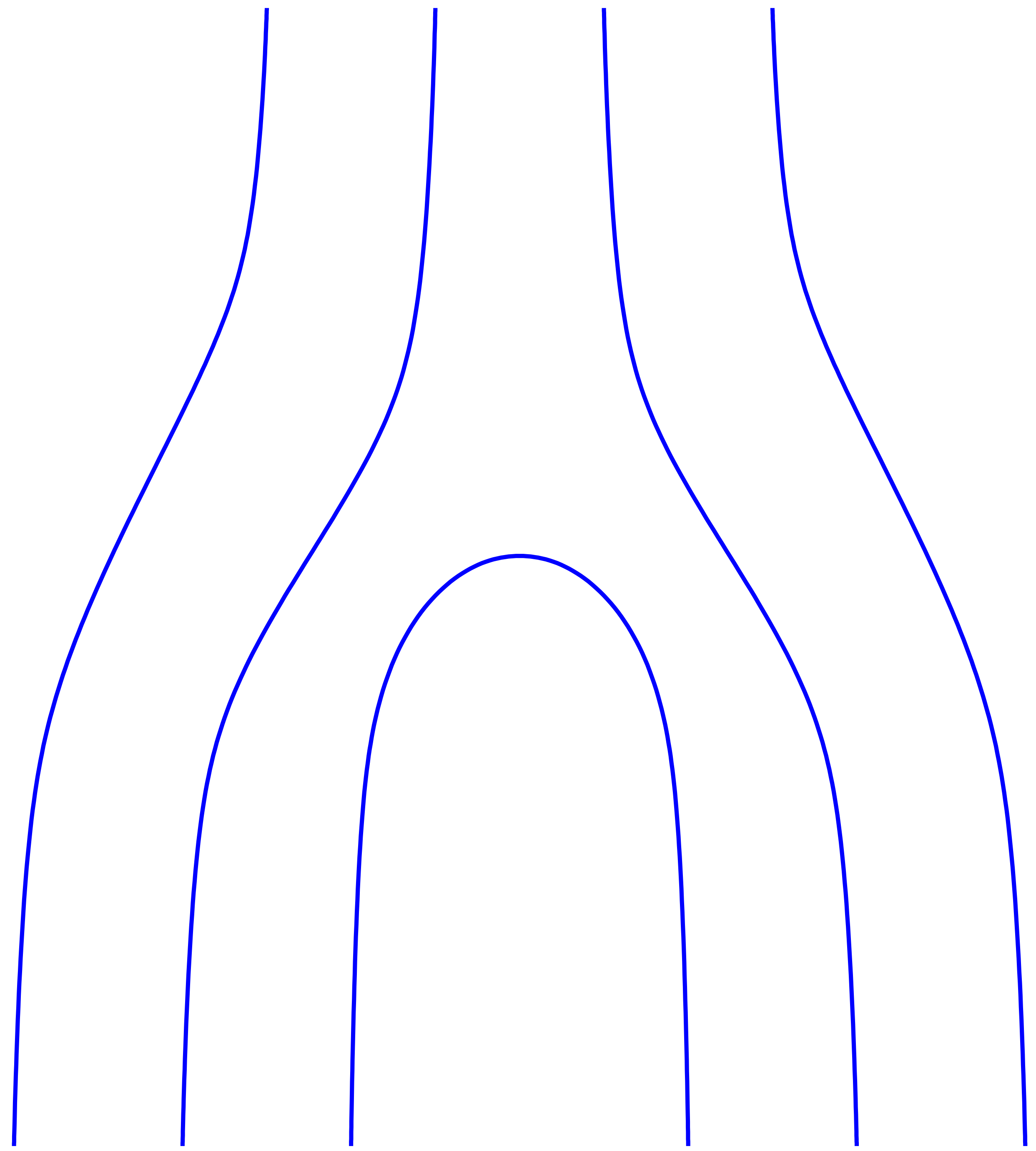
- Strongly layered system: 2D thermal *phase fluctuations* play a key role
- Unidirectional PDW SC
- Two phase fields: the SC phase $\theta_+=(\theta_Q+\theta_{-Q})/2$ and the CDW phase $\phi=\theta_Q-\theta_{-Q}$
- $H=(\rho_s (\nabla\theta_+)^2 + \kappa (\nabla\phi)^2)/2$
- SC vortex with $\Delta\theta_+ = 2\pi$ and $\Delta\phi=0$
- Bound state of a 1/2 vortex and a CDW dislocation

  $\Delta\theta_+ = \pi, \Delta\phi= 2\pi$
- Double CDW dislocation, $\Delta\theta_+ = 0, \Delta\phi= 4\pi$

Topological Excitations of the PDW



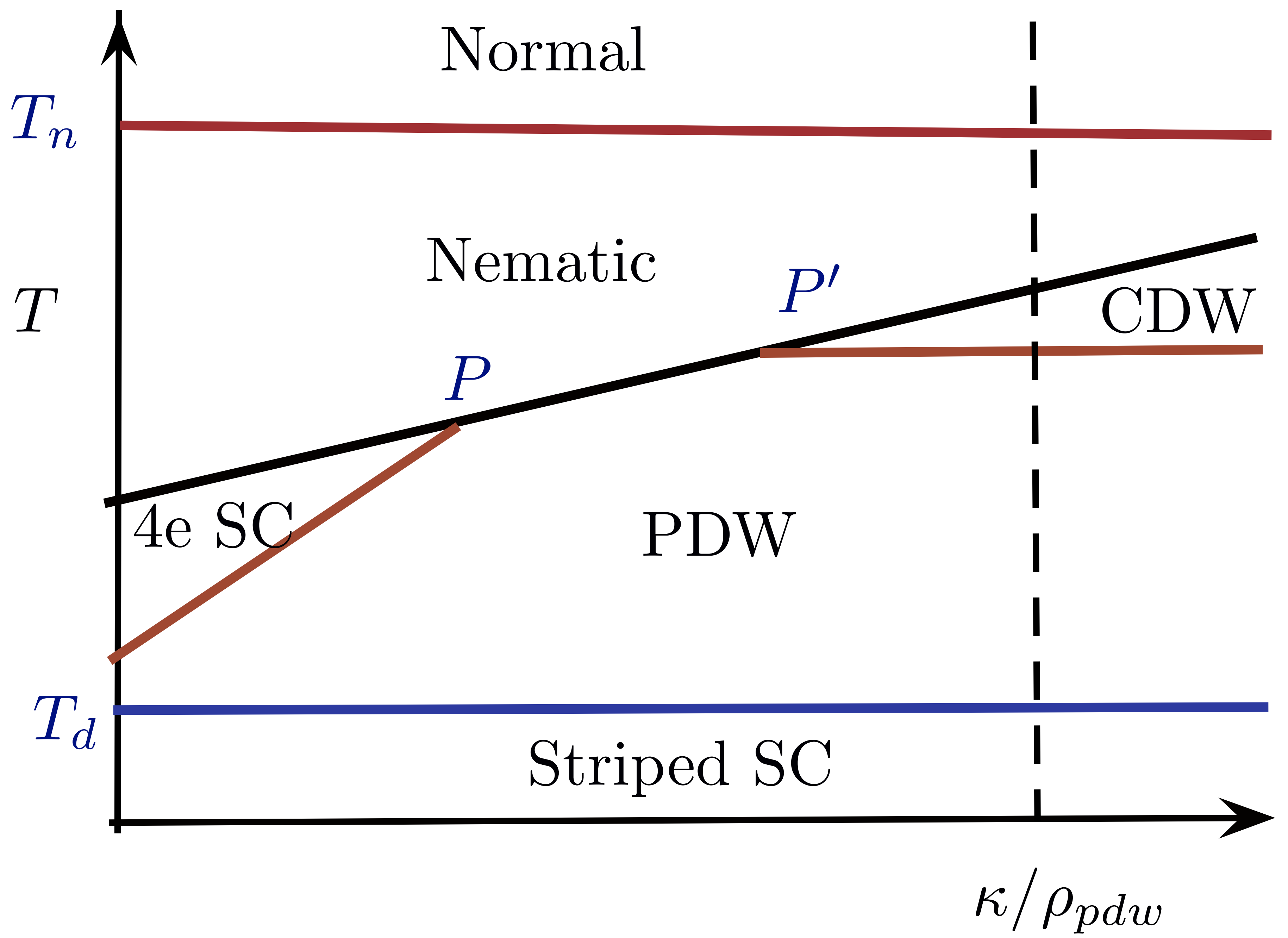
Half-vortex and a Dislocation



Double Dislocation

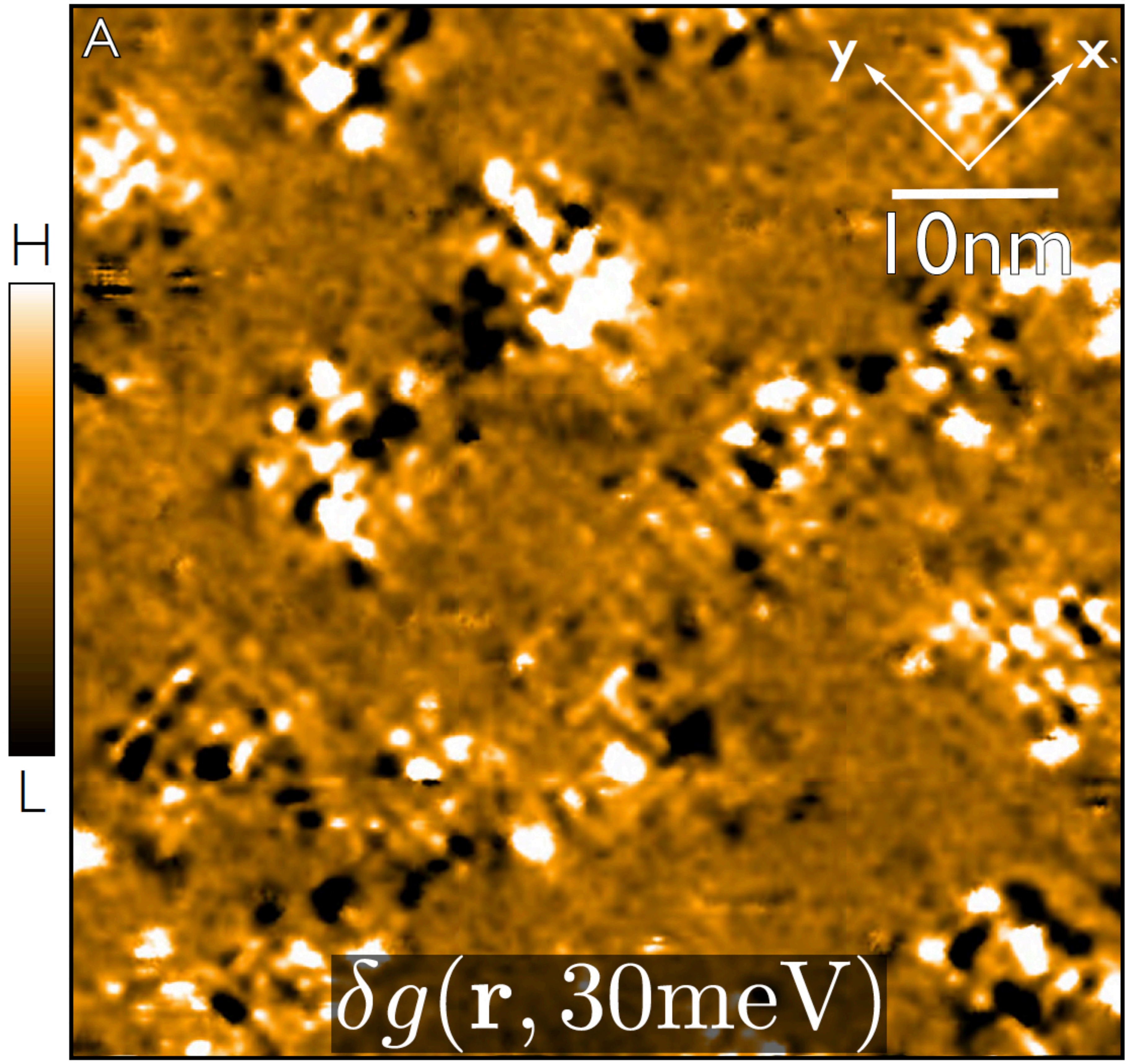
Thermal melting of the PDW state

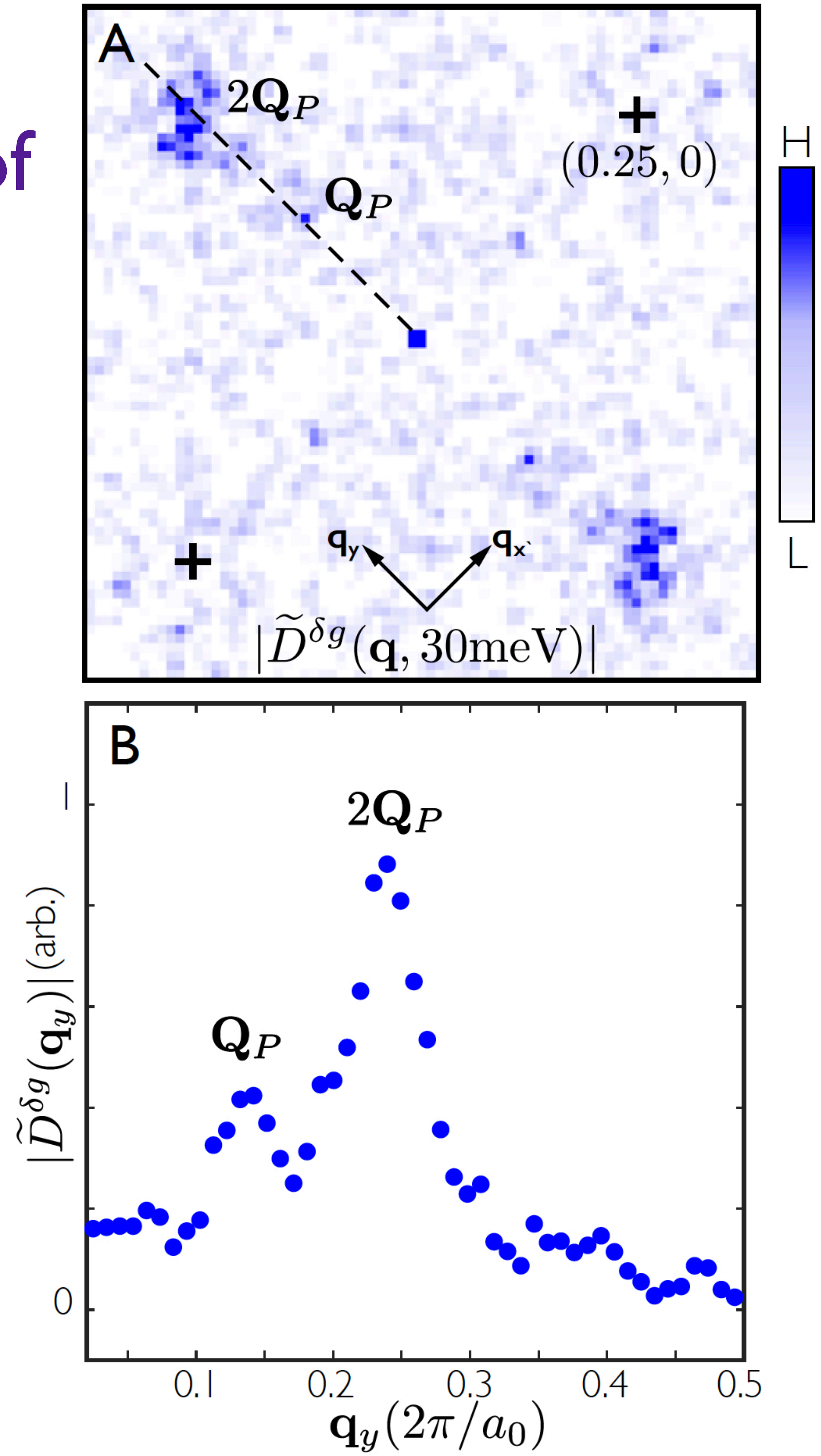
- Three types of topological excitations: SC vortex), double dislocation, 1/2 vortex-single dislocation
- Three pathways for thermal melting
- KT phase transitions by proliferation of topological defects
- Phases: PDW, Charge 4e SC, CDW, and normal (Ising nematic)



Comparable T_c's without fine tuning!

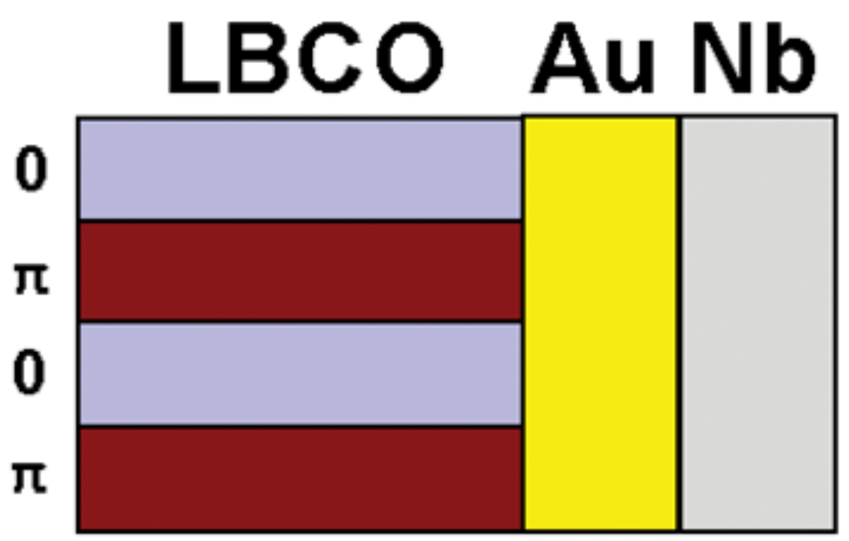
STM spectra in the vortex halos of BSCCO (Edkins et al, 2018)

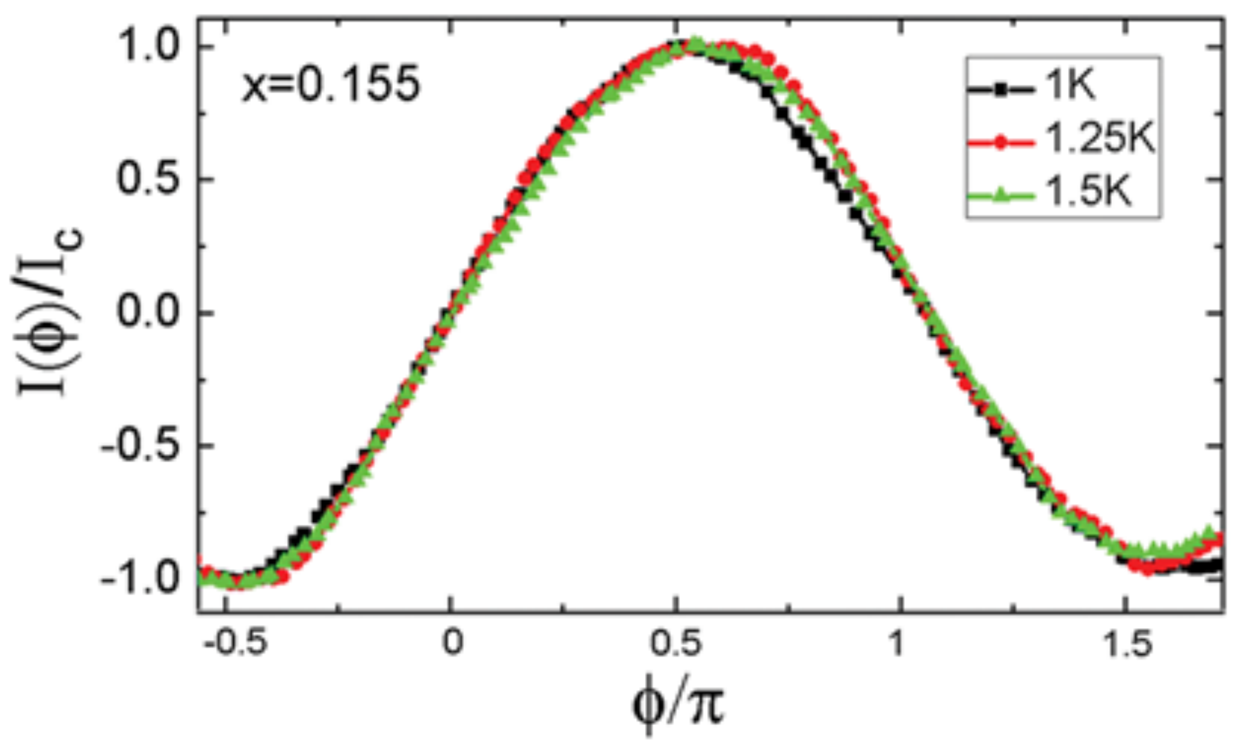


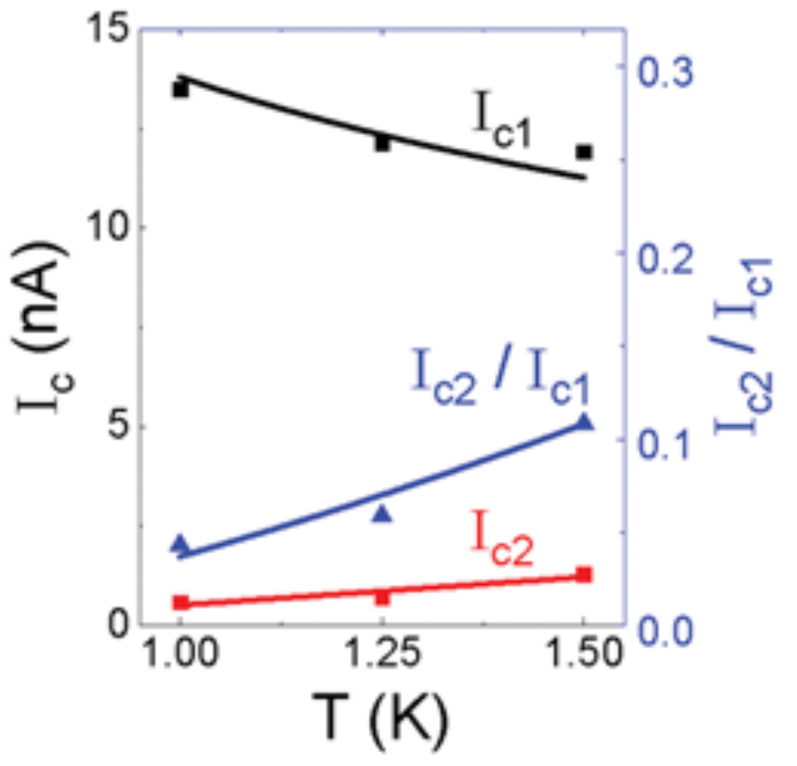


Measurement of the CPR relation and detection of D-wave and PDW SC Order in Low T LBCO @ x=1/8

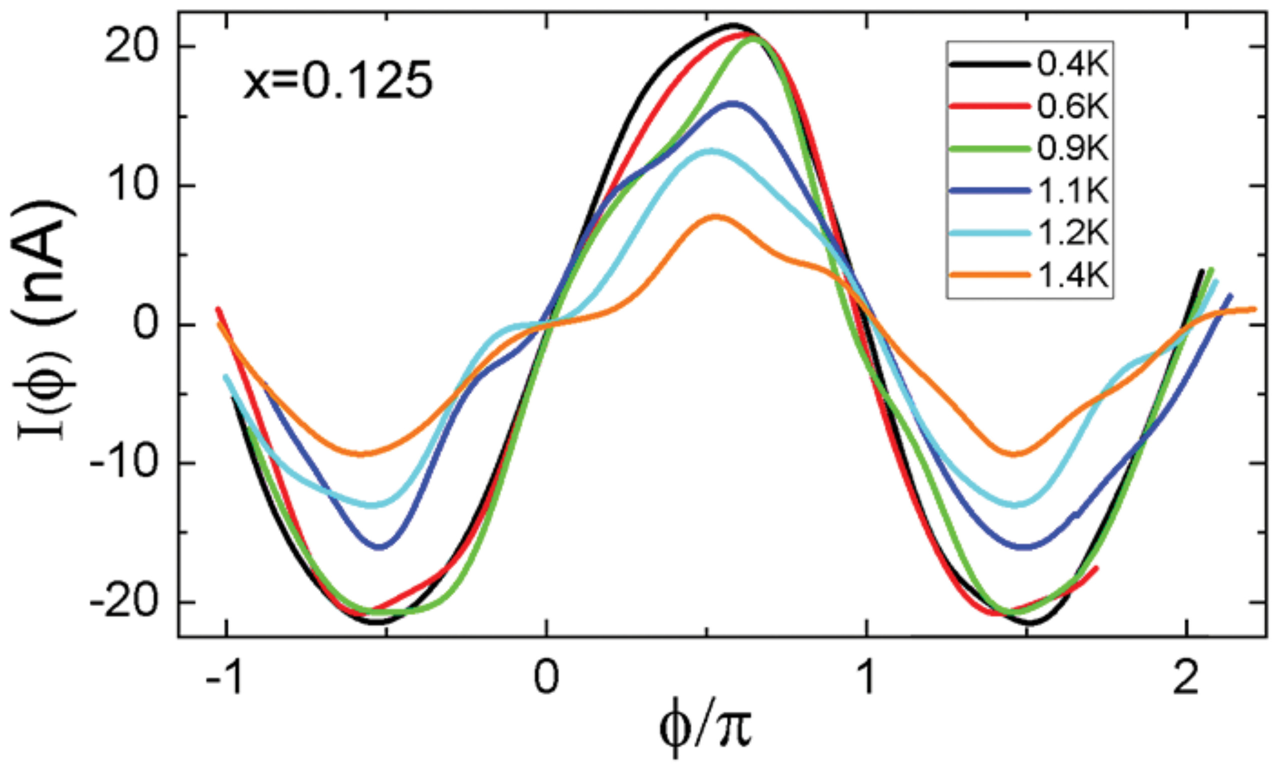
D. Hamilton, G. Gu, E. F., D. Van Harlingen, 2018

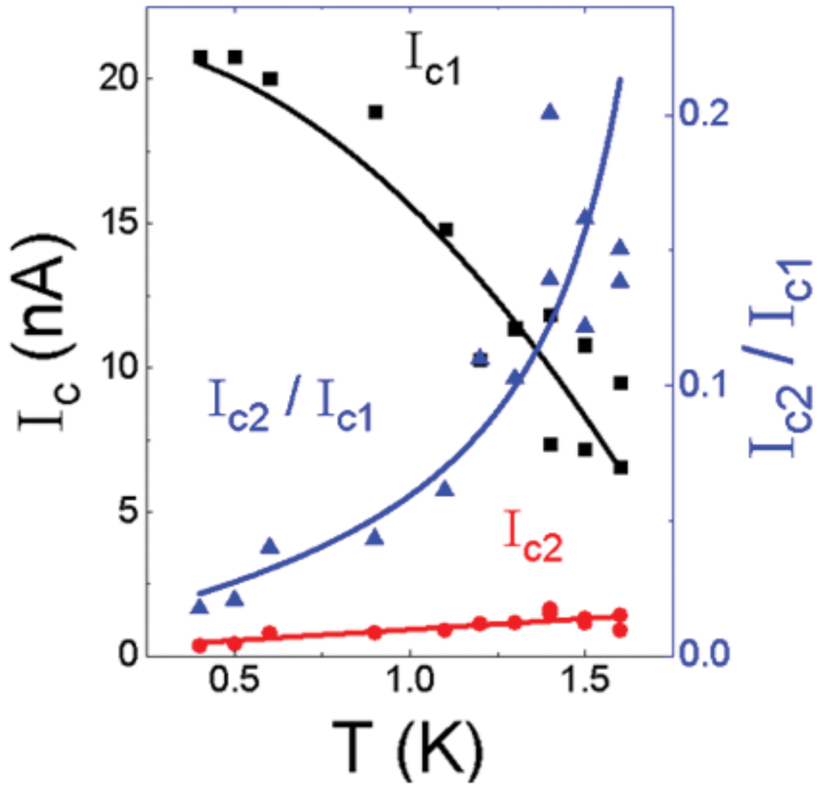






- Current-Phase relation in the Meisner state of LBCO
- d-wave SC
- @1/8 the $\sin 2\phi$ component grows as T↑
- the $\sin\phi$ decreases as T↓
- uniform d-wave SC and PDW coexist below Tc~4K



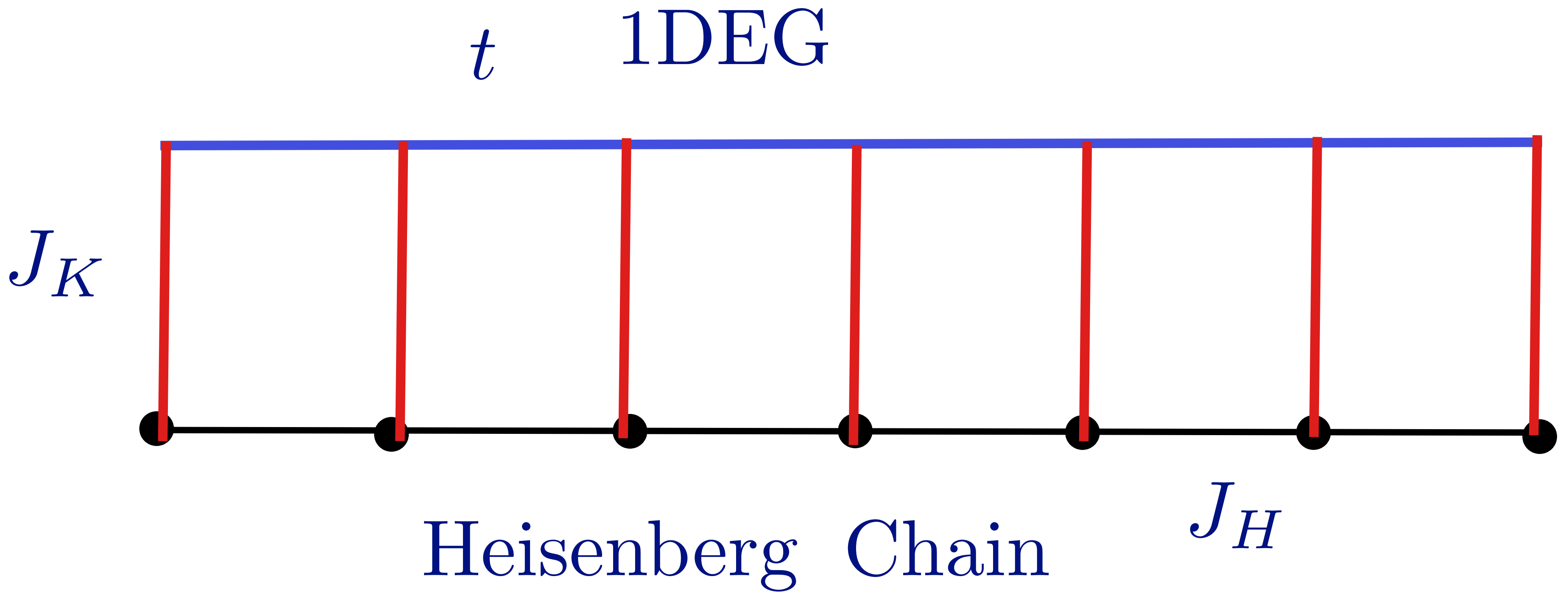


What is the status of microscopic theory?

- Intertwined ordres require theories in the medium to strong interaction regime
- BCS type approaches only work at weak coupling
- PDW states are found in BCS approaches only if the interactions are larger than the bandwidth of the lattice model where it should not work
- Determinantal QMC sees short range stripe order in the 2D Hubbard model at relatively high temperatures (sign problem)
- DMRG sees stripe phases and superconductivity in Hubbard and t-J models on cylinders of up to 6 legs
- iPEPS simulations in the 2D t-J model find three phases (uniform d-wave SC, SC coexisting with a stripe/CDW phase and a PDW) with energies that differ only in the 4th decimal place
- Nevertheless there is evidence for PDW phases in some models

The Kondo-Heisenberg chain and the in 2-leg extended Hubbard model

E. Berg, E. Fradkin, and S. Kivelson, PRL 2010; Jaefari & EF, PRB 2012



$$H_{\mathrm{KH}} = -t\sum_j \left(c^\dagger_{j\sigma}c_{j+1,\sigma} + \mathrm{h.c.}\right) + J_H\sum_j \mathbf{S}_j\cdot\mathbf{S}_{j+1} + J_K\sum_j \mathbf{S}_j\cdot\mathbf{s}(x_j)$$

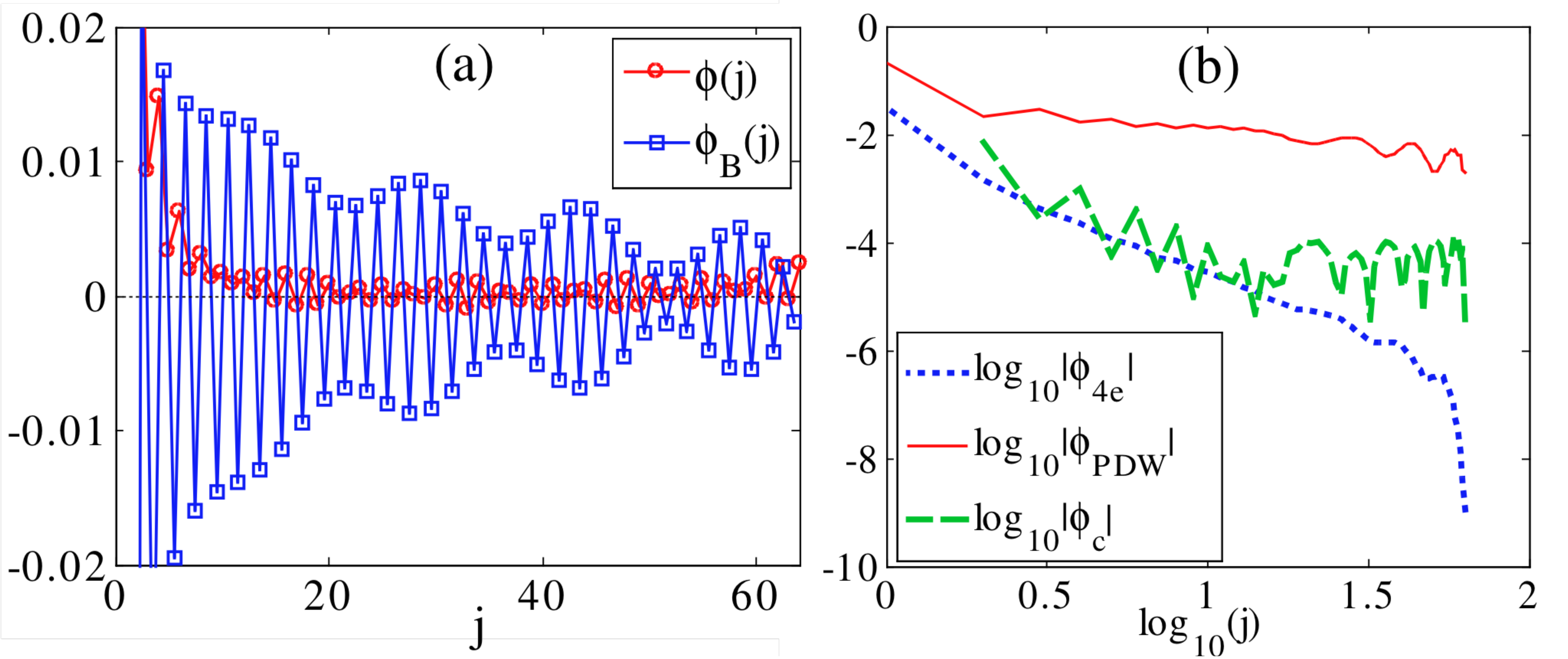
where $$\mathbf{s}(x_j) = \frac{1}{2}\sum_{\alpha,\beta=\uparrow,\downarrow} c^\dagger_{j,\alpha}\boldsymbol{\sigma}_{\alpha\beta}c_{j\beta}$$

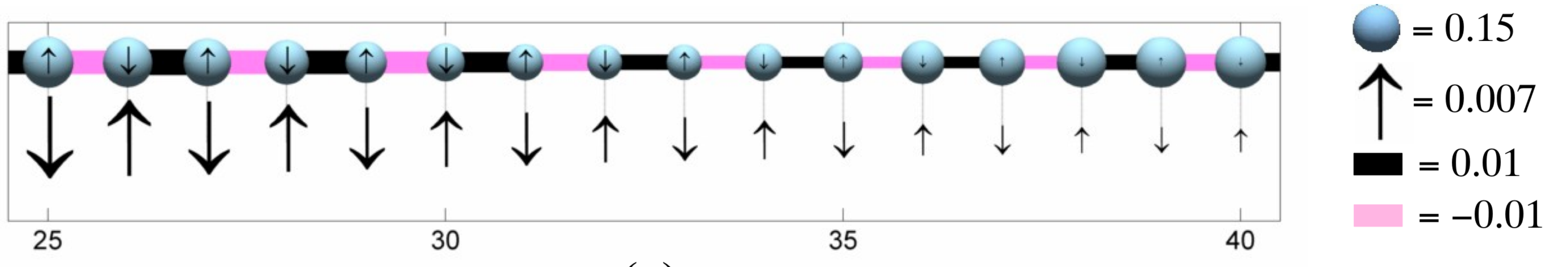
J_K is the Kondo coupling and J_H is the Heisenberg coupling

DMRG finds a commensurate PDW in a broad phase when $J_H > J_K$

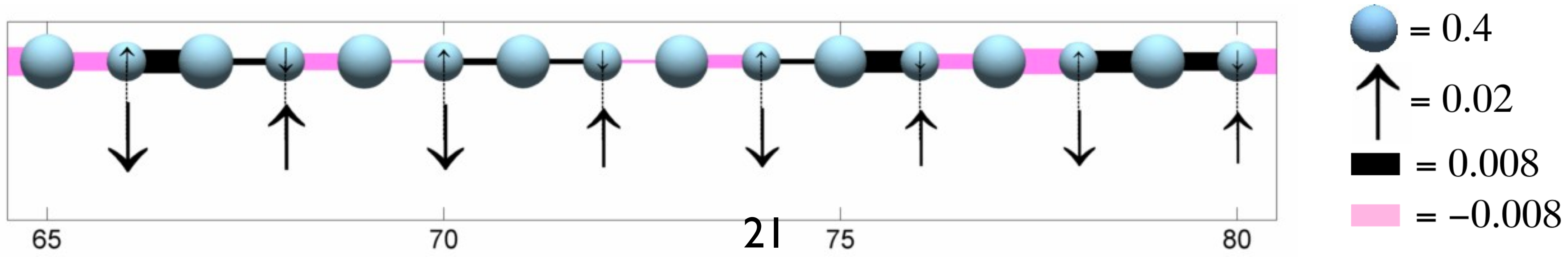
Bosonization yields the same result

Expectation Values of the Order Parameters as a function of distance from the left edge





(a)

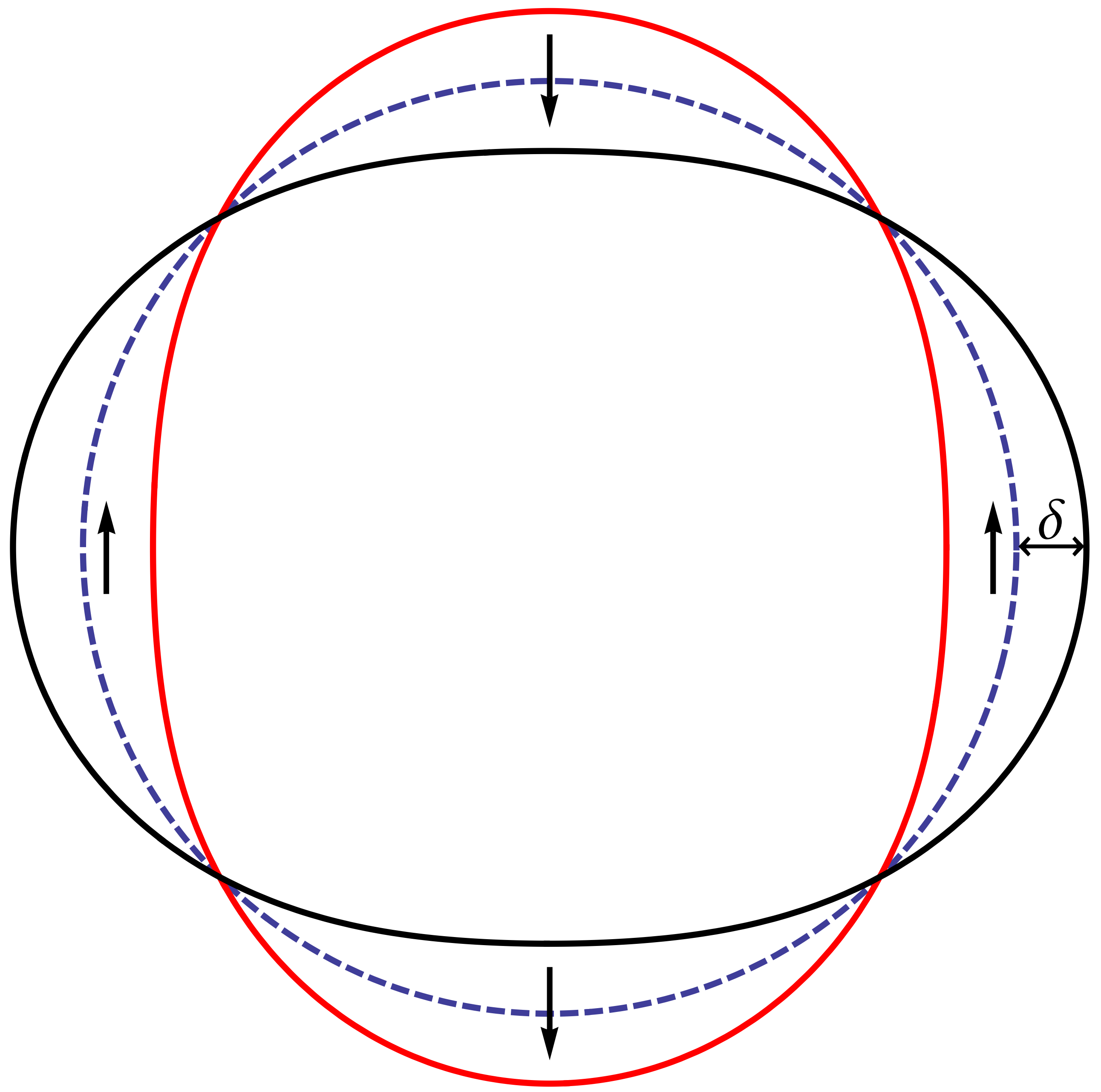


(b)

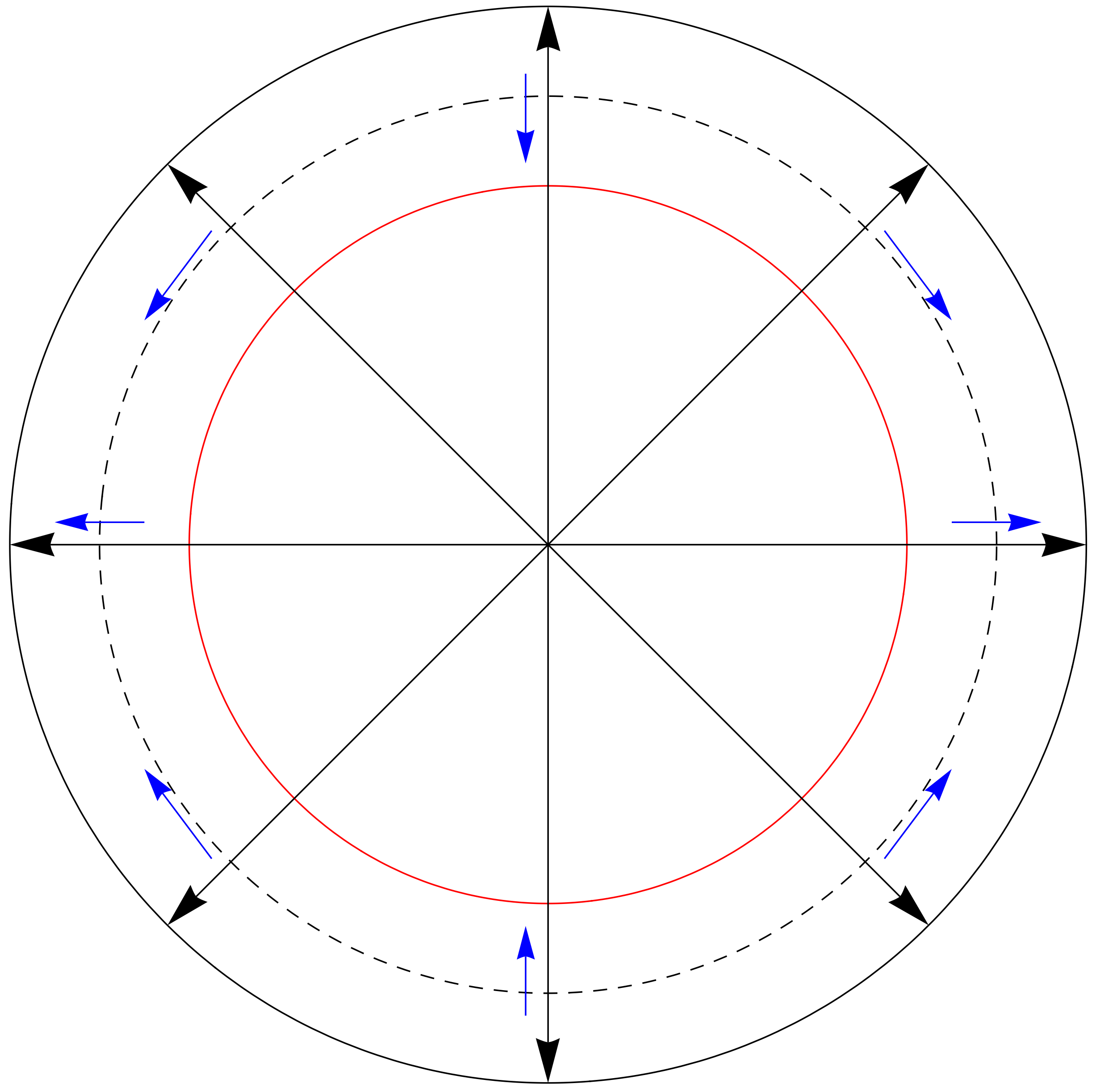
PDW phases from a triplet Pomeranchuk instability

C. Wu, K. Sun, EF, and S.C. Zhang (2007)

Nematic states in the triplet channel with orbital angular momentum l=2



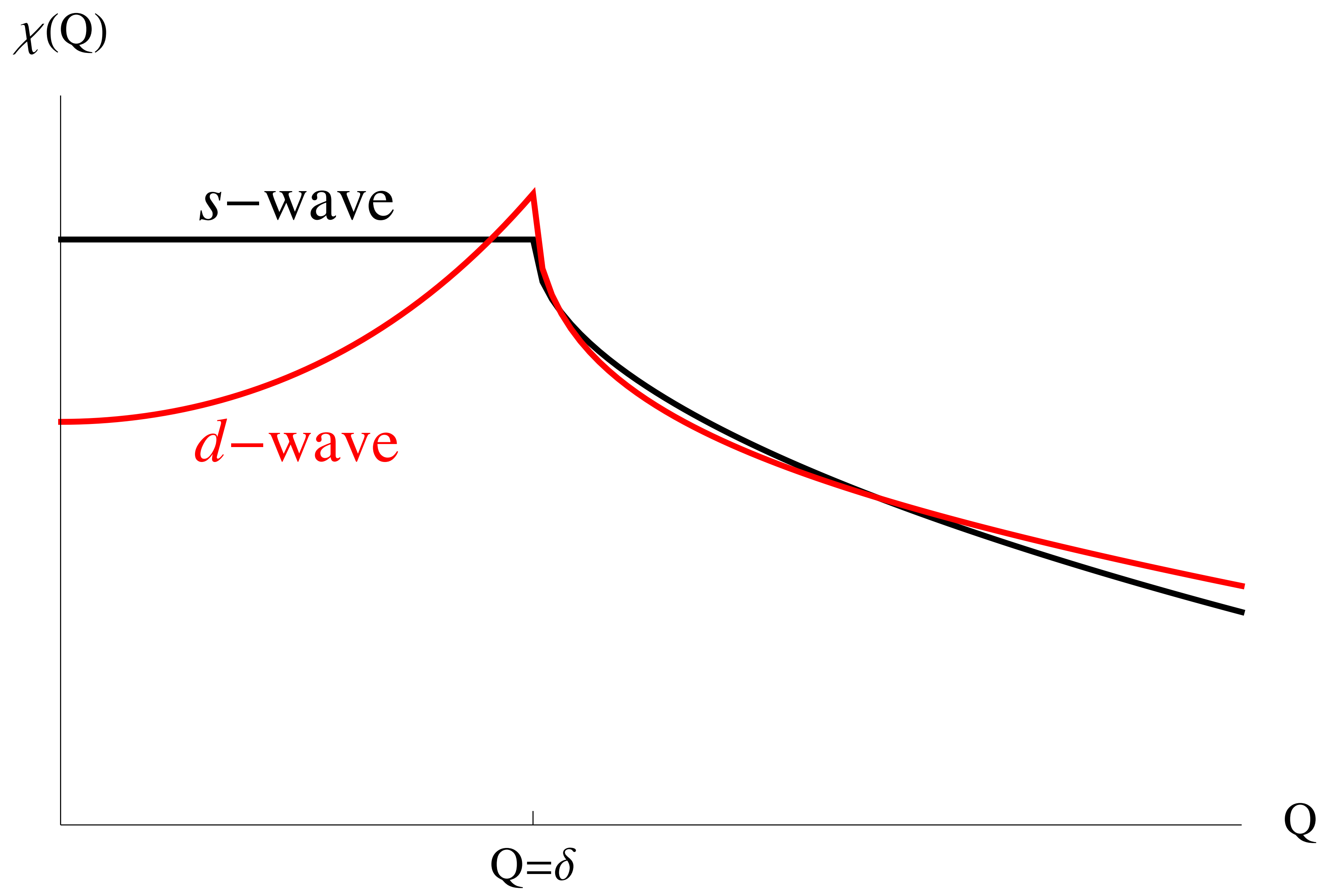
"altermagnet"



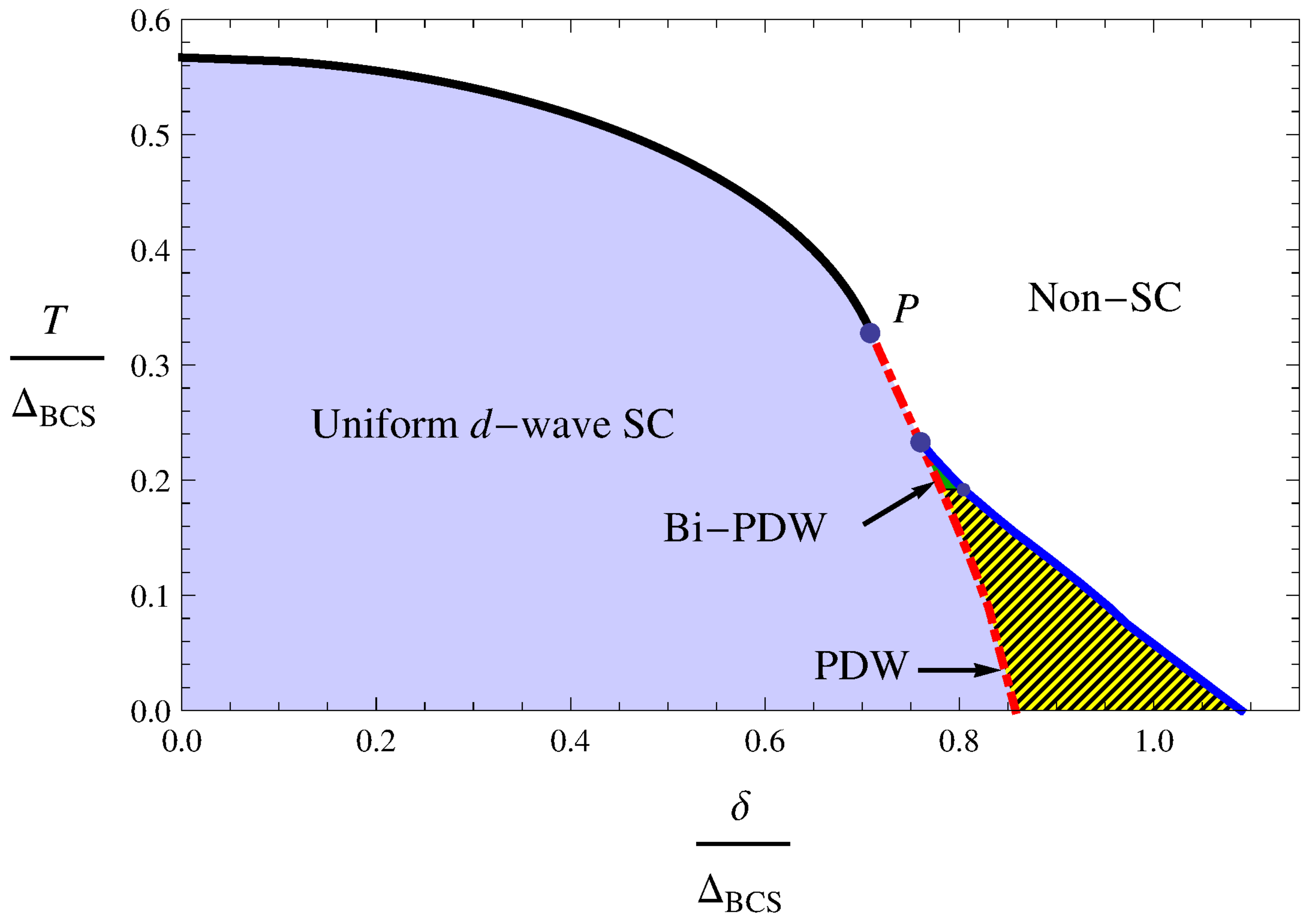
dynamical d-wave "Rashba"

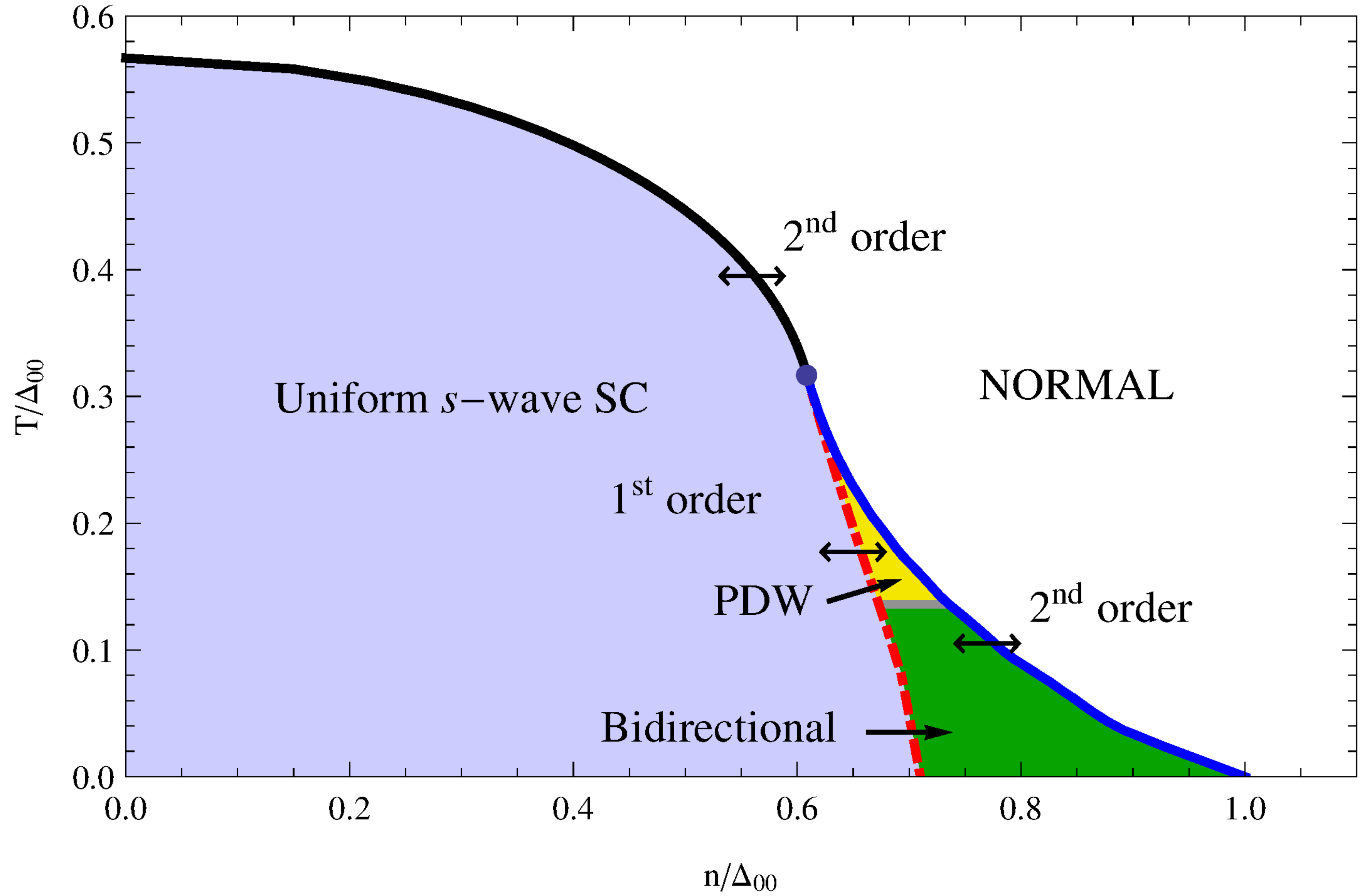
Superconducting Phases in the Nematic Triplet Channel

Soto-Garrido and EF, PRB 2014



Pair Field susceptibility in the s-wave and d-wave channels





Conclusions

- Intertwined orders in HTSC!
- The orders melt in different sequences, they appear essentially with similar strength
- In quasi 2D systems it is natural to get complex phase diagrams with comparable critical temperatures!
- The PDW is a new state that can explain many intriguing features of HTSC
- Big question: how generic is the PDW?
- Encouraging results in some models
- Theoretical Challenge: construct a microscopic theory of Intertwined Orders (and PDW states)!