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Continuous-Variable Optimization
- Quantum vs Classical -

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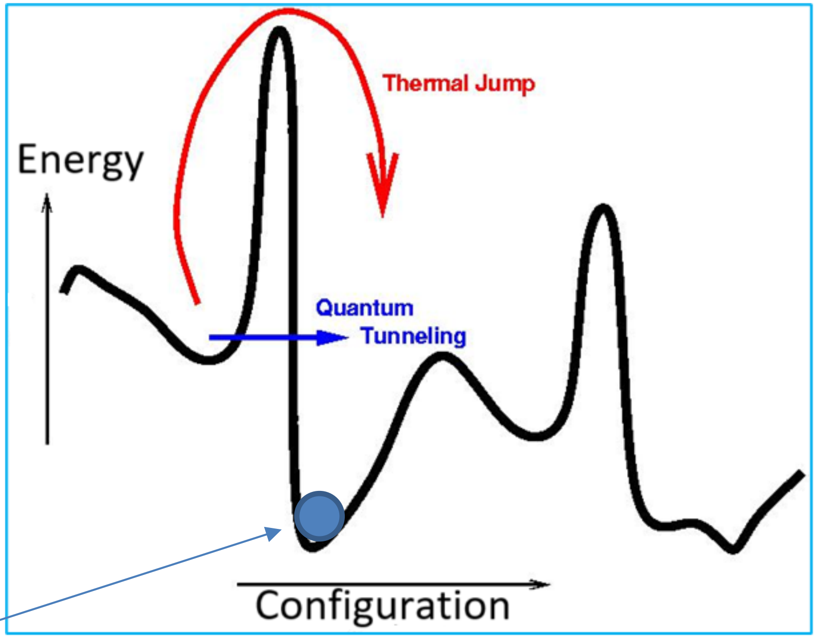
Quantum Annealing

A brief overview for discrete variables

What is quantum annealing?

- Quantum algorithm for the ground-state search of the Ising model, typically the spin glass.
- Quantum counterpart of simulated annealing.

$$H = -\sum J_{ij}\sigma_i^z\sigma_j^z$$



To find the lowest-energy state

From Wikipedia

Classical simulated annealing uses thermal hopping.
Quantum annealing employs tunneling.

Ground-state search of the Ising model

Ising model

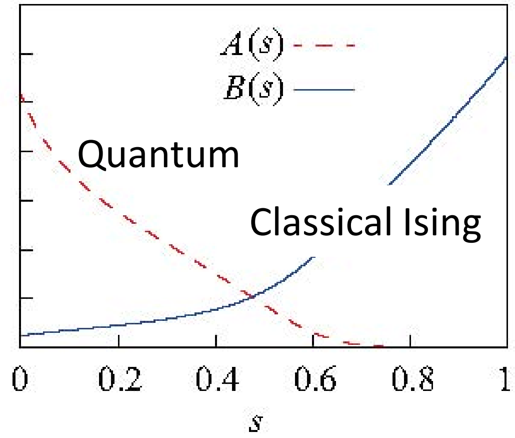
$$H(s) = - \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z$$

Ground-state search of the Ising model

Transverse-field Ising model

Quantum Classical

$$H(s) = -\frac{A(s)}{2}\sum_i \sigma_i^x - \frac{B(s)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$



Ground-state search of the Ising model

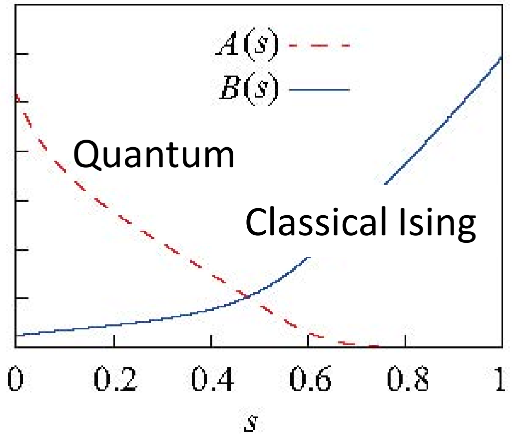
Quantum Classical

Transverse-field Ising model

$$H(s) = -\frac{A(s)}{2}\sum_i \sigma_i^x - \frac{B(s)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$

Time evolution

$$H(0) = -\frac{A(0)}{2}\sum_i \sigma_i^x \Rightarrow H(1) = -\frac{B(1)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$



Formulation of quantum annealing

Quantum　　Classical

Transverse-field Ising model

$$H(s) = -\frac{A(s)}{2}\sum_i \sigma_i^x - \frac{B(s)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$

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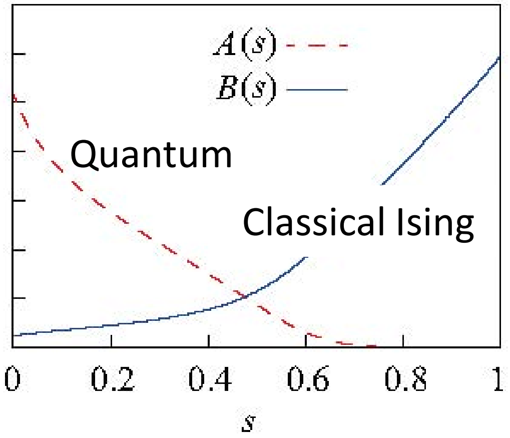

Ground state　　Ground state　　Ground state

Superposition of all states with equal probability

$|+>_x$
=|00000>+|00001>
+|00010>+|00011>
+….
+|11110>+|11111>

→ 10110 　Solution

→ Time-dependent Schrödinger equation

Formulation of quantum annealing

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Quantum — Classical

Transverse-field Ising model

$$H(s) = -\frac{A(s)}{2}\sum_i \sigma_i^x - \frac{B(s)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$

Time evolution

$$H(0) = -\frac{A(0)}{2}\sum_i \sigma_i^x \;\rightarrow\; H(1) = -\frac{B(1)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$

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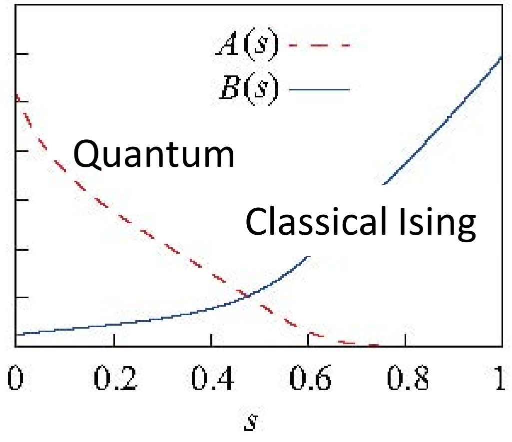

Superposition of all states with equal probability

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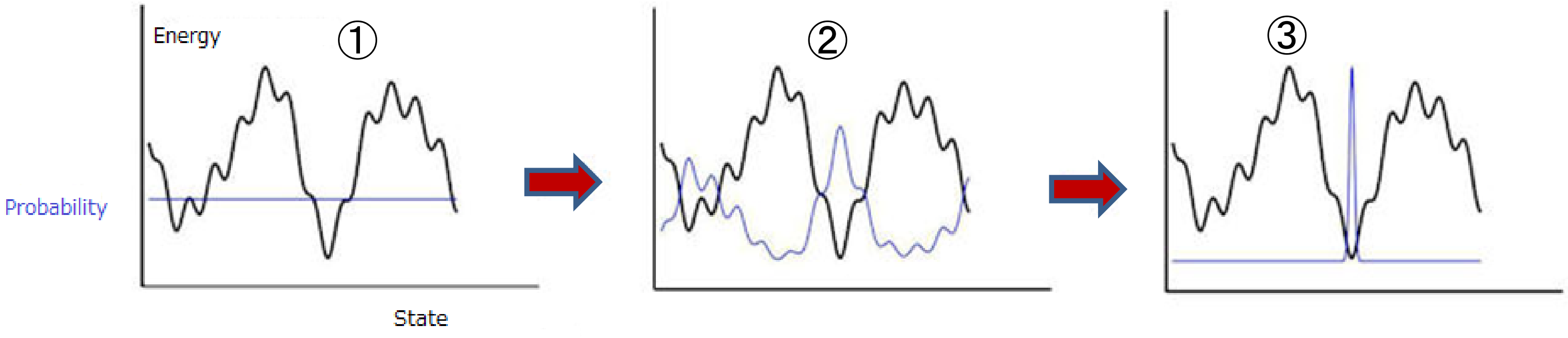
Time-dependent Schrödinger equation



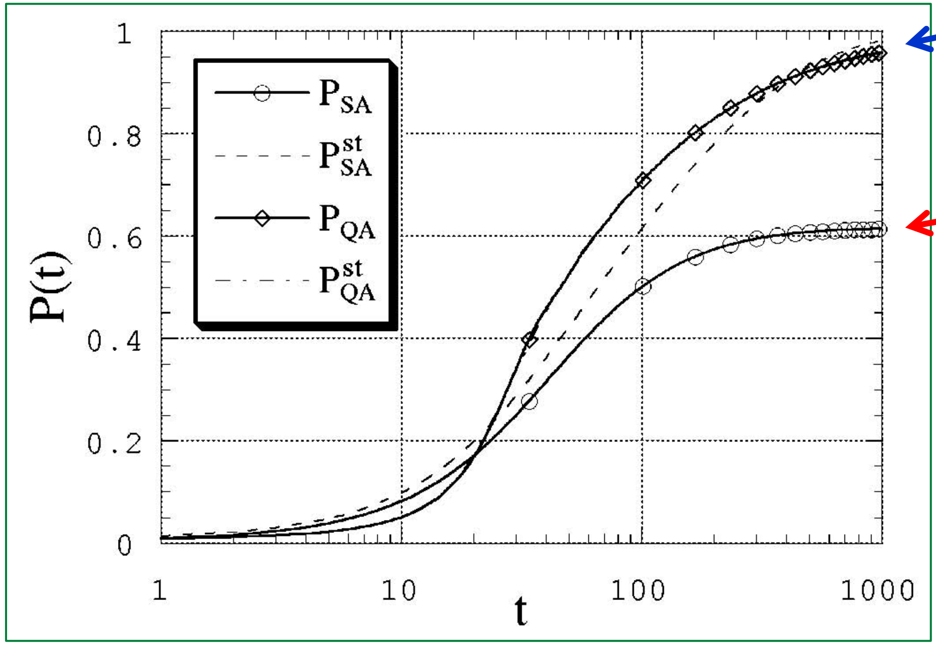
All states with equal probability

Less quantum effects
= higher probability of the ground state.

Largest probability of the ground state

Comparison of quantum annealing and simulated annealing

Small spin glass problem (8 spins)



Probability of ground state

Time

Quantum annealing

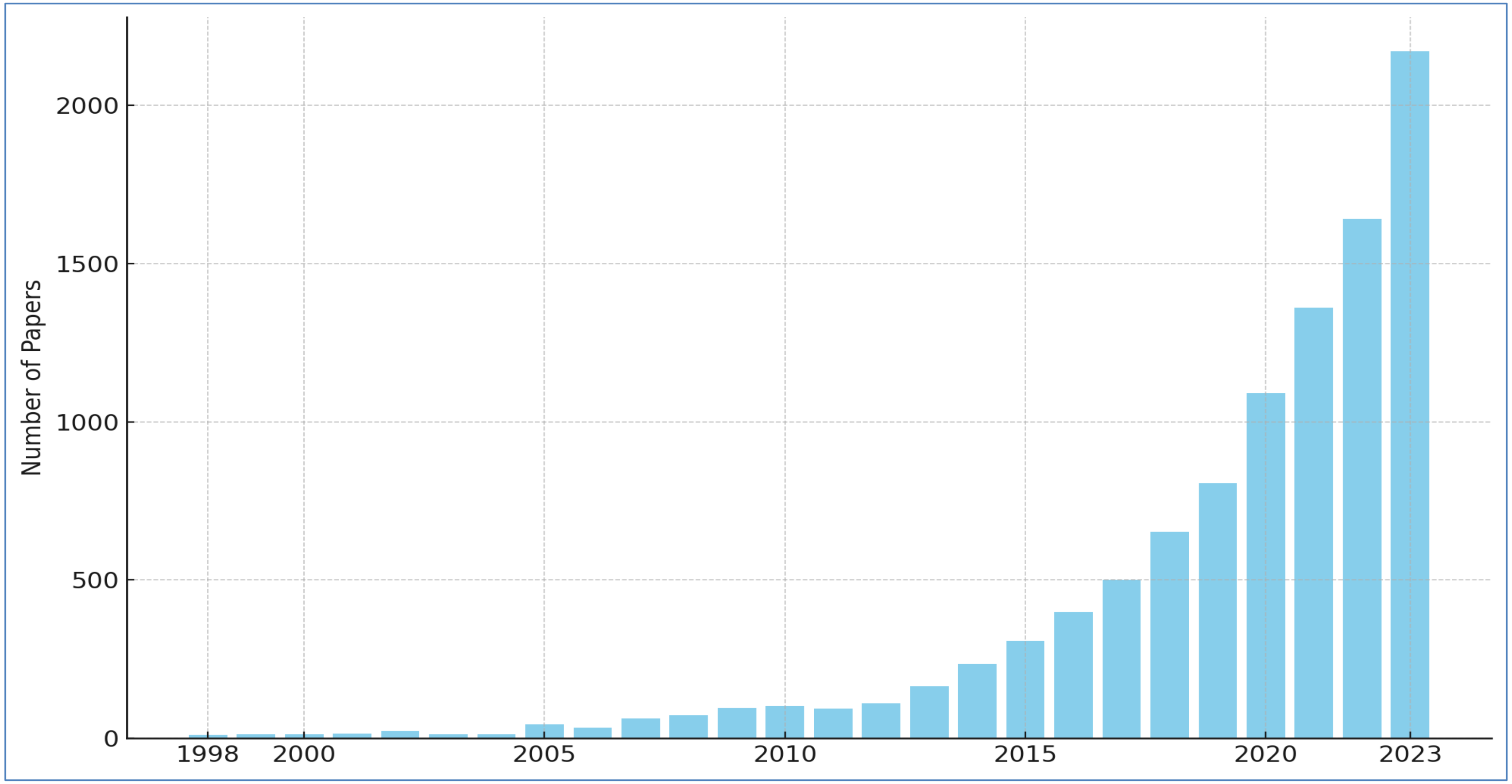
Classical simulated annealing

- QA reaches higher probability than SA.
- Similar results for all examples studied.

Kadowaki and Nishimori, Phys. Rev. E **58**, 5355 (1998)

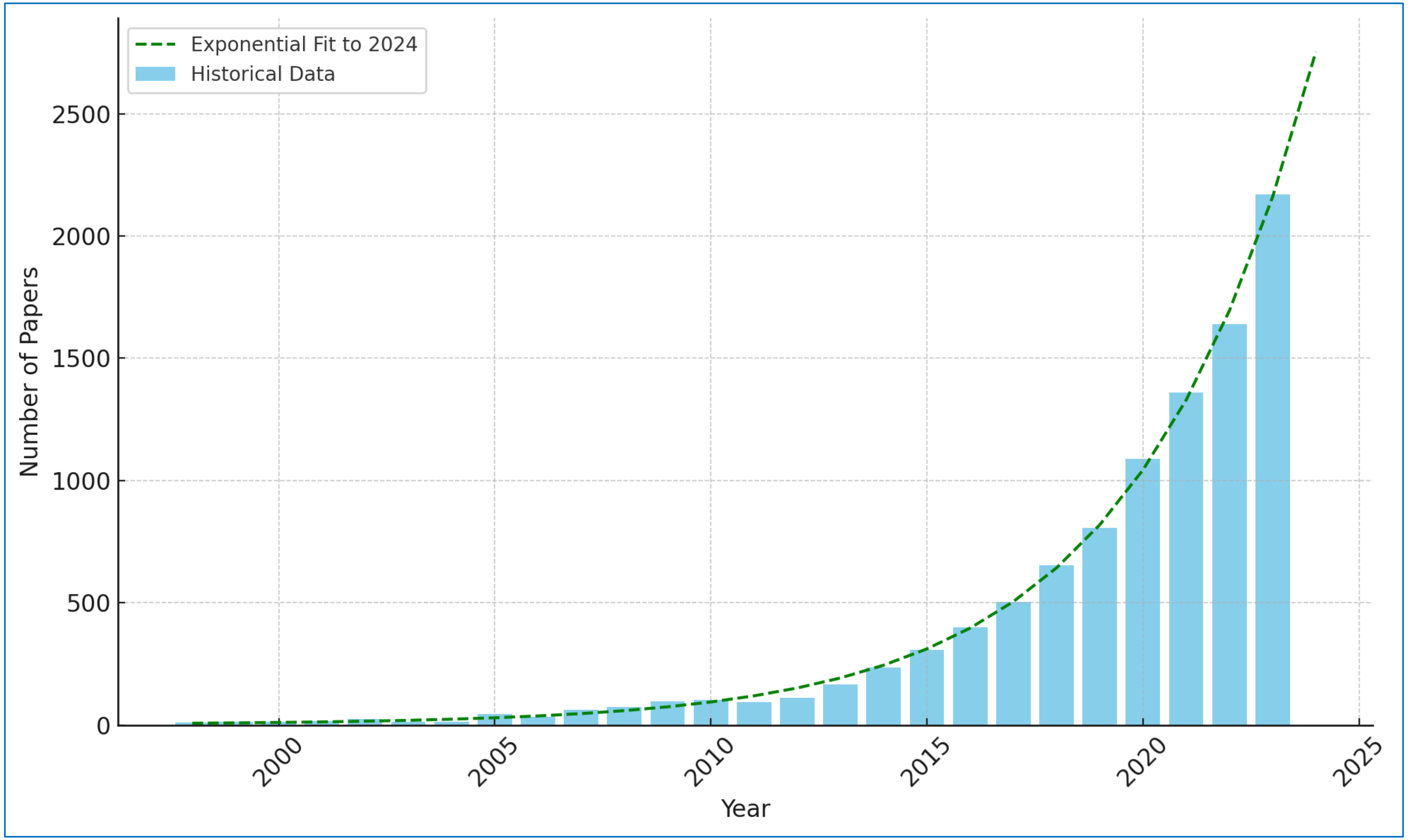
Research activities of quantum annealing is expanding rapidly

Number of papers with the keyword “quantum annealing”



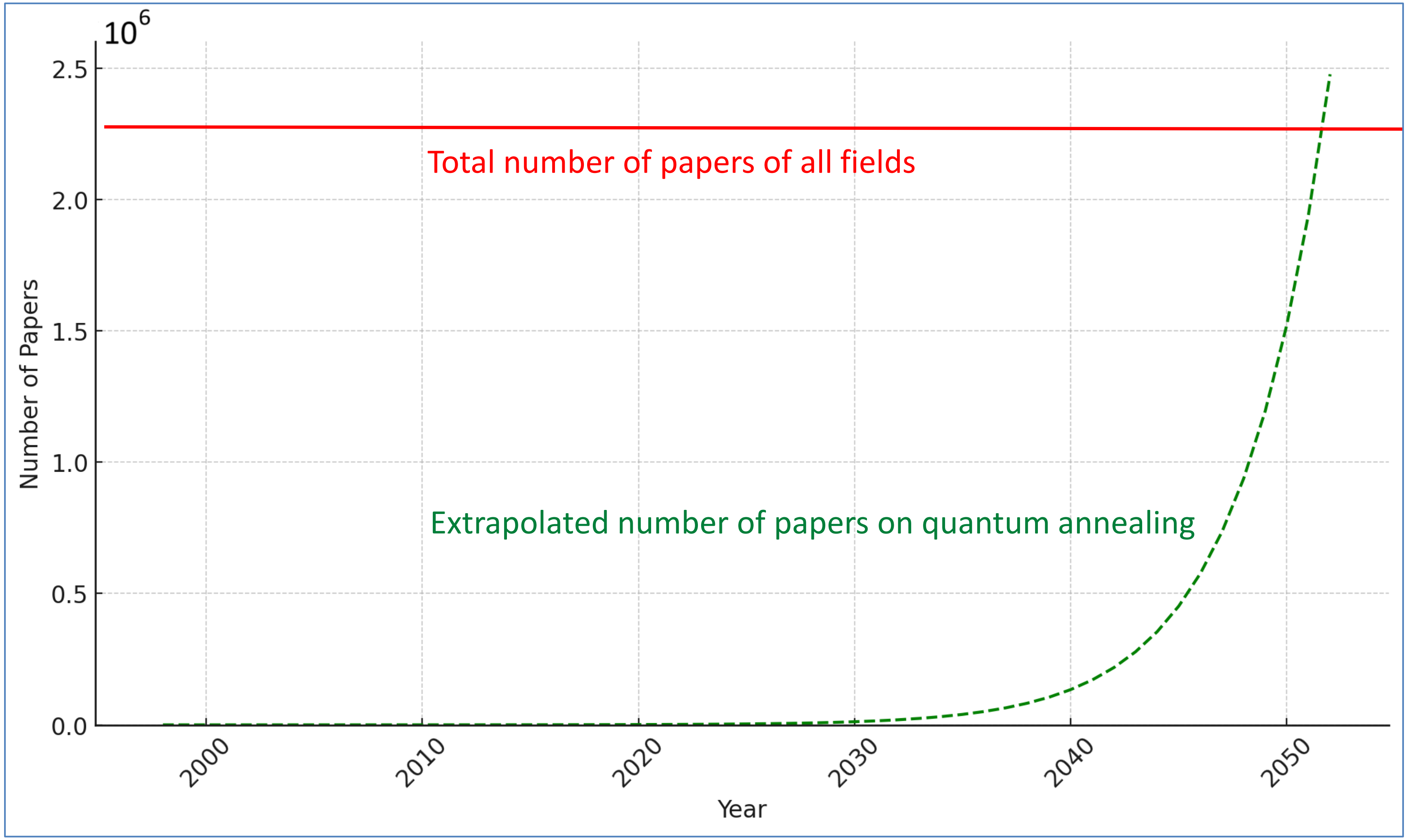
Data from Google Scholar

Exponential growth



Number of papers with the keyword “quantum annealing”

Singularity at 2052



Number of papers with the keyword “quantum annealing”

Hardware implementation

Hardware implementation of moderately-large scale (5000 qubits) is commercialized.

“D-Wave Advantage” device at Jülich, Germany

Hardware implementation

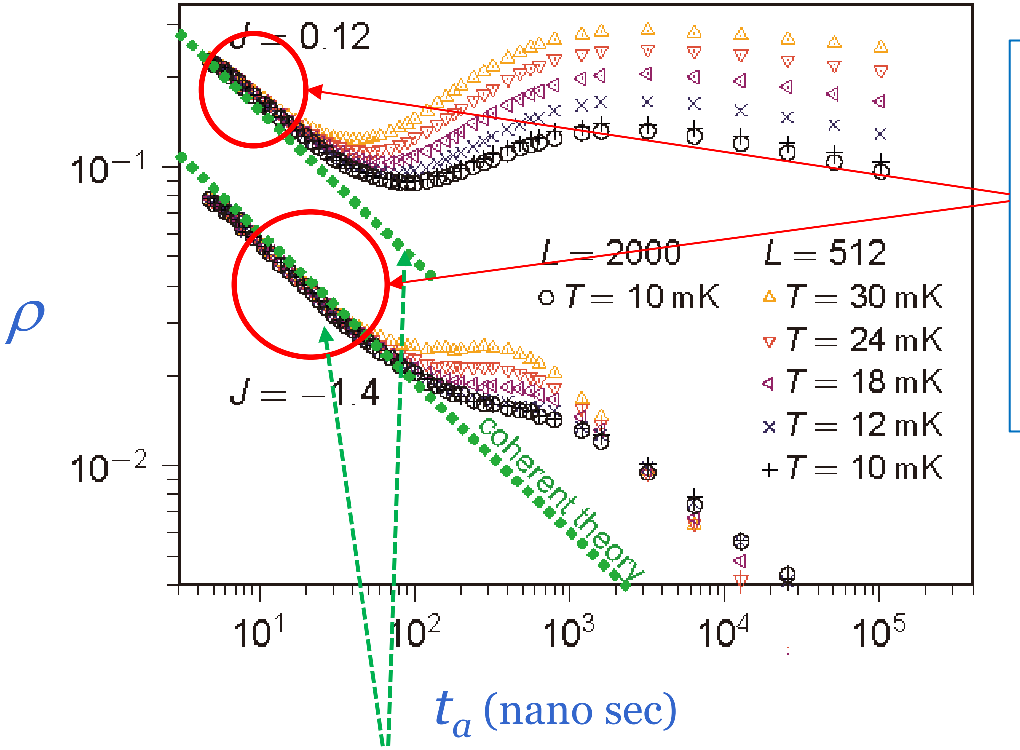
"D-Wave Advantage" device

Building to house D-Wave Advantage

Hardware implementation

Emergency power backup operated by Schrödinger’s cat

Quantum simulation of the Kibble-Zurek mechanism



$$\rho = \frac{1}{2\pi}\sqrt{\frac{\hbar}{2J}}\, t_a^{-1/2}$$

slope

offset

- Data from 2000-qubit system agrees with the theory including the quantum coefficient at short time scale up to about 50 ns.
- No fitting parameters.
- The device runs coherently up to 50 ns.
- Environment (thermal effect) affects after about 50 ns.

The Kibble-Zurek theory has been confirmed including the coefficient.

King et al, Nature Phys. **18**, 1324 (2022)

Quantum annealing used solve real-world problem

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To minimize waiting time of trucks by optimization of the motion of cranes.

Port of Los Angeles

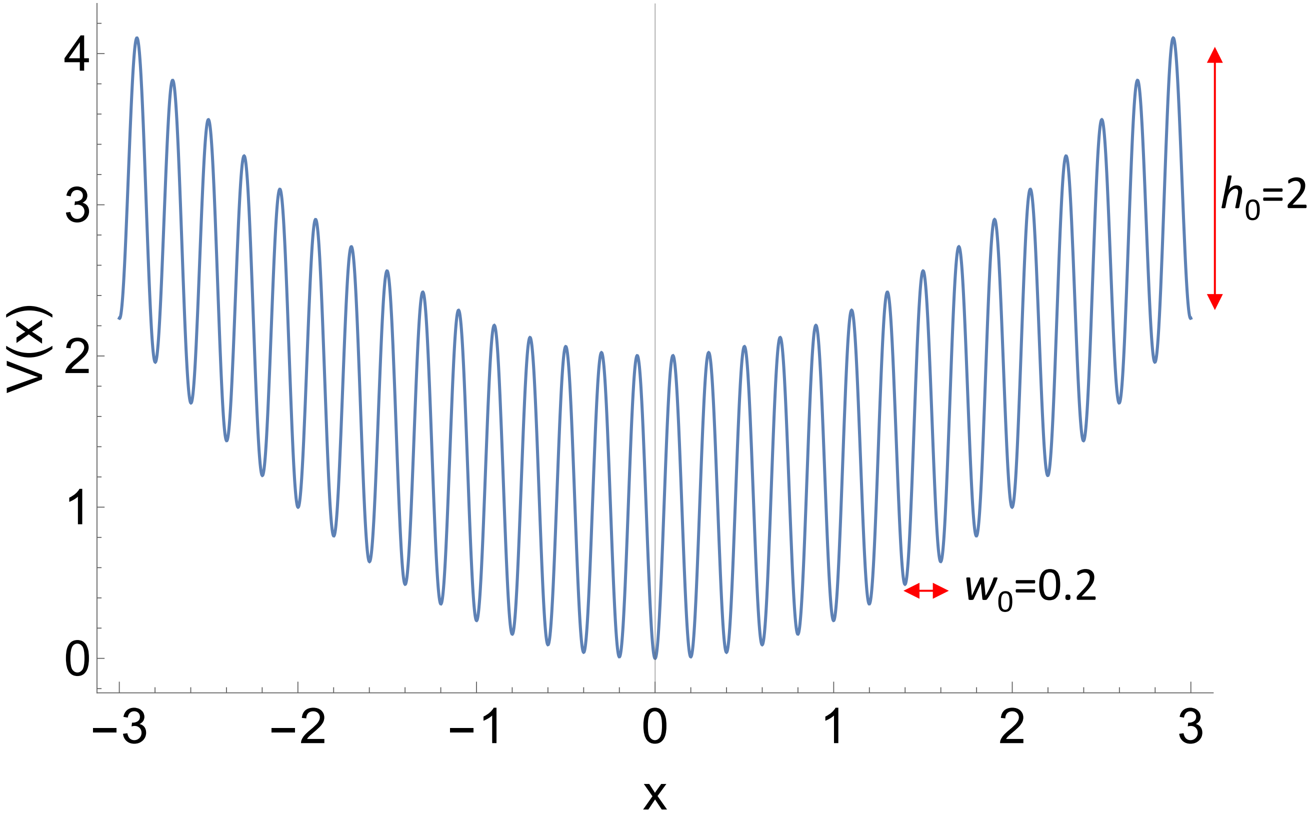
- Instructions to the cranes should be sent out about every 10 seconds.
- D-Wave sends back its solution within a few seconds.
- Classical methods take longer than 10 seconds, thus unpractical.
- Constant speedup is sufficient for this type of real-time processing.

Continuous-variable optimization

S. Arai, H. Oshiyama and H. Nishimori, Phys. Rev. A **108**, 042403 (2023)

Y. W. (Patrick) Koh and H. Nishimori, Phys. Rev. A **105**, 062435 (2022)

1d problem with rugged energy landscape



Rastrigin function

One of standard benchmark functions in the field of continuous-variable optimization

$$V(x) = \frac{1}{2}kx^2 + \frac{h_0}{2}\left[1 - \cos\left(\frac{2\pi x}{w_0}\right)\right]$$

- Hardness is controllable by h_0 and w_0(=0.2).
- N=211, Δx=0.02857

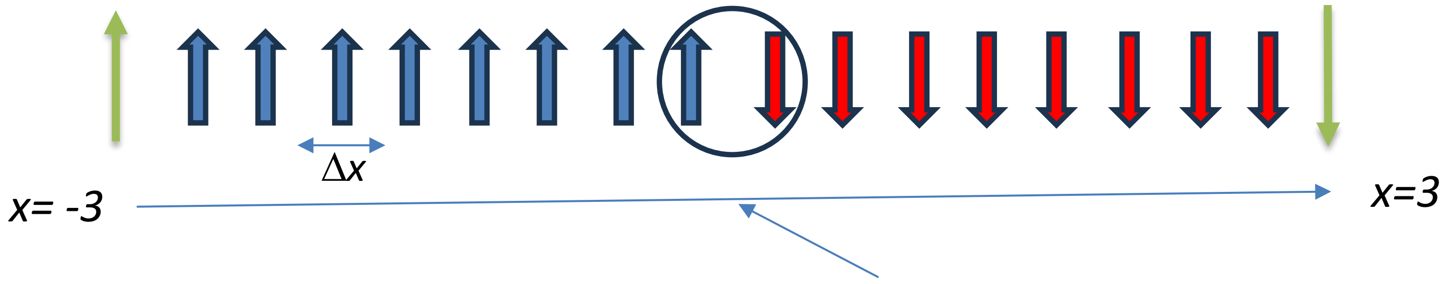
Discretization of continuous variable
Domain-wall encoding

Chancellor (2019)

1d ferromagnetic Ising model with strong bias fields at both edges

$$\frac{\sigma_i - \sigma_{i+1}}{2} = 1 \quad \text{only at the domain wall}$$

Strong +*h* field on the left edge



Strong -*h* field on the right edge

Continuous *x* is discretized to the position x_{dw} of domain wall

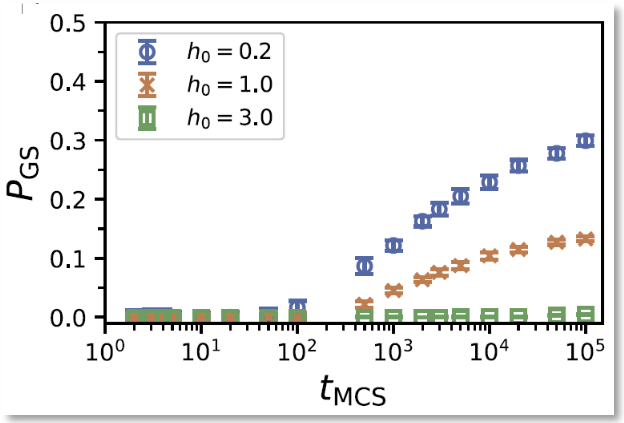
$$H_{\text{cost}} = \sum_{i=1}^{N-1} V(x_i) \frac{\sigma_i - \sigma_{i+1}}{2} = V(x_{\text{dw}})$$

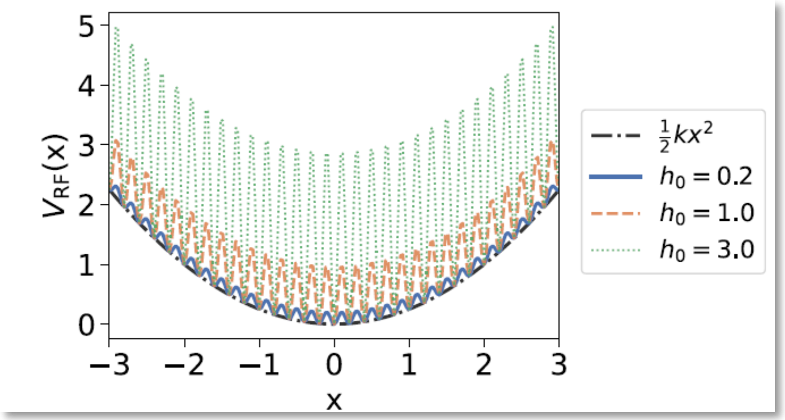
$$H_{\text{DW}} = -J \sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z + h(\sigma_1^z - \sigma_N^z)$$

$$H_0 = \lambda H_{\text{cost}} + H_{\text{DW}}$$

Ground-state probability (success rate)

Simulated annealing



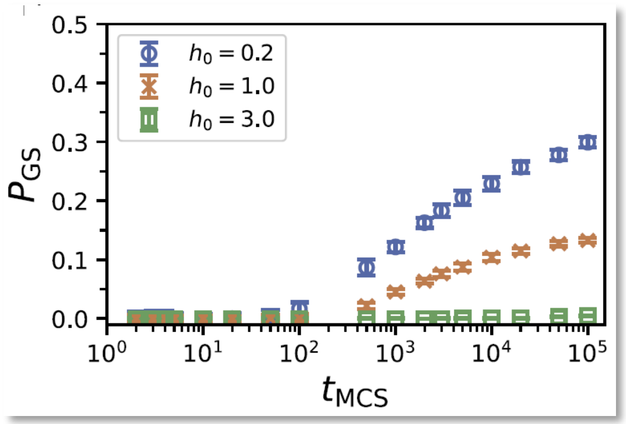


Barrier height: Green: highest Brown: Intermediate Blue: Lowest

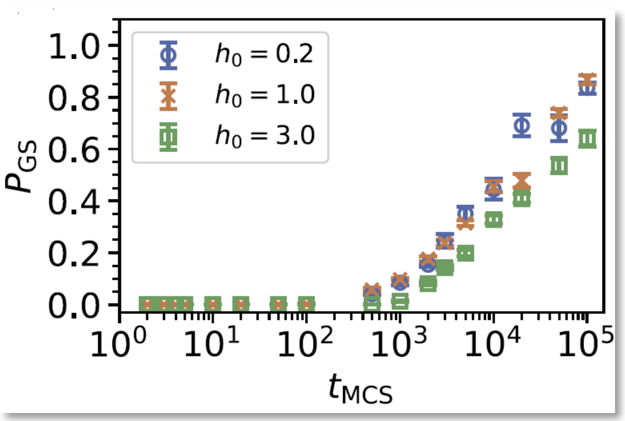
- Simulated annealing is strongly affected by the barrier height.

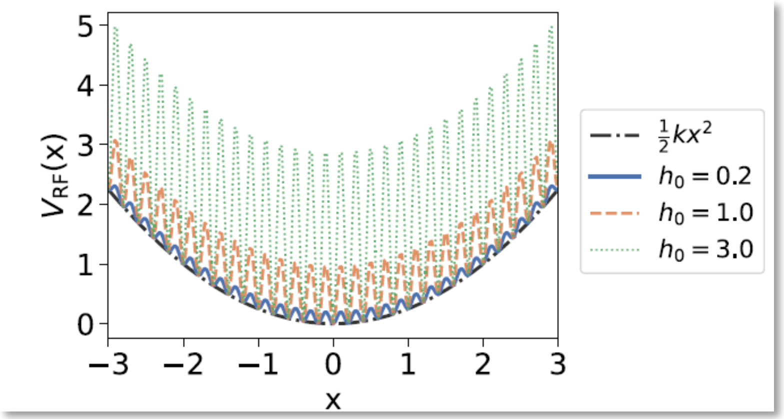
Ground-state probability (success rate)

Simulated annealing



Simulated QA



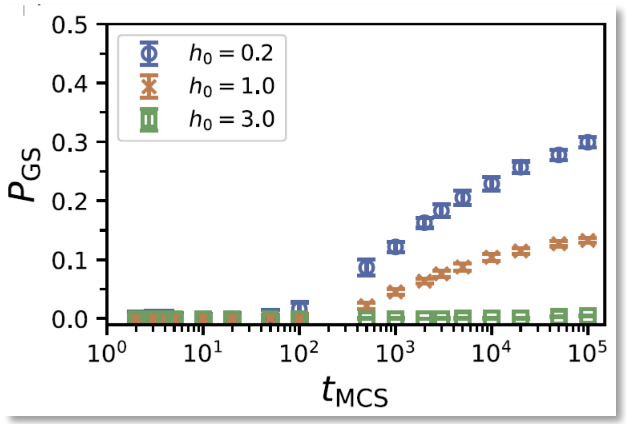


Barrier height: Green: highest Brown: Intermediate Blue: Lowest

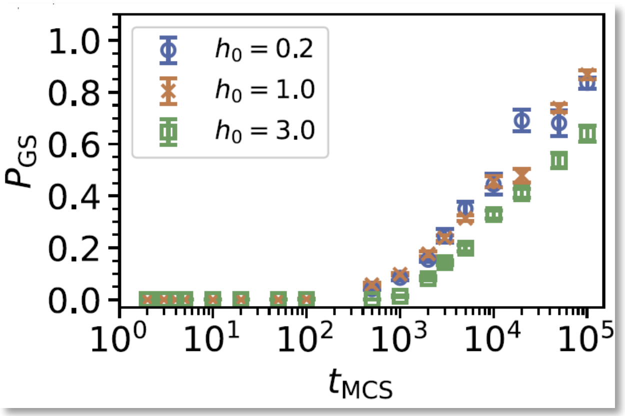
- Simulated annealing is strongly affected by the barrier height.
- Simulated quantum annealing is less affected by the height. Partly simulating tunneling effects.

Ground-state probability (success rate)

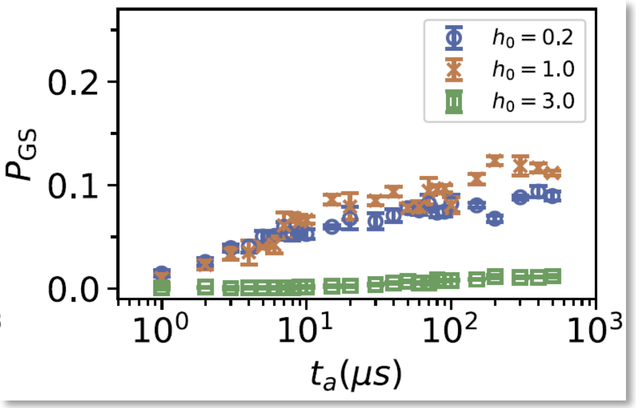
Simulated annealing

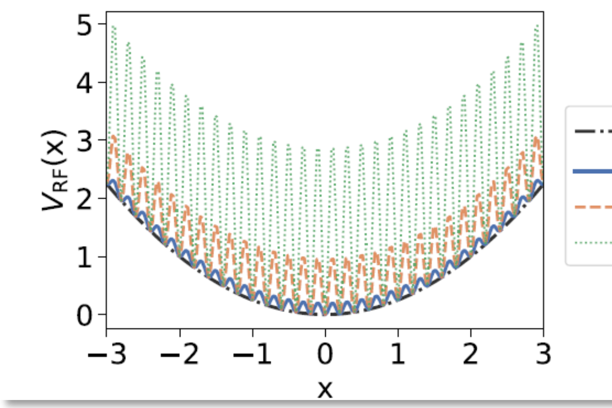


Simulated QA



D-Wave





Barrier height: Green: highest Brown: Intermediate Blue: Lowest

- Simulated annealing is strongly affected by the barrier height.
- Simulated quantum annealing is less affected by the height. Partly simulating tunneling effects.
- D-Wave is less susceptible to the height than simulated annealing but shows evidence of thermal effects.

Ground-state probability (success rate)

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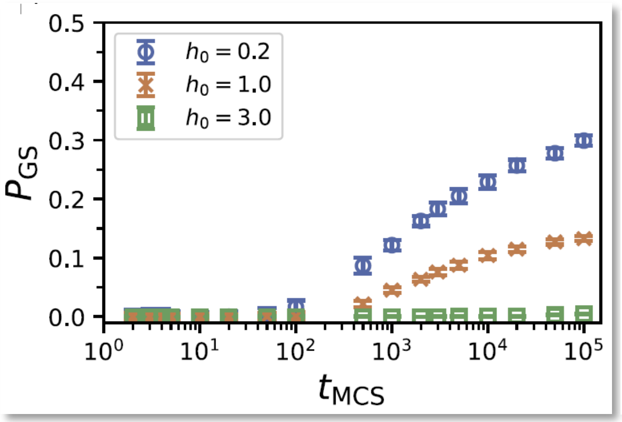
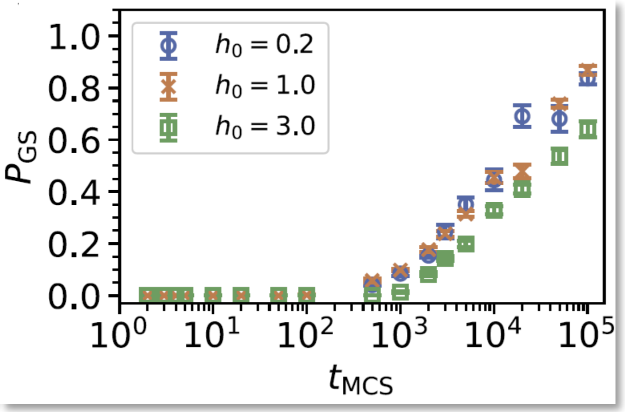
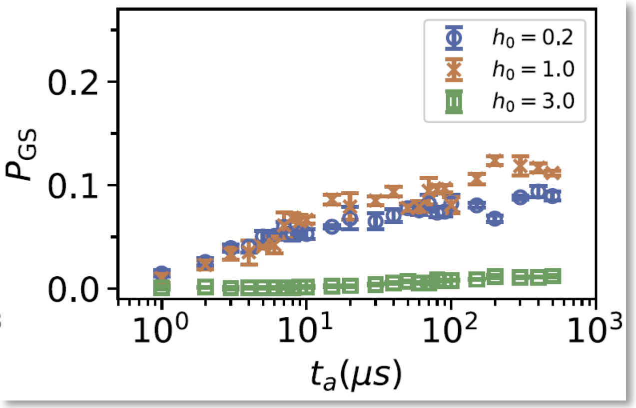
Simulated annealing



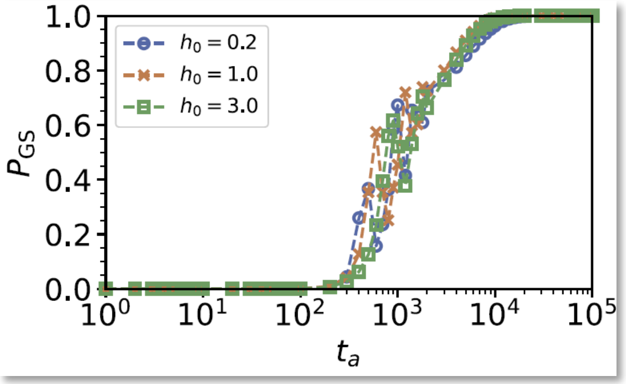
Simulated QA



D-Wave

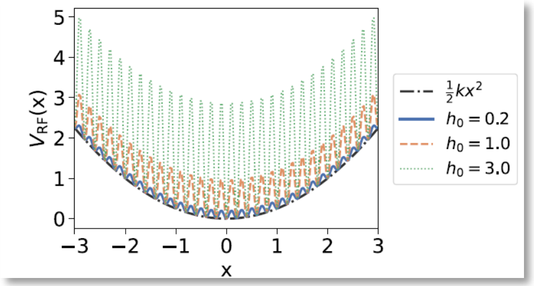


Coherent QA by TEBD



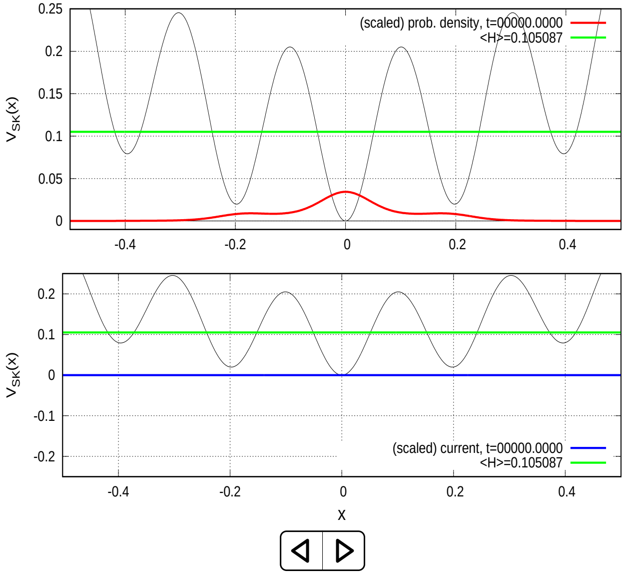
Barrier height: Green: highest Brown: Intermediate Blue: Lowest

- Simulated annealing is strongly affected by the barrier height.
- Simulated quantum annealing is less affected by the height. Partly simulating tunneling effects.
- D-Wave is less susceptible to the height than simulated annealing but shows evidence of thermal effects.
- Coherent QA (thermal-noise free) is independent of the height. Quantum tunneling in action.
- Building QA device with minimal thermal noise will greatly enhance the performance of QA.

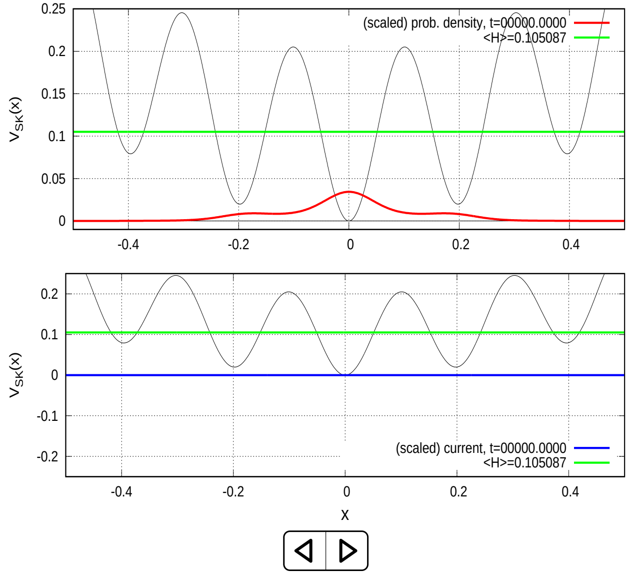


Stage 2

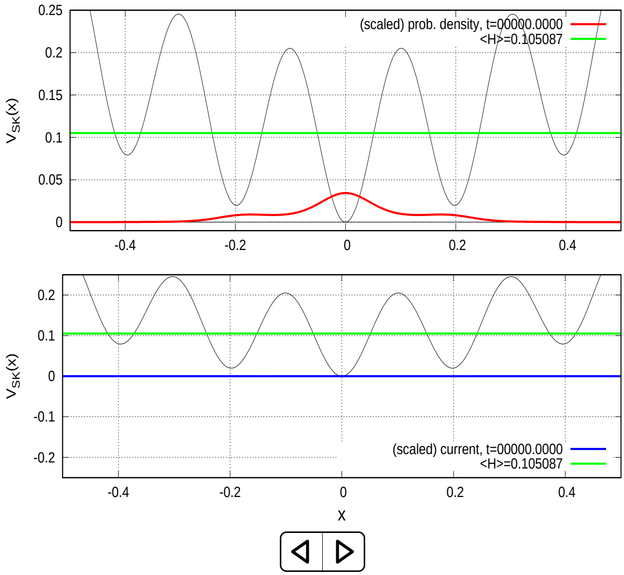
Video 1: $T = 320$



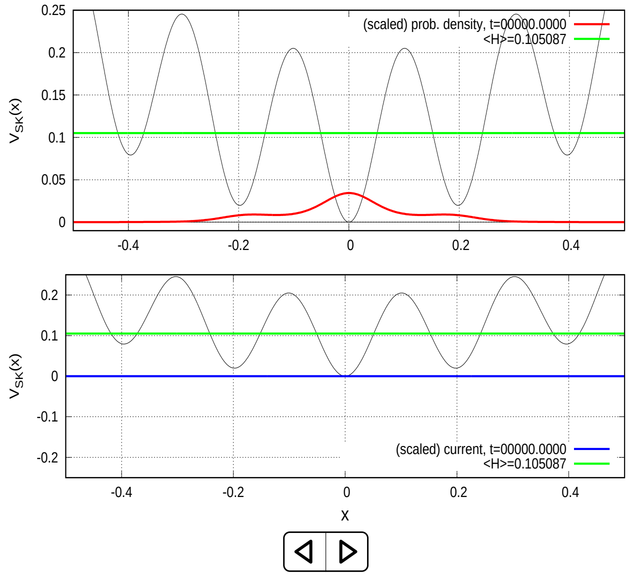
Video 2: $T = 3250$



Video 3: $T = 5500$



Video 4: $T = 10000$



Conclusion

- Continuous-variable optimization has been solved via the Ising model.
- Quantum tunneling helps the system reach the optimal state more efficiently than classical hill-climing processes.