"Recent Progress in Many-Body Theories (RPMBT22)",
Sep 23-27, 2024, Kasuga Campus, University of Tsukuba, Tsukuba, Japan

Boson-fermion pairing in resonant Bose-Fermi mixtures

Pierbiagio Pieri

Dept. of Physics and Astronomy, University of Bologna
& INFN, Sezione di Bologna, Italy

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Bose-Fermi mixtures with a tunable BF attraction

- System of bosons of one species interacting with one-component fermions through a **tunable boson-fermion attraction**.

- For weak attraction, weakly interacting Bose-Fermi mixture: at sufficiently low temperature bosons condense, while fermions fill a Fermi sphere.

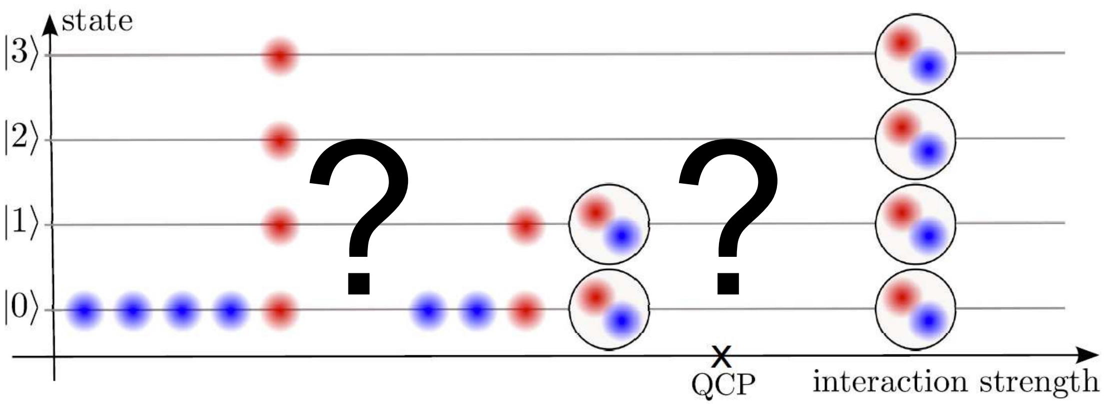
- For **strong attraction** bosons pair with fermions to **form molecules**. Condensation suppressed in favor of molecule formation. Fermi sphere of molecules coexisting with Fermi sphere of unpaired fermions for $n_F \geq n_B$.

Bose-Fermi mixtures with a tunable BF attraction

• System of bosons of one species interacting with one-component fermions through a **tunable boson-fermion attraction**.

• For weak attraction, weakly interacting Bose-Fermi mixture: at sufficiently low temperature bosons condense, while fermions fill a Fermi sphere.

• For **strong attraction** bosons pair with fermions to **form molecules**. Condensation suppressed in favor of molecule formation. Fermi sphere of molecules coexisting with Fermi sphere of unpaired fermions for $n_F \geq n_B$.



• How does the system evolves from one limit to the other one?

• How to describe this evolution?

PHYSICAL REVIEW A **91**, 023603 (2015)

Condensed phase of Bose-Fermi mixtures with a pairing interaction

Andrea Guidini,[1] Gianluca Bertaina,[2] Davide Emilio Galli,[2] and Pierbiagio Pieri[1]

The model

• **Two-component Hamiltonian** with attractive contact interaction between bosons and fermions.

$$H_{\mathrm{BF}} = \sum_{s=\mathrm{B,F}} \int d\mathbf{r} \psi_s^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m_s} - \mu_s \right) \psi_s(\mathbf{r}) + v_0 \int d\mathbf{r} \psi_B^\dagger(\mathbf{r}) \psi_F^\dagger(\mathbf{r}) \psi_F(\mathbf{r}) \psi_B(\mathbf{r})$$

• Bare contact-interaction strength between bosons and fermions expressed in terms of the **boson-fermion** scattering length a_{BF} .

$$\frac{1}{v_0} = \frac{m_r}{2\pi a_{\mathrm{BF}}} - \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{2m_r}{\mathbf{k}^2}$$

$$m_r = \frac{m_{\mathrm{B}} m_{\mathrm{F}}}{m_{\mathrm{B}} + m_{\mathrm{F}}}$$

The model

- **Two-component Hamiltonian** with attractive contact interaction between bosons and fermions.

$$H_{\mathrm{BF}} = \sum_{s=\mathrm{B,F}} \int d\mathbf{r} \psi_s^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m_s} - \mu_s \right) \psi_s(\mathbf{r}) + v_0 \int d\mathbf{r} \psi_B^\dagger(\mathbf{r}) \psi_F^\dagger(\mathbf{r}) \psi_F(\mathbf{r}) \psi_B(\mathbf{r})$$

- Bare contact-interaction strength between bosons and fermions expressed in terms of the **boson-fermion** scattering length a_{BF} .

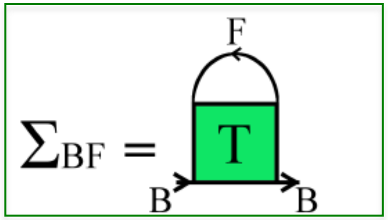
$$\frac{1}{v_0} = \frac{m_r}{2\pi a_{\mathrm{BF}}} - \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{2m_r}{\mathbf{k}^2} \qquad m_r = \frac{m_{\mathrm{B}} m_{\mathrm{F}}}{m_{\mathrm{B}} + m_{\mathrm{F}}}$$

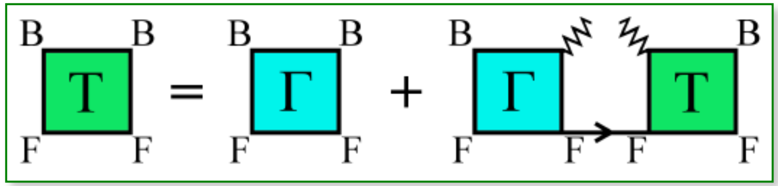
- No Fermi-Fermi interaction (fermions are identical: short-range interaction suppressed). Some **boson-boson repulsion** is required for stability.

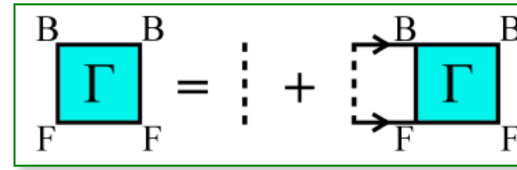
$$H = H_{\mathrm{BF}} + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' V_{\mathrm{BB}}(\mathbf{r} - \mathbf{r}') \psi_B^\dagger(\mathbf{r}) \psi_B^\dagger(\mathbf{r}') \psi_B(\mathbf{r}') \psi_B(\mathbf{r})$$

- We focus on systems with $n_F \geq n_B$.

Bosonic and fermionic self-energy diagrams for the **condensed** phase







Boson self-energy

$$\Sigma_{\mathrm{B}}^{11}(\bar{k}) = \frac{8\pi a_{\mathrm{BB}}}{m_B} n_0 + \Sigma_{\mathrm{BF}}(\bar{k})$$

$$\Sigma_{\mathrm{B}}^{12}(\bar{k}) = \frac{4\pi a_{\mathrm{BB}}}{m_B} n_0$$

$$\Sigma_{\mathrm{BF}}(\bar{k}) = \int \frac{d\mathbf{P}}{(2\pi)^3} \int \frac{d\Omega}{2\pi} T(\bar{P}) G_{\mathrm{F}}^0(\bar{P} - \bar{k})$$

$\bar{P} \equiv (\mathbf{P}, i\Omega)$, $\bar{k} \equiv (\mathbf{k}, i\omega)$ $\quad G_s^0(\bar{k})^{-1} = i\omega - \xi_{\mathbf{k}}^s$

Boson-fermion T-matrix

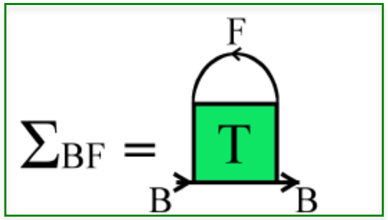
$$T(\bar{P})^{-1} = \Gamma(\bar{P})^{-1} - n_0 G_{\mathrm{F}}^0(\bar{P})$$

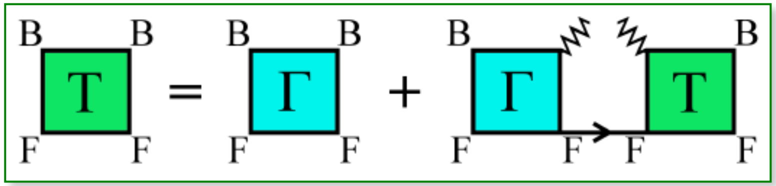
$$\Gamma(\bar{P})^{-1} = \frac{m_r}{2\pi a_{\mathrm{BF}}} - \frac{m_r^{\frac{3}{2}}}{\sqrt{2}\,\pi} \left[\frac{P^2}{2M} - 2\mu - i\Omega \right]^{\frac{1}{2}} - I_{\mathrm{F}}(\bar{P})$$

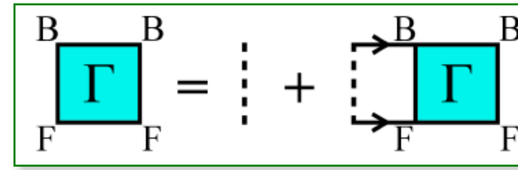
$$I_{\mathrm{F}}(\bar{P}) \equiv \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{\Theta(-\xi_{\mathbf{P}-\mathbf{p}}^{\mathrm{F}})}{\xi_{\mathbf{P}-\mathbf{p}}^{\mathrm{F}} + \xi_{\mathbf{p}}^{\mathrm{B}} - i\Omega}$$

$$\mu = (\mu_{\mathrm{B}} + \mu_{\mathrm{F}})/2 \qquad \xi_{\mathbf{p}}^s = p^2/2m_s - \mu_s$$

Bosonic and fermionic self-energy diagrams for the **condensed** phase







Boson self-energy

$$\Sigma_{\rm B}^{11}(\bar{k}) = \frac{8\pi a_{\rm BB}}{m_B} n_0 + \Sigma_{\rm BF}(\bar{k})$$

$$\Sigma_{\rm B}^{12}(\bar{k}) = \frac{4\pi a_{\rm BB}}{m_B} n_0$$

$$\Sigma_{\rm BF}(\bar{k}) = \int \frac{d\mathbf{P}}{(2\pi)^3} \int \frac{d\Omega}{2\pi} T(\bar{P}) G_{\rm F}^0(\bar{P} - \bar{k})$$

$\bar{P} \equiv (\mathbf{P}, i\Omega)$, $\bar{k} \equiv (\mathbf{k}, i\omega)$ $\quad G_s^0(\bar{k})^{-1} = i\omega - \xi_{\mathbf{k}}^s$

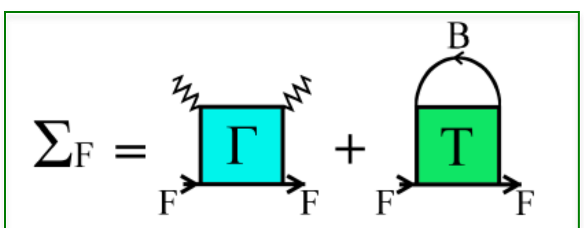
Boson-fermion T-matrix

$$T(\bar{P})^{-1} = \Gamma(\bar{P})^{-1} - n_0 G_{\rm F}^0(\bar{P})$$

$$\Gamma(\bar{P})^{-1} = \frac{m_r}{2\pi a_{\rm BF}} - \frac{m_r^{\frac{3}{2}}}{\sqrt{2}\,\pi} \left[\frac{P^2}{2M} - 2\mu - i\Omega \right]^{\frac{1}{2}} - I_{\rm F}(\bar{P})$$

$$I_{\rm F}(\bar{P}) \equiv \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{\Theta(-\xi_{\mathbf{P}-\mathbf{p}}^{\rm F})}{\xi_{\mathbf{P}-\mathbf{p}}^{\rm F} + \xi_{\mathbf{p}}^{\rm B} - i\Omega}$$

$$\mu = (\mu_{\rm B} + \mu_{\rm F})/2 \qquad \xi_{\mathbf{p}}^s = p^2/2m_s - \mu_s$$



Fermion self-energy

$$\Sigma_{\rm F}(\bar{k}) = n_0 \Gamma(\bar{k}) - \int \frac{d\mathbf{P}}{(2\pi)^3} \int \frac{d\Omega}{2\pi} T(\bar{P}) G_{\rm B}^0(\bar{P} - \bar{k})$$

Coupled equations for chemical potentials and condensate density n_0

Green's functions obtained from the self-energies through Dyson's equations:

$$G_{\rm F}(\bar{k})^{-1} = G^0_{\rm F}(\bar{k})^{-1} - \Sigma_{\rm F}(\bar{k})$$

$$G'_{\rm B}(\bar{k}) = \frac{i\omega + \xi^{\rm B}_{\mathbf{k}} + \Sigma^{11}_{\rm B}(-\bar{k})}{[i\omega + \xi^{\rm B}_{\mathbf{k}} + \Sigma^{11}_{\rm B}(-\bar{k})][i\omega - \xi^{\rm B}_{\mathbf{k}} - \Sigma^{11}_{\rm B}(\bar{k})] + \Sigma^{12}_{\rm B}(\bar{k})^2}$$

Coupled equations for chemical potentials and condensate density n_0

Green's functions obtained from the self-energies through Dyson's equations:

$$G_{\rm F}(\bar{k})^{-1} = G^0_{\rm F}(\bar{k})^{-1} - \Sigma_{\rm F}(\bar{k})$$

$$G'_{\rm B}(\bar{k}) = \frac{i\omega + \xi^{\rm B}_{\mathbf{k}} + \Sigma^{11}_{\rm B}(-\bar{k})}{[i\omega + \xi^{\rm B}_{\mathbf{k}} + \Sigma^{11}_{\rm B}(-\bar{k})][i\omega - \xi^{\rm B}_{\mathbf{k}} - \Sigma^{11}_{\rm B}(\bar{k})] + \Sigma^{12}_{\rm B}(\bar{k})^2}$$

Momentum distributions obtained from the Green's functions:

$$n_{\rm F}(\mathbf{k}) = \int \frac{d\omega}{2\pi} G_{\rm F}(\bar{k})\, e^{i\omega 0^+}$$

$$n_{\rm B}(\mathbf{k}) = -\int \frac{d\omega}{2\pi} G'_{\rm B}(\bar{k})\, e^{i\omega 0^+}$$

Coupled equations for chemical potentials and condensate density n_0

Green's functions obtained from the self-energies through Dyson's equations:

$$G_{\rm F}(\bar{k})^{-1} = G^0_{\rm F}(\bar{k})^{-1} - \Sigma_{\rm F}(\bar{k})$$

$$G'_{\rm B}(\bar{k}) = \frac{i\omega + \xi^{\rm B}_{\mathbf{k}} + \Sigma^{11}_{\rm B}(-\bar{k})}{[i\omega + \xi^{\rm B}_{\mathbf{k}} + \Sigma^{11}_{\rm B}(-\bar{k})][i\omega - \xi^{\rm B}_{\mathbf{k}} - \Sigma^{11}_{\rm B}(\bar{k})] + \Sigma^{12}_{\rm B}(\bar{k})^2}$$

Momentum distributions obtained from the Green's functions:

$$n_{\rm F}(\mathbf{k}) = \int \frac{d\omega}{2\pi} G_{\rm F}(\bar{k})\, e^{i\omega 0^+} \qquad n_{\rm B}(\mathbf{k}) = -\int \frac{d\omega}{2\pi} G'_{\rm B}(\bar{k})\, e^{i\omega 0^+}$$

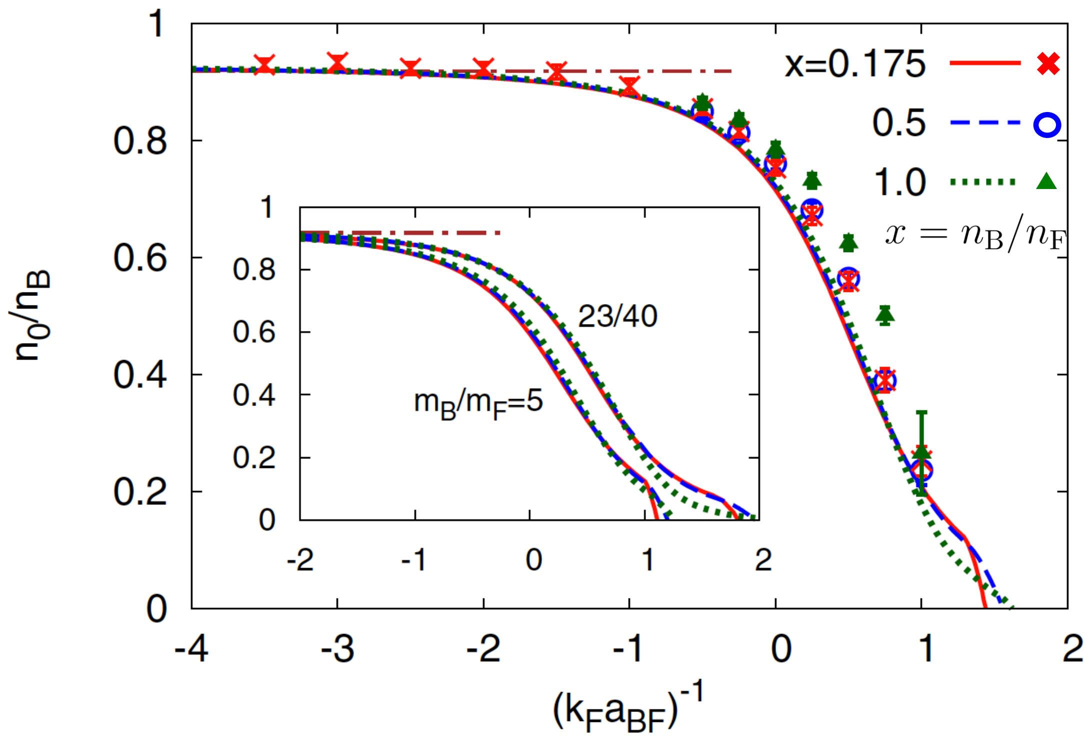
Integration over **k** + Hugenholtz-Pines relation ⇨ coupled eqs. for μ_B, μ_F, n_0:

$$n_{\rm F} = \int \frac{d\mathbf{k}}{(2\pi)^3} n_{\rm F}(\mathbf{k}) \qquad n_{\rm B} = n_0 + \int \frac{d\mathbf{k}}{(2\pi)^3} n_{\rm B}(\mathbf{k})$$

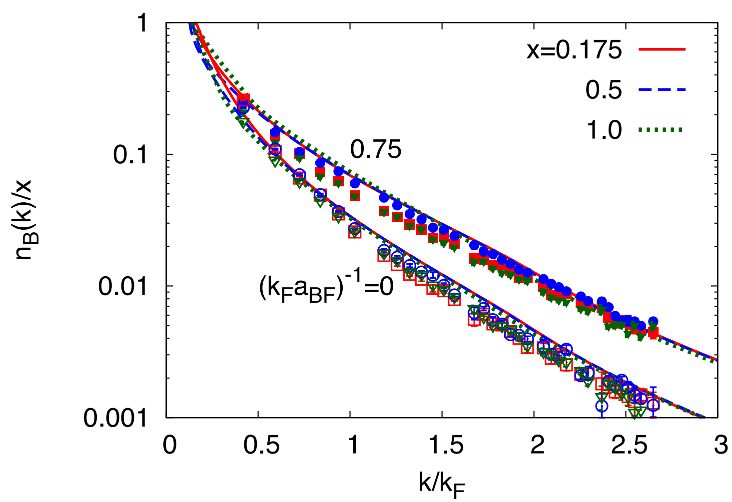
$$\mu_{\rm B} = \Sigma^{11}_{\rm B}(0) - \Sigma^{12}_{\rm B}(0)$$

Universality of condensate fraction and boson momentum distribution

$n_B a_{BB}^3 = 3 \times 10^{-3} \leftrightarrow k_F a_{BB} \gtrsim 0.5$

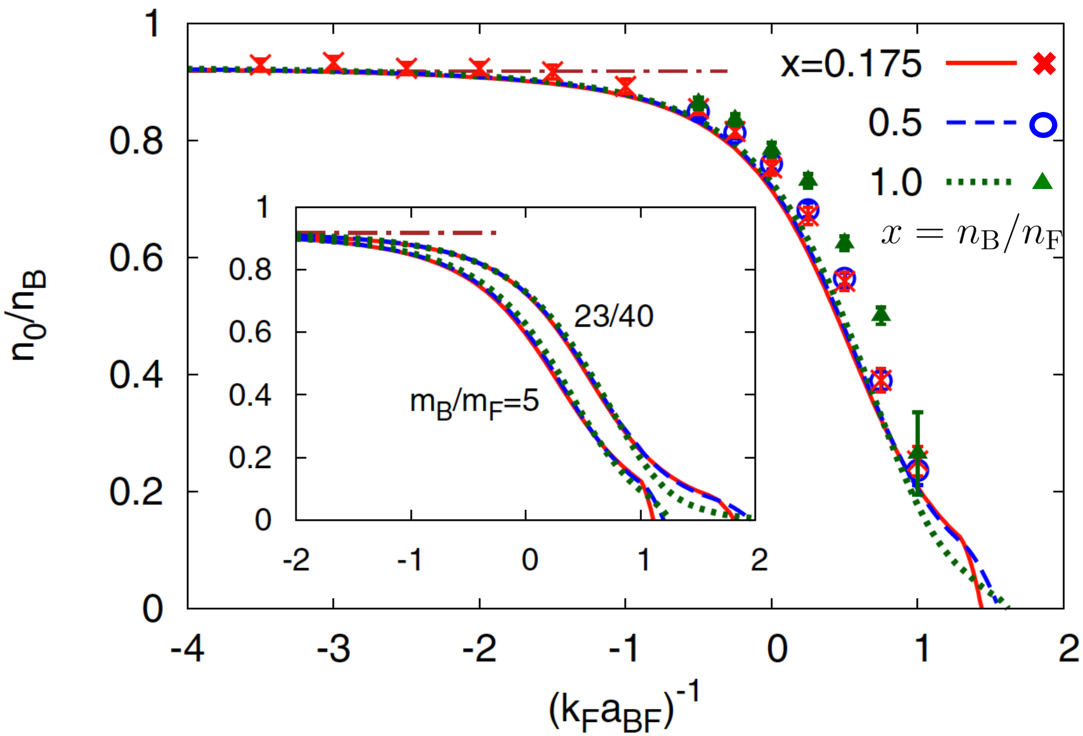


Lines: our diagrammatic calculations
Symbols: FN-DMC by G. Bertaina
Dashed-dotted line: Bogoliubov.

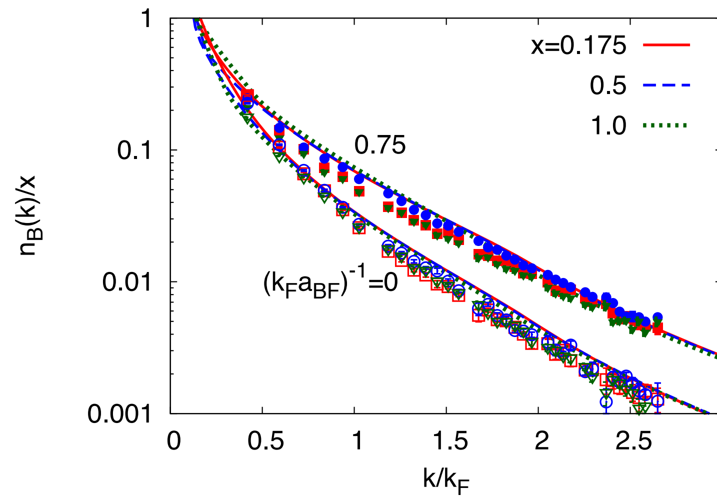


Universality of condensate fraction and boson momentum distribution

$$n_B a_{BB}^3 = 3 \times 10^{-3} \leftrightarrow k_F a_{BB} \gtrsim 0.5$$



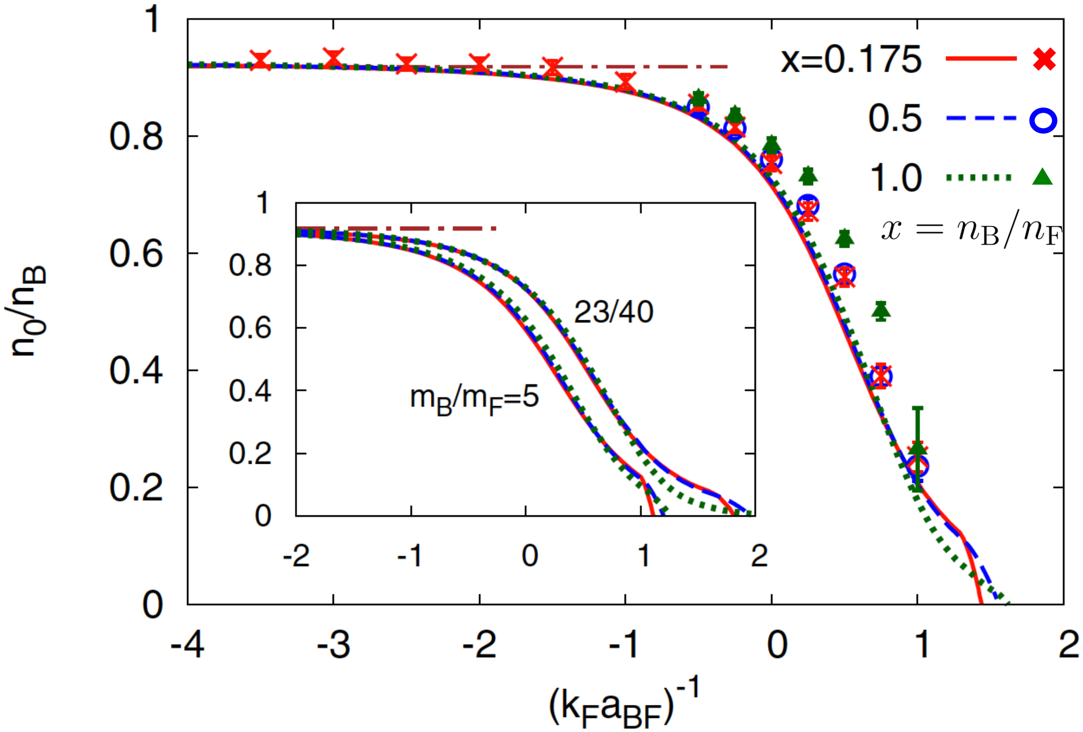
Lines: our diagrammatic calculations
Symbols: FN-DMC by G. Bertaina
Dashed-dotted line: Bogoliubov.



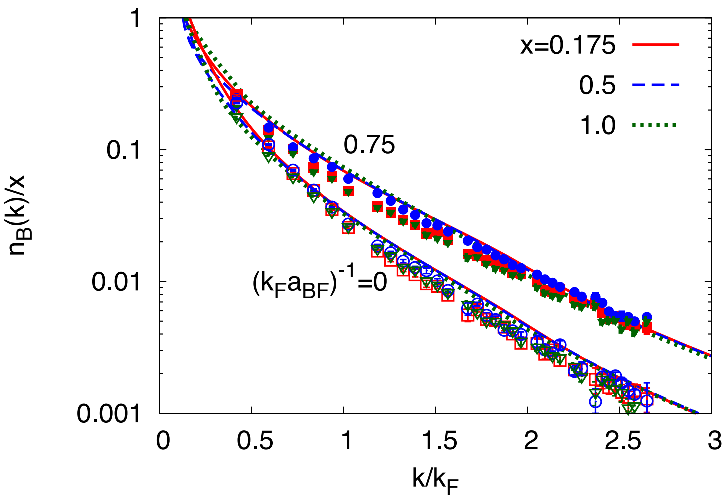
- Condensate fraction vanishes at a critical coupling: **quantum phase transition**.

Universality of condensate fraction and boson momentum distribution

$$n_B a_{BB}^3 = 3 \times 10^{-3} \leftrightarrow k_F a_{BB} \gtrsim 0.5$$



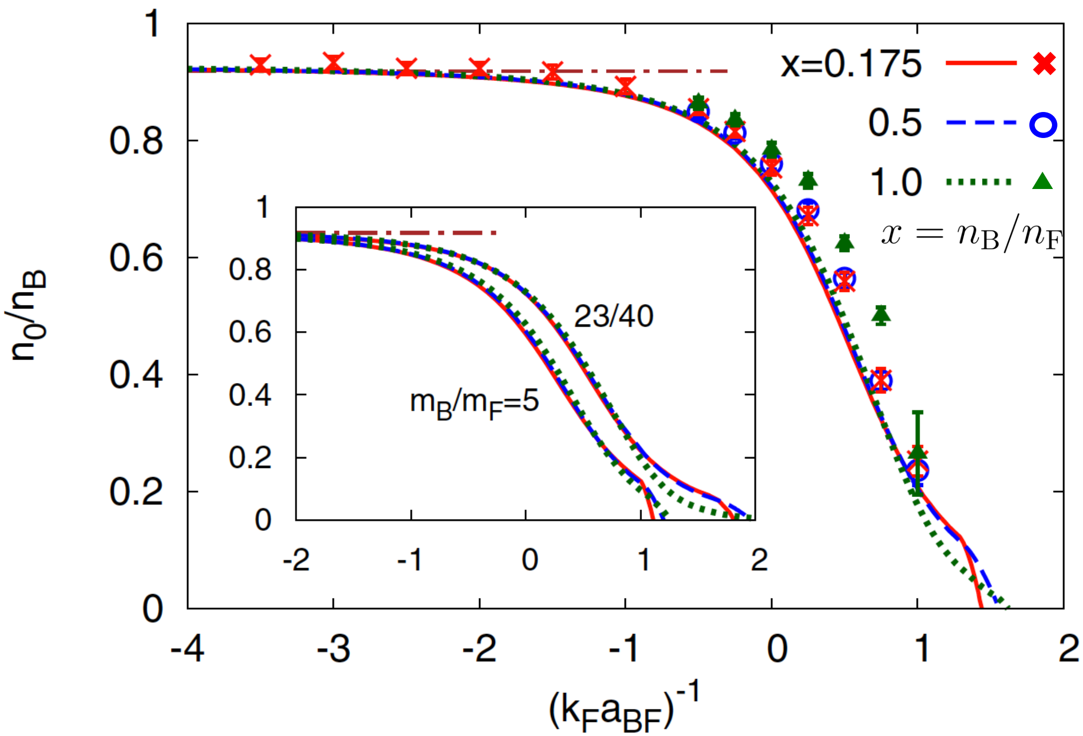
Lines: our diagrammatic calculations
Symbols: FN-DMC by G. Bertaina
Dashed-dotted line: Bogoliubov.



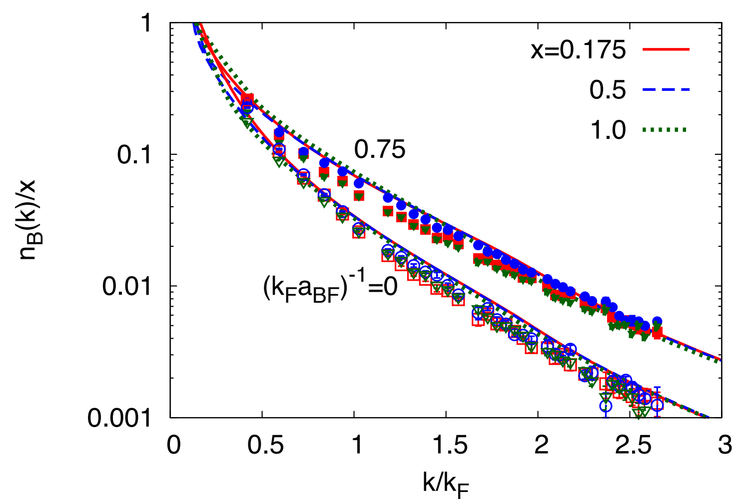
- Condensate fraction vanishes at a critical coupling: **quantum phase transition**.
- Condensate fraction almost **independent of the boson concentration** $x=n_B/n_F$.

Universality of condensate fraction and boson momentum distribution

$$n_B a_{BB}^3 = 3\times 10^{-3} \leftrightarrow k_F a_{BB} \gtrsim 0.5$$



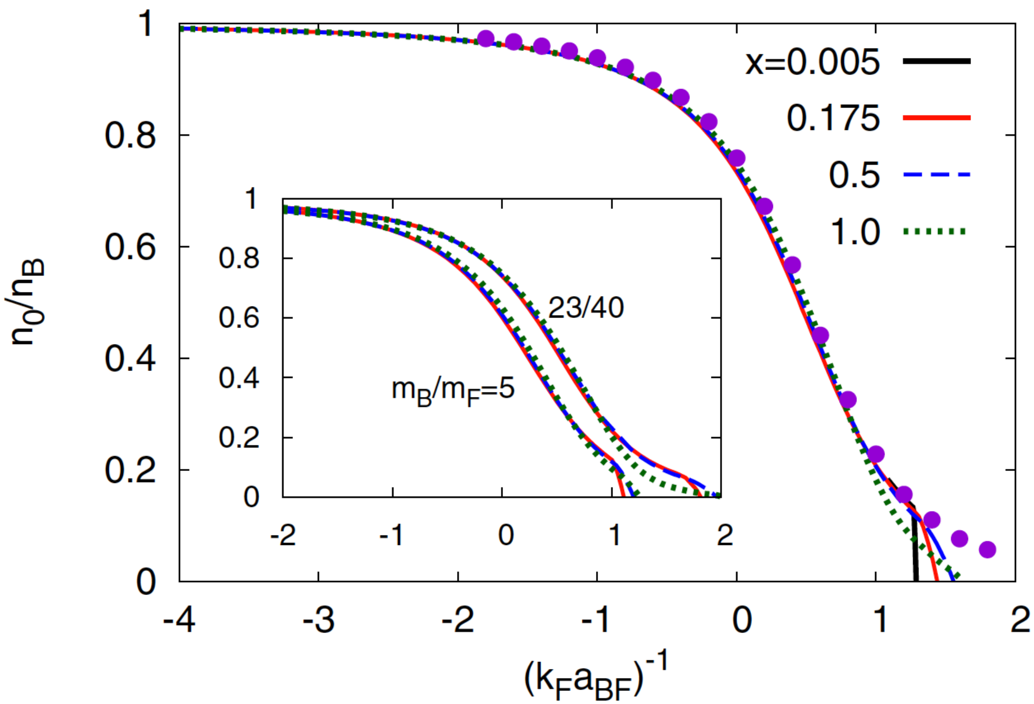
Lines: our diagrammatic calculations
Symbols: FN-DMC by G. Bertaina
Dashed-dotted line: Bogoliubov.



- Condensate fraction vanishes at a critical coupling: **quantum phase transition**.
- Condensate fraction almost **independent of the boson concentration** $x=n_B/n_F$.
- Universality also for the momentum distribution (once normalized by n_B). It suggests:

$$n_B(k) = n_B \times V n_{\mathrm{pol}}(k) = N_B \times n_{\mathrm{pol}}(k)$$

Comparison with (Fermi) polaron quasiparticle residue Z



Lines: our calculations at four different concentrations for **zero Bose repulsion**.

Circles: Diagrammatic MC results for Z [J. Vlietinck, J. Ryckebusch, K. Van Houcke, PRB **87**, 115133 (2013)]

Polaron problem: single (mobile) impurity interacting with a Fermi sea.

Quasiparticle residue for the polaron $Z = \left[1 - \frac{\partial}{\partial\omega}\mathrm{Re}\Sigma_R(k=0,\omega)\right]^{-1}_{\omega=0}$ where Σ_R is the (retarded) self-energy of the impurity.

Surprising agreement between ‘universal condensate fraction’ and Fermi polaron quasiparticle residue.

nature physics

Article

https://doi.org/10.1038/s41567-023-01948-1

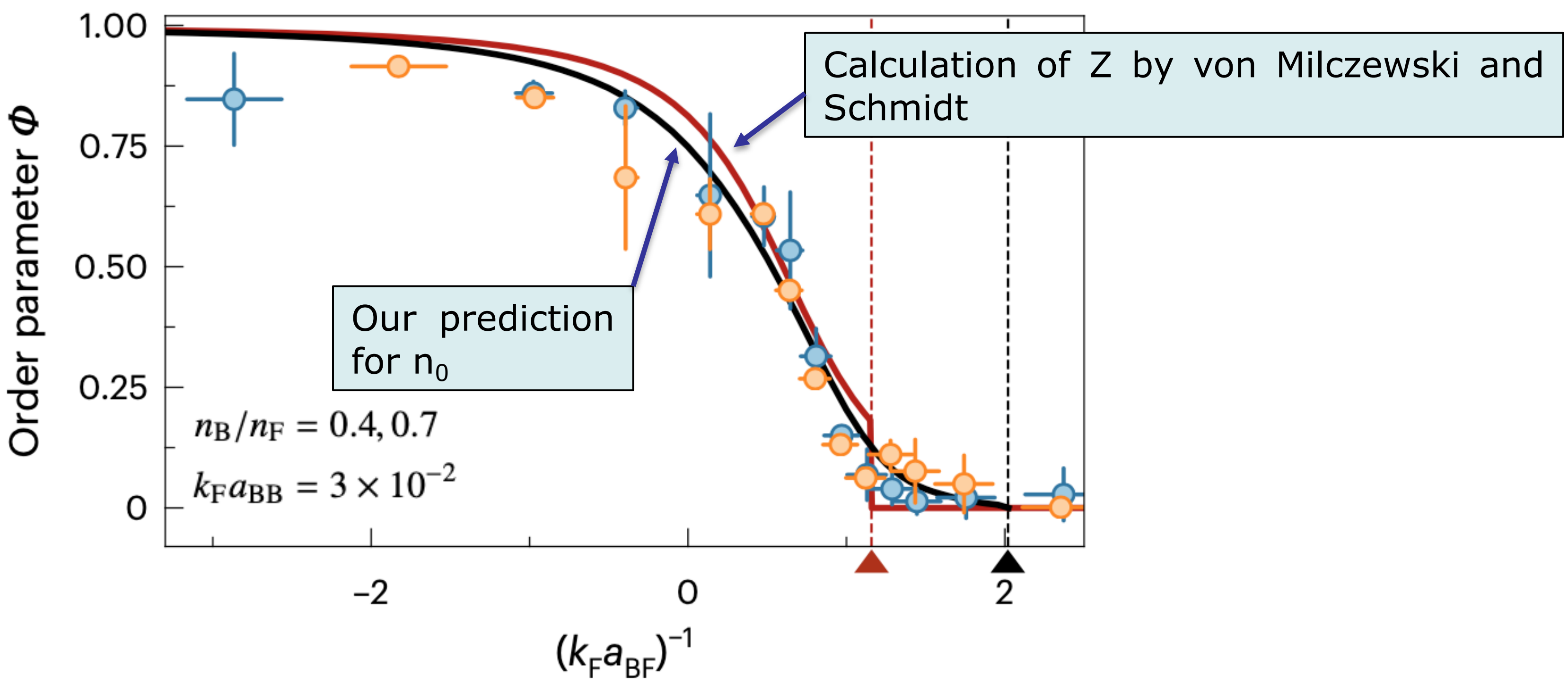
Transition from a polaronic condensate to a degenerate Fermi gas of heteronuclear molecules

Received: 28 July 2022

Accepted: 6 January 2023

Marcel Duda [1,2], Xing-Yan Chen [1,2], Andreas Schindewolf [1,2], Roman Bause [1,2], Jonas von Milczewski [1,2], Richard Schmidt [1,2,3,4], Immanuel Bloch [1,2,5] & Xin-Yu Luo [1,2]

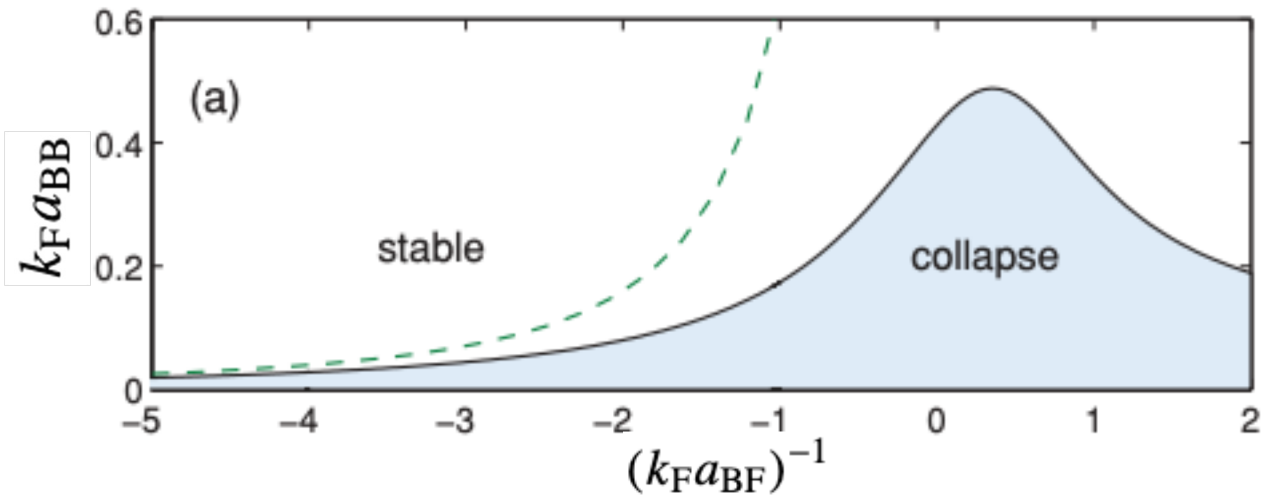
^{23}Na-^{40}K Bose-Fermi mixture with broad Feshbach resonance ($k_F R^* = 0.08$).



PHYSICAL REVIEW A **83**, 041603(R) (2011)

Stability condition of a strongly interacting boson-fermion mixture across an interspecies Feshbach resonance

Zeng-Qiang Yu,[1] Shizhong Zhang,[2] and Hui Zhai[1]

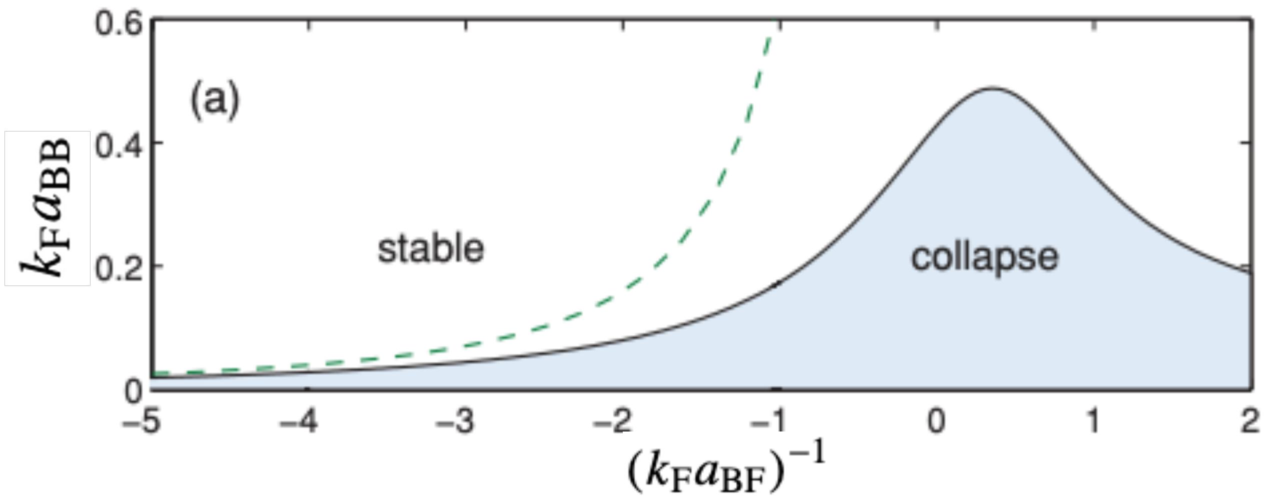


- Stability condition calculated for $n_B/n_F << 1$ and $m_B = m_F$ with lowest-order constrained variational approximation over Jastrow-Slater wave-function.

PHYSICAL REVIEW A **83**, 041603(R) (2011)

Stability condition of a strongly interacting boson-fermion mixture across an interspecies Feshbach resonance

Zeng-Qiang Yu,[1] Shizhong Zhang,[2] and Hui Zhai[1]



- Stability condition calculated for $n_B/n_F << 1$ and $m_B = m_F$ with lowest-order constrained variational approximation over Jastrow-Slater wave-function.

- BB repulsion required for stability of BF mixture at unitarity more than **one order of magnitude** larger than BB repulsion in the experiment.

Stability from compressibility matrix

We have studied the stability by calculating the compressibility matrix $\mathcal{M}$ within our diagrammatic approach.

$$\mathcal{M} = \begin{pmatrix} \frac{\partial \mu_F}{\partial n_F} & \frac{\partial \mu_F}{\partial n_B} \\ \frac{\partial \mu_B}{\partial n_F} & \frac{\partial \mu_B}{\partial n_B} \end{pmatrix}$$

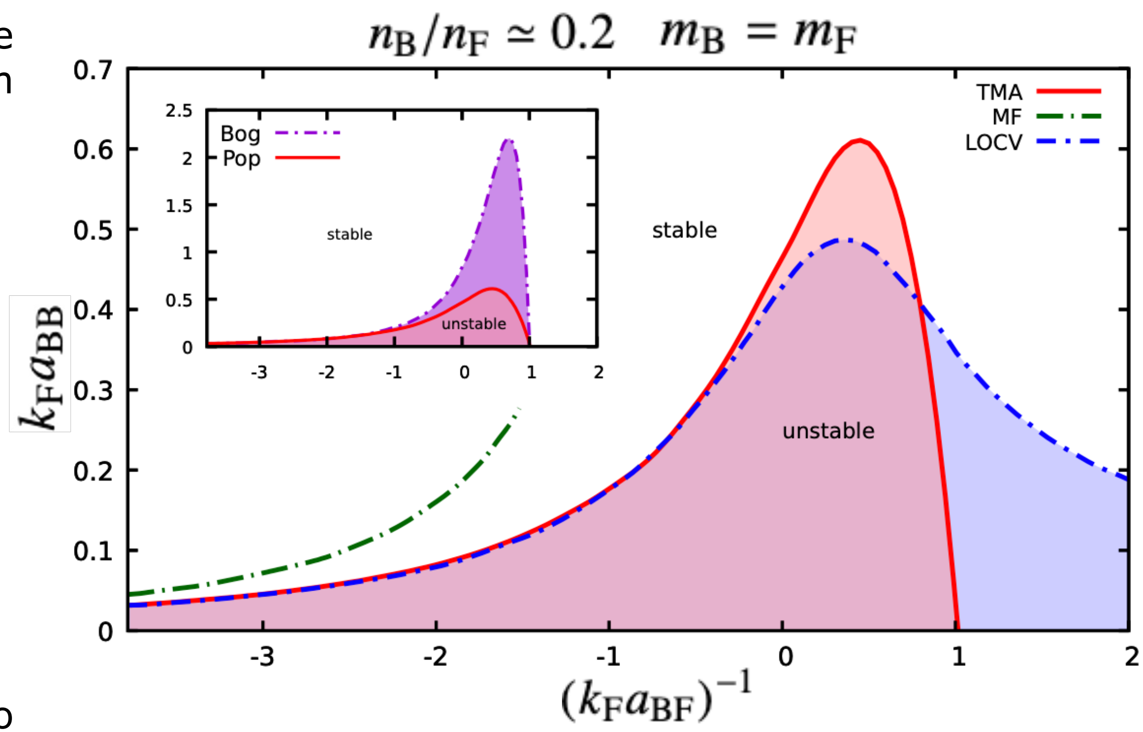
We have improved the description of BB repulsion from Bogoliubov:

$$\Sigma_B^{11} = 8\pi n_0 a_{BB}/m_B$$
$$\Sigma_B^{12} = 4\pi n_0 a_{BB}/m_B$$

to Popov:

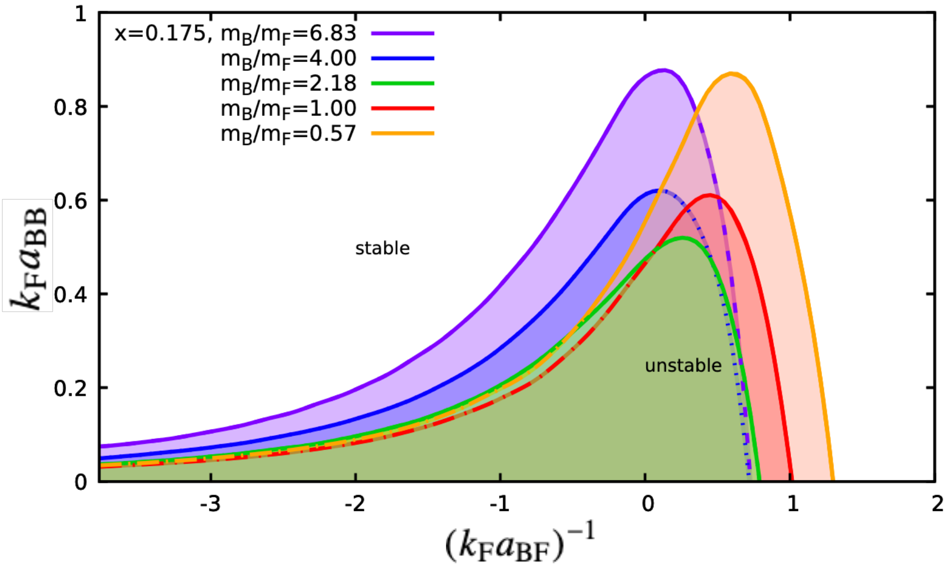
$$\Sigma_B^{11} = 8\pi n_B a_{BB}/m_B$$
$$\Sigma_B^{12} = 4\pi n_0 a_{BB}/m_B$$

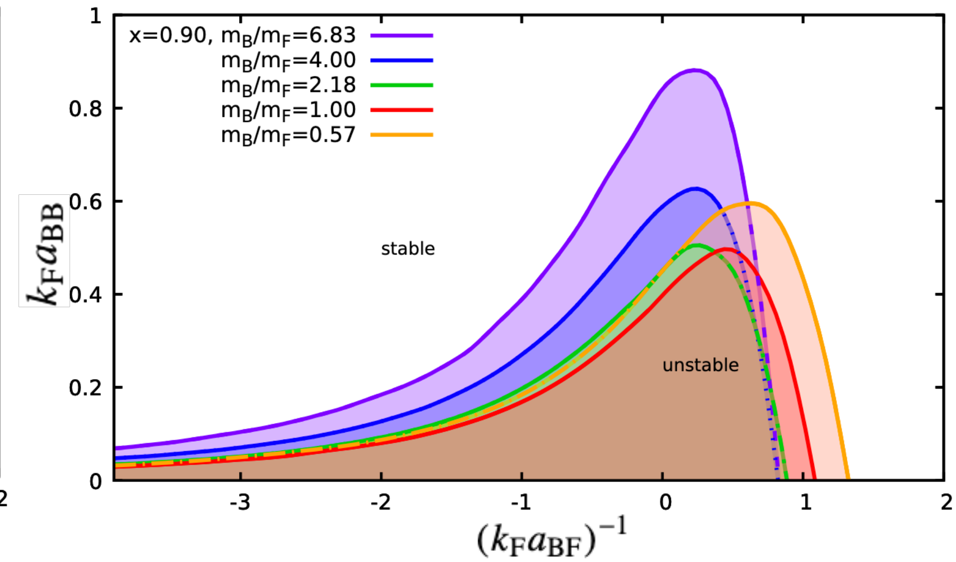
This improvement leads to negligible differences in the condensate fraction and momentum distributions, but to significant differences in the stability when n_0 is small.

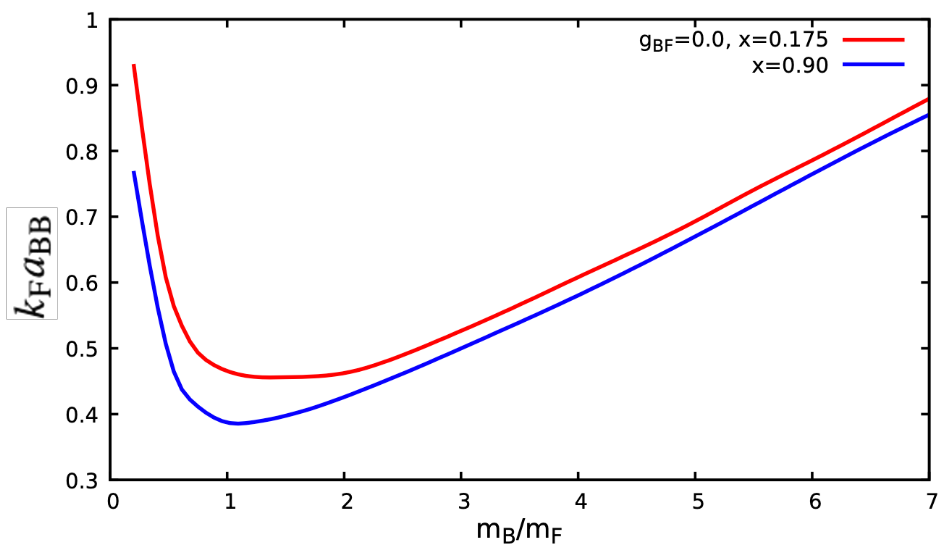


C. Gualerzi, L. Pisani, P. Pieri (in preparation)

Analysis of stability for different mass ratios







C. Gualerzi, L. Pisani, P. Pieri (in preparation)

$m_B/m_F=0.57$ for ^{23}Na-^{40}K mixture.

A repulsion $k_F a_{BB} \gtrsim 0.5$ would be required to guarantee stability for all values of $(k_F a_{BF})^{-1}$.

Collapse did not occur during the timescale of the experiment:
→ **metastable long-lived many-body phase**!

arXiv:2405.05029

Boson-fermion pairing and condensation in two-dimensional Bose-Fermi mixtures

Leonardo Pisani,[1,2] Pietro Bovini,[1,2] Fabrizio Pavan,[3] and Pierbiagio Pieri[1,2]

[1]*Dipartimento di Fisica e Astronomia "Augusto Righi",*
Università di Bologna, Via Irnerio 46, I-40126, Bologna, Italy
[2]*INFN, Sezione di Bologna, Viale Berti Pichat 6/2, I-40127, Bologna, Italy*
[3]*Dipartimento di Fisica E. Pancini - Università di Napoli Federico II - I-80126 Napoli, Italy*

Motivation for the 2D case

- 2D confinement provides an **extra knob**: BB repulsion could be controlled by varying the confinement length (**confinement induced resonance**) while the BF attraction is varied with a (3D) **Feshbach resonance**, or vice versa.

Motivation for the 2D case

- 2D confinement provides an **extra knob**: BB repulsion could be controlled by varying the confinement length (**confinement induced resonance**) while the BF attraction is varied with a (3D) **Feshbach resonance**, or vice versa.

•Fermi-Bose dimers could be a **platform** to realize a **p-wave superfluid** according to a proposal by Bazak & Petrov:

PHYSICAL REVIEW LETTERS **121**, 263001 (2018)

Stable *p*-Wave Resonant Two-Dimensional Fermi-Bose Dimers

B. Bazak[1] and D. S. Petrov[2]

We consider two-dimensional weakly bound heterospecies molecules formed in a Fermi-Bose mixture with attractive Fermi-Bose and repulsive Bose-Bose interactions. Bosonic exchanges lead to an intermolecular attraction, which can be controlled and tuned to a *p*-wave resonance. Such attractive fermionic molecules can be realized in quasi-two-dimensional ultracold isotopic mixtures. We show that they are stable with respect to the recombination to deeply bound molecular states and with respect to the formation of higher-order clusters (trimers, tetramers, etc.)

The model (2d case)

- **Two-component Hamiltonian** with attractive contact interaction between bosons and fermions.

$$H_{\mathrm{BF}} = \sum_{s=\mathrm{B,F}} \int d\mathbf{r}\, \psi_s^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m_s} - \mu_s \right) \psi_s(\mathbf{r}) + v_0^{\mathrm{BF}} \int d\mathbf{r}\, \psi_{\mathrm{B}}^\dagger(\mathbf{r}) \psi_{\mathrm{F}}^\dagger(\mathbf{r}) \psi_{\mathrm{F}}(\mathbf{r}) \psi_{\mathrm{B}}(\mathbf{r})$$

- Bare contact-interaction strength between bosons and fermions expressed in terms of 2D **boson-fermion** scattering length a_{BF} .

$$\frac{1}{v_0^{\mathrm{BF}}} = -\int \frac{d\mathbf{k}}{(2\pi)^2} \frac{1}{\varepsilon_0 + \frac{k^2}{2m_r}} \qquad \varepsilon_0 = 1/(2m_r a_{\mathrm{BF}}^2)$$

The model (2d case)

- **Two-component Hamiltonian** with attractive contact interaction between bosons and fermions.

$$H_{\rm BF} = \sum_{s={\rm B,F}} \int d\mathbf{r}\, \psi_s^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m_s} - \mu_s\right) \psi_s(\mathbf{r}) + v_0^{\rm BF} \int d\mathbf{r}\, \psi_{\rm B}^\dagger(\mathbf{r})\psi_{\rm F}^\dagger(\mathbf{r})\psi_{\rm F}(\mathbf{r})\psi_{\rm B}(\mathbf{r})$$

- Bare contact-interaction strength between bosons and fermions expressed in terms of 2D **boson-fermion** scattering length $a_{\rm BF}$.

$$\frac{1}{v_0^{\rm BF}} = -\int \frac{d\mathbf{k}}{(2\pi)^2} \frac{1}{\varepsilon_0 + \frac{k^2}{2m_r}}$$

$$\varepsilon_0 = 1/(2m_r a_{\rm BF}^2)$$

- Boson-boson short-range (weak) repulsion:

$$H = H_{\rm BF} + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' V_{\rm BB}(\mathbf{r}-\mathbf{r}')\psi_B^\dagger(\mathbf{r})\psi_B^\dagger(\mathbf{r}')\psi_B(\mathbf{r}')\psi_B(\mathbf{r})$$

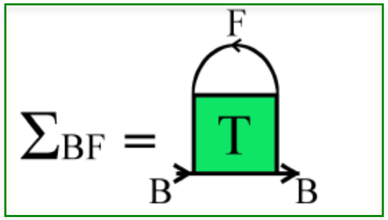
- We focus on equal masses $m_{\rm F}=m_{\rm B}$.

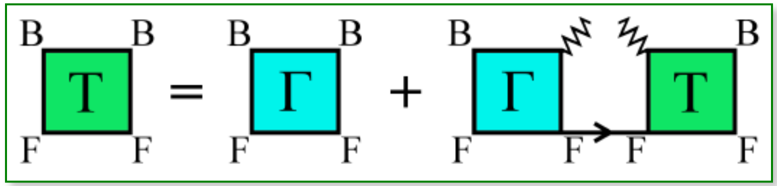
Dimensionless coupling strengths in 2D:

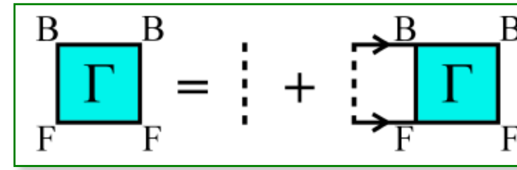
$g_{\rm BF} = -\ln(k_{\rm F} a_{\rm BF})$ for the (resonant) BF attraction

$1/\left|\ln\left(n_{\rm B} a_{\rm BB}^2\right)\right|$ for the (weak) BB repulsion.

Bosonic and fermionic self-energy diagrams for the **condensed** phase







Boson self-energy

$$\Sigma_B^{11}\left(\bar{k}\right) = \frac{8\pi n_0}{m_B\left|\ln\left(n_B a_{BB}^2\right)\right|} + \Sigma_{BF}\left(\bar{k}\right)$$

$$\Sigma_B^{12} = \frac{4\pi n_0}{m_B\left|\ln\left(n_B a_{BB}^2\right)\right|}$$

$$\Sigma_{BF}\left(\bar{k}\right) = \int \frac{d\mathbf{P}}{(2\pi)^2}\int \frac{d\Omega}{2\pi} T(\bar{P}) G_F^0\left(\bar{P}-\bar{k}\right) e^{i\Omega 0^+}$$

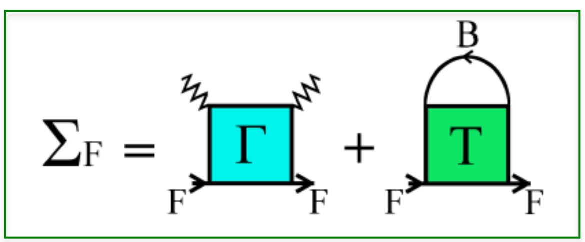
Boson-fermion T-matrix

$$T(\bar{P})^{-1} = \Gamma(\bar{P})^{-1} - n_0 G_F^0(\bar{P})$$

$$\Gamma(\bar{P})^{-1} = -\frac{m_r}{2\pi}\ln\left(\frac{\frac{P^2}{2M}-\mu_F-\mu_B-i\Omega}{\varepsilon_0}\right) - I_F(\bar{P})$$

$$I_F\left(\bar{P}\right) = \int \frac{d\mathbf{k}}{(2\pi)^2}\frac{\Theta\left(-\xi^F_{\mathbf{P-k}}\right)}{\xi^F_{\mathbf{P-k}}+\xi^B_{\mathbf{k}}-i\Omega}$$

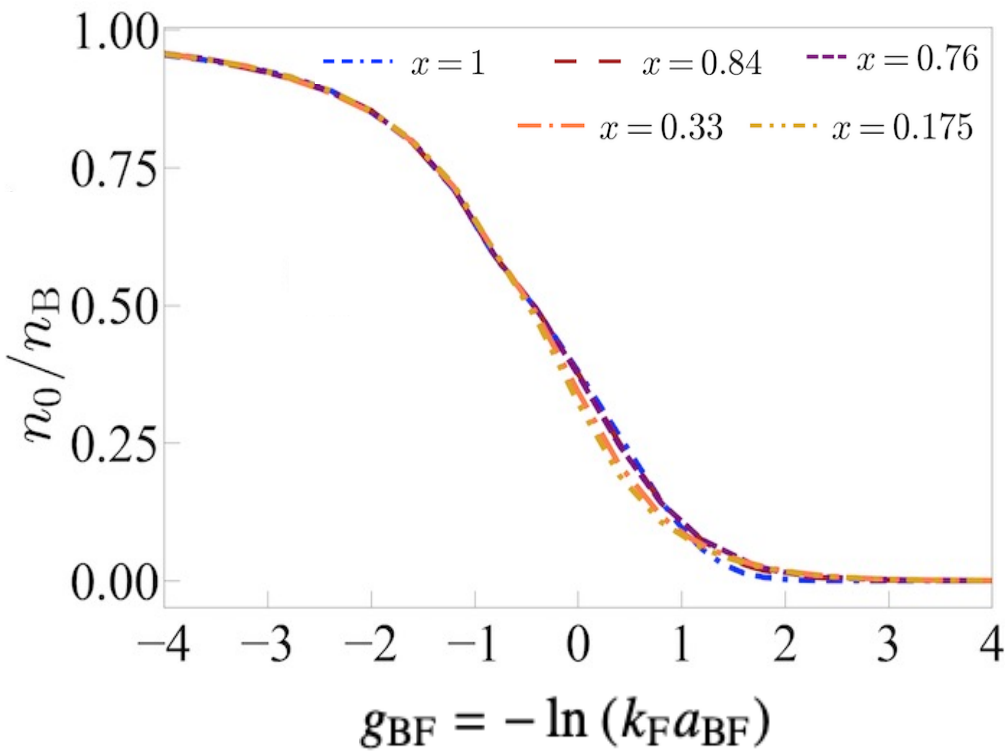
$\varepsilon_0 = 1/(2m_r a_{BF}^2)$

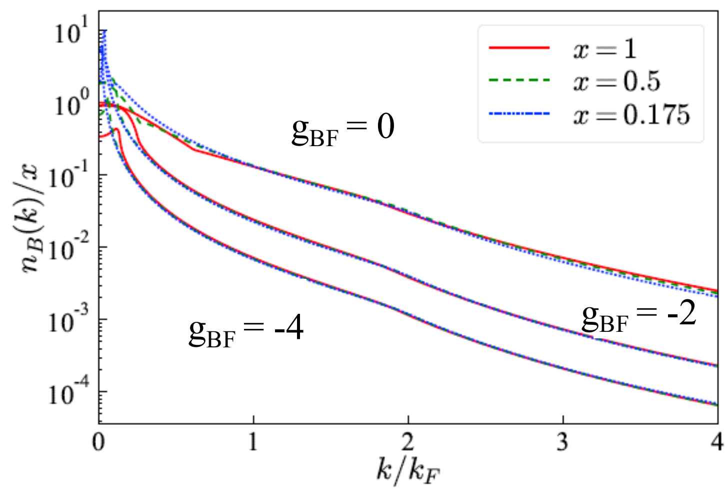


Fermion self-energy

$$\Sigma_F(\bar{k}) = n_0\Gamma(\bar{k}) - \int \frac{d\mathbf{P}}{(2\pi)^3}\int \frac{d\Omega}{2\pi} T(\bar{P}) G_B^0(\bar{P}-\bar{k})$$

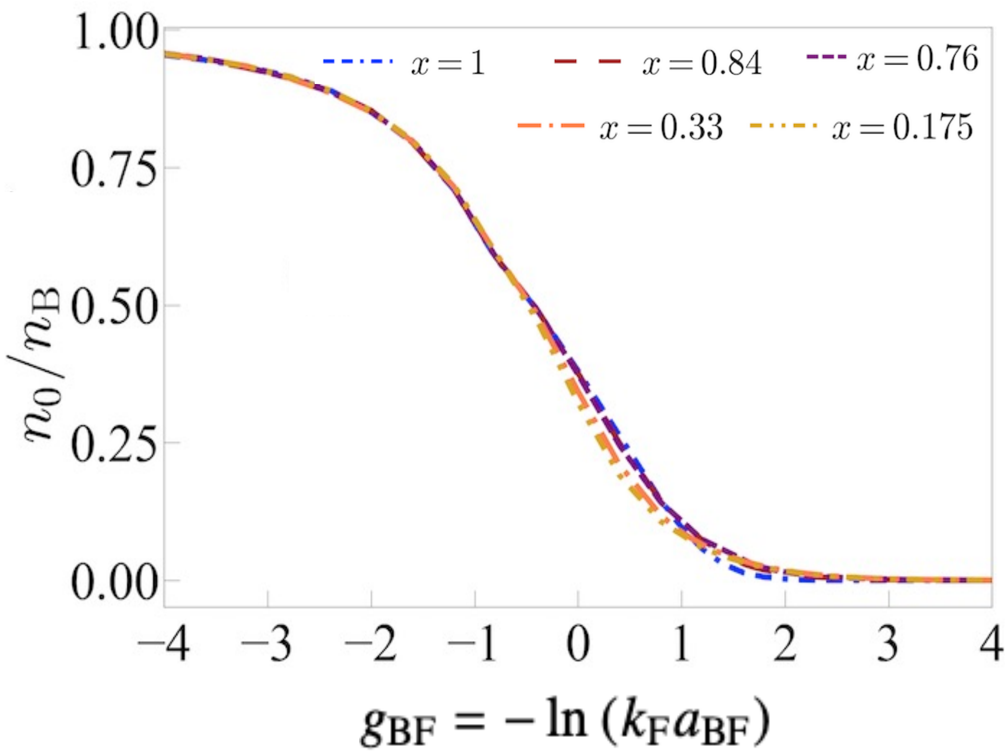
Universality of condensate fraction and boson momentum distribution

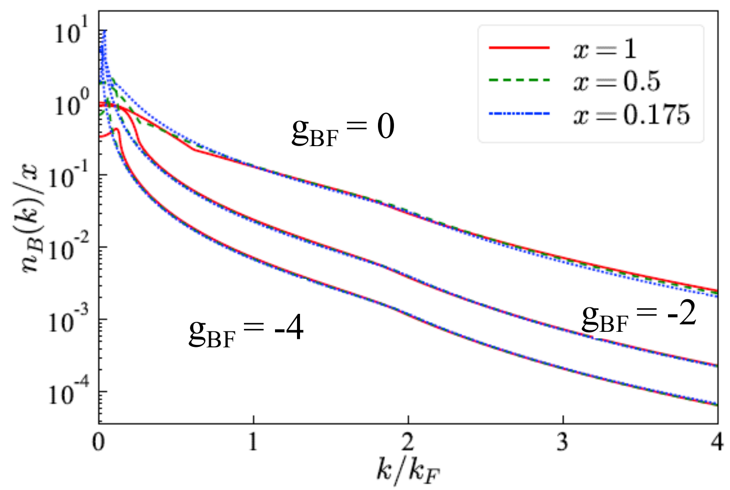




- Like in 3D, condensate fraction and momentum distribution display **universal behavior**.

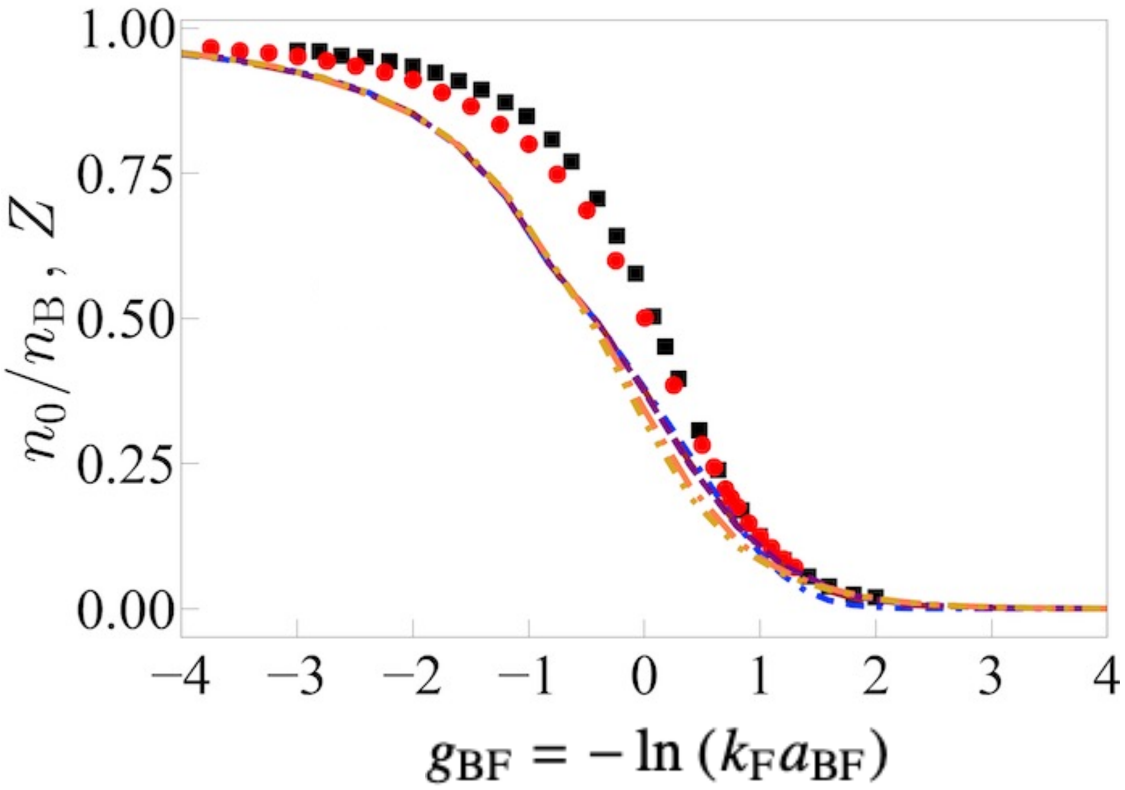
Universality of condensate fraction and boson momentum distribution





- Like in 3D, condensate fraction and momentum distribution display **universal behavior**.
- However, in contrast with 3D, the condensate **does not exactly vanish** beyond a critical coupling. It remains finite (**albeit exponentially small** at large BF coupling strength).

Comparison with (Fermi) polaron quasiparticle residue

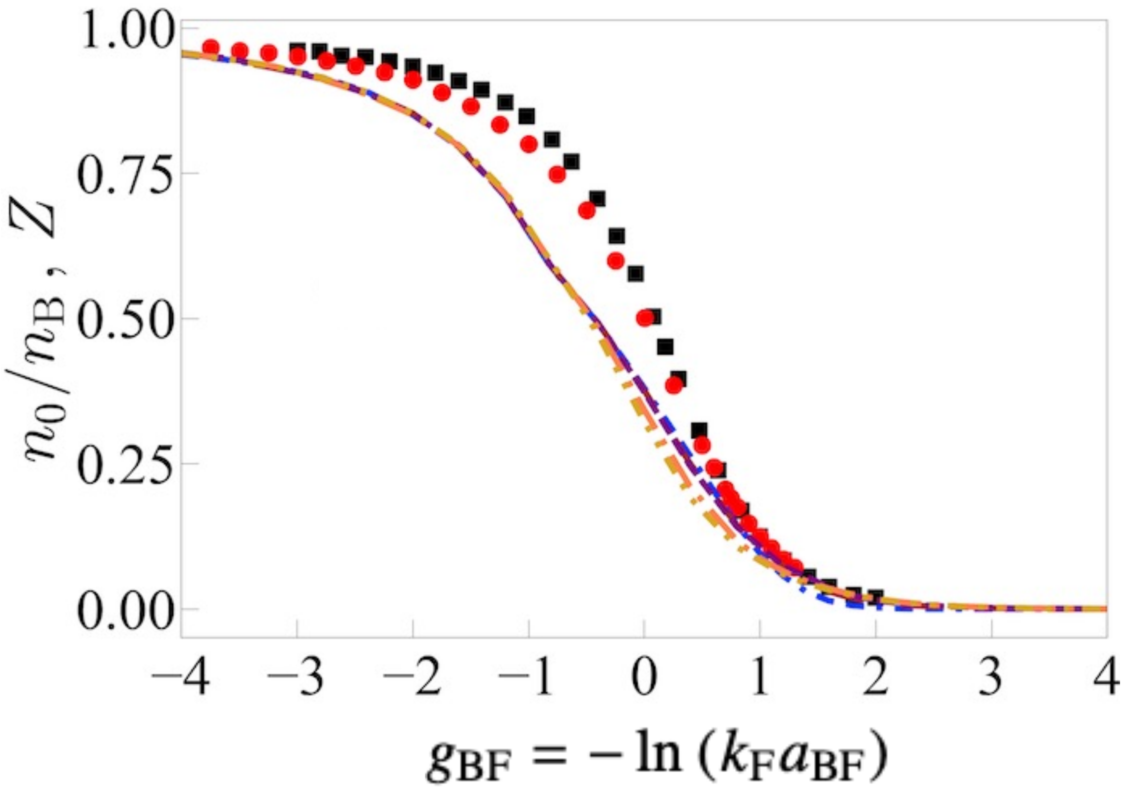


■ T-matrix results for Z [R. Schmidt, T. Enss, V. Pietilä, E. Demler, PRA **85**, 021602 (2012)]

● Diagrammatic MC results for Z [J. Vlietinck, J. Ryckebusch, K. Van Houcke PRB **89**, 085119 (2014)]

- **Agreement** with polaron residue **disappears in 2D**.

Comparison with (Fermi) polaron quasiparticle residue

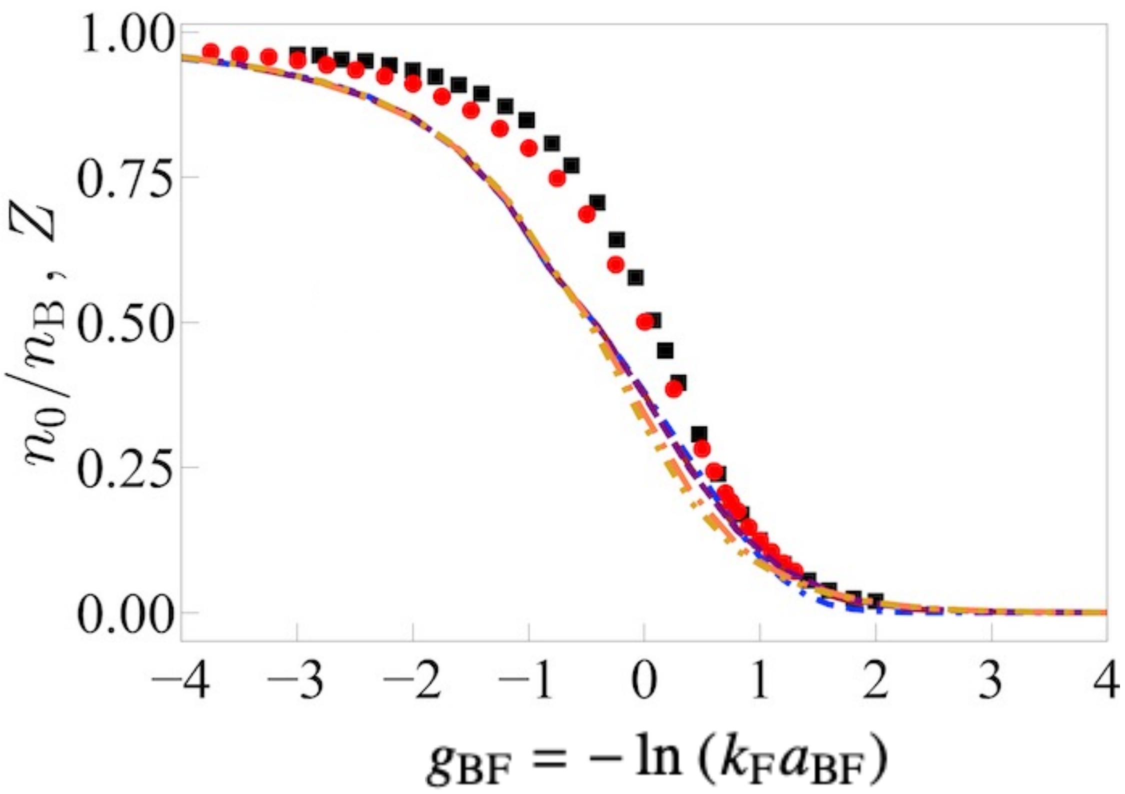


■ T-matrix results for Z [R. Schmidt, T. Enss, V. Pietilä, E. Demler, PRA **85**, 021602 (2012)]

● Diagrammatic MC results for Z [J. Vlietinck, J. Ryckebusch, K. Van Houcke PRB **89**, 085119 (2014)]

- **Agreement** with polaron residue **disappears in 2D**.
- The T-matrix results for Z (■) are based on the same self-energy as ours when restricted to the polaron limit → difference between condensate fraction and Z is **not due to different levels of approximation**.

Comparison with (Fermi) polaron quasiparticle residue



■ T-matrix results for Z [R. Schmidt, T. Enss, V. Pietilä, E. Demler, PRA **85**, 021602 (2012)]

● Diagrammatic MC results for Z [J. Vlietinck, J. Ryckebusch, K. Van Houcke PRB **89**, 085119 (2014)]

- **Agreement** with polaron residue **disappears in 2D**.
- The T-matrix results for Z (■) are based on the same self-energy as ours when restricted to the polaron limit → difference between condensate fraction and Z is **not due to different levels of approximation**.
- Was the 'degeneracy' between condensate fraction and Z found in 3D just accidental?

Conclusions

Conclusions

- Our predictions for condensate fraction as a function of BF attraction in a 3D BF mixture recently confirmed experimentally by Duda et al. (2023), including the apparent connection with the polaron quasiparticle residue.

Conclusions

- Our predictions for condensate fraction as a function of BF attraction in a 3D BF mixture recently confirmed experimentally by Duda et al. (2023), including the apparent connection with the polaron quasiparticle residue.

- BB repulsion in experiment was way below the threshold to guarantee mechanical stability: metastable (nonequilibrium) phase has been observed in the exp, which is however well described by our equilibrium theory.

Conclusions

- Our predictions for condensate fraction as a function of BF attraction in a 3D BF mixture recently confirmed experimentally by Duda et al. (2023), including the apparent connection with the polaron quasiparticle residue.

- BB repulsion in experiment was way below the threshold to guarantee mechanical stability: metastable (nonequilibrium) phase has been observed in the exp, which is however well described by our equilibrium theory.

- Calculations in 2D confirm ‘universal behavior’ of condensate fraction and bosonic momentum distribution found in 3D.

Conclusions

- Our predictions for condensate fraction as a function of BF attraction in a 3D BF mixture recently confirmed experimentally by Duda et al. (2023), including the apparent connection with the polaron quasiparticle residue.

- BB repulsion in experiment was way below the threshold to guarantee mechanical stability: metastable (nonequilibrium) phase has been observed in the exp, which is however well described by our equilibrium theory.

- Calculations in 2D confirm ‘universal behavior’ of condensate fraction and bosonic momentum distribution found in 3D.

- Important differences with respect to 3D: i) ‘degeneracy’ with polaron quasiparticle residue removed in 2D ii) Quantum phase transition to molecular phase becomes a (rapid) crossover in 2D.

Conclusions

- Our predictions for condensate fraction as a function of BF attraction in a 3D BF mixture recently confirmed experimentally by Duda et al. (2023), including the apparent connection with the polaron quasiparticle residue.

- BB repulsion in experiment was way below the threshold to guarantee mechanical stability: metastable (nonequilibrium) phase has been observed in the exp, which is however well described by our equilibrium theory.

- Calculations in 2D confirm ‘universal behavior’ of condensate fraction and bosonic momentum distribution found in 3D.

- Important differences with respect to 3D: i) ‘degeneracy’ with polaron quasiparticle residue removed in 2D ii) Quantum phase transition to molecular phase becomes a (rapid) crossover in 2D.

Thank you!

More material…

PHYSICAL REVIEW A **109**, 053302 (2024)

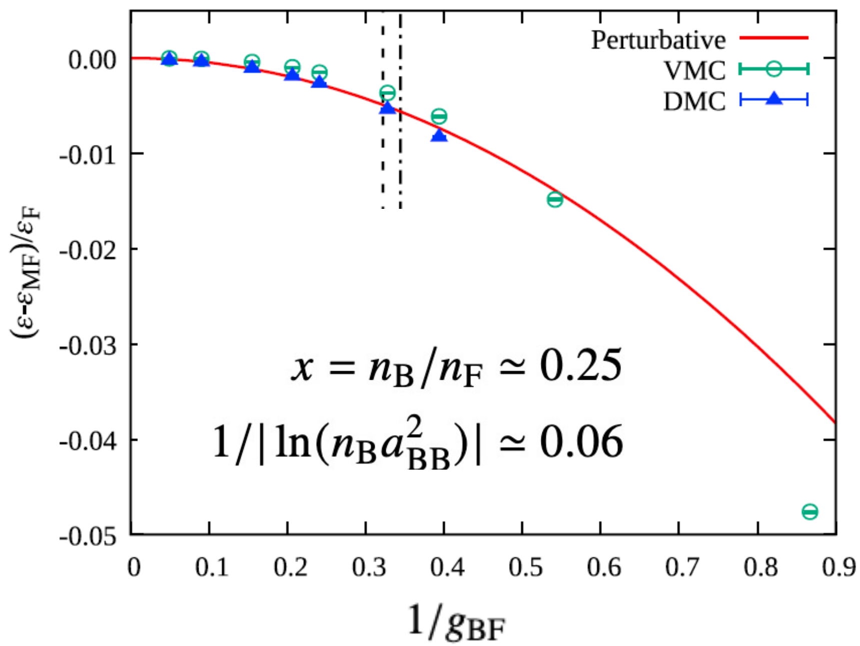
Editors' Suggestion

Quantum Monte Carlo and perturbative study of two-dimensional Bose-Fermi mixtures

Jacopo D'Alberto,[1] Lorenzo Cardarelli,[2] Davide Emilio Galli,[1] Gianluca Bertaina,[3,*] and Pierbiagio Pieri[4,5,†]

[1] *Dipartimento di Fisica "Aldo Pontremoli", Università degli Studi di Milano, via Celoria 16, I-20133 Milano, Italy*
[2] *PASQAL, 7 rue Léonard de Vinci, 91300 Massy, France*
[3] *Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, I-10135 Torino, Italy*
[4] *Dipartimento di Fisica e Astronomia "Augusto Righi", Università di Bologna, Via Irnerio 46, I-40126, Bologna, Italy*
[5] *INFN, Sezione di Bologna, Viale Berti Pichat 6/2, I-40127, Bologna, Italy*

Quantum Monte Carlo + perturbative calculation



Equal masses

$$\mu_{\rm B} = \frac{4\pi n_0 \eta_{\rm B}}{m_{\rm B}} + \frac{E_{\rm F}}{g_{\rm BF}}\left[1 - \frac{1}{g_{\rm BF}}\left(\ln 2 - \frac{1}{2}\right)\right]$$

$$\mu_{\rm F} = E_{\rm F} + x\frac{E_{\rm F}}{g_{\rm BF}}\left[1 + \frac{1}{g_{\rm BF}}(1 - \ln 2)\right]$$

Generic mass ratio $\alpha = m_{\rm B}/m_{\rm F}$

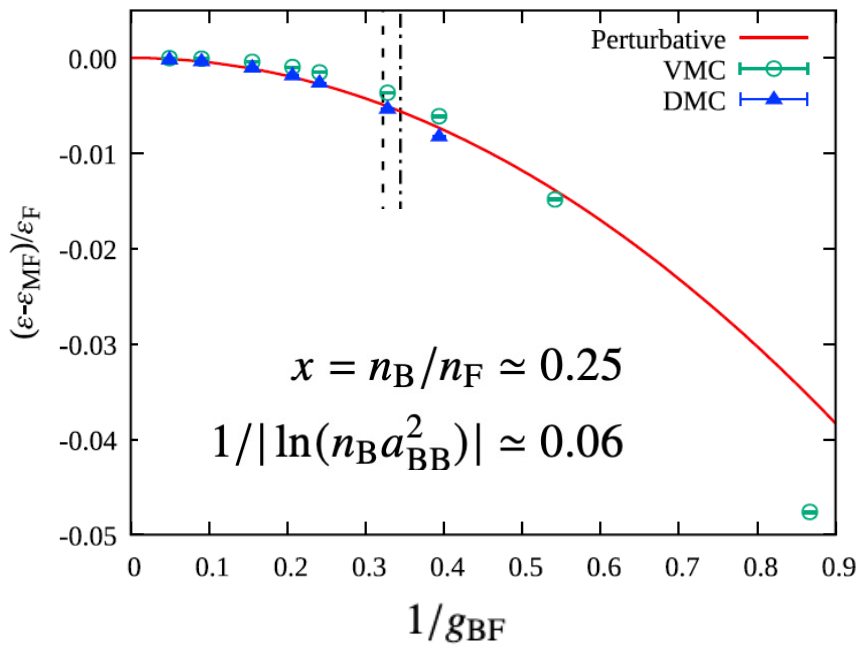
$$\mu_{\rm B} = \frac{4\pi n_0 \eta_{\rm B}}{m_{\rm B}} + \frac{\alpha+1}{2\alpha}\frac{E_{\rm F}}{g_{\rm BF}}\left[1 + \frac{1}{g_{\rm BF}}\ln\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{\sqrt{e}(\alpha+1)}\right]$$

$$\mu_{\rm F} = E_{\rm F} + \frac{\alpha+1}{2\alpha}x\frac{E_{\rm F}}{g_{\rm BF}}\left[1 + \frac{1}{g_{\rm BF}}\ln\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{\alpha+1}\right]$$

Second-order perturbative calculation valid for both attractive and repulsive interaction.
QMC performed for repulsive BF mixture.

Perturbative curve works well till line of instability (and even slightly above it).

Quantum Monte Carlo + perturbative calculation



Equal masses

$$\mu_B = \frac{4\pi n_0 \eta_B}{m_B} + \frac{E_F}{g_{BF}}\left[1 - \frac{1}{g_{BF}}\left(\ln 2 - \frac{1}{2}\right)\right]$$

$$\mu_F = E_F + x\frac{E_F}{g_{BF}}\left[1 + \frac{1}{g_{BF}}(1 - \ln 2)\right]$$

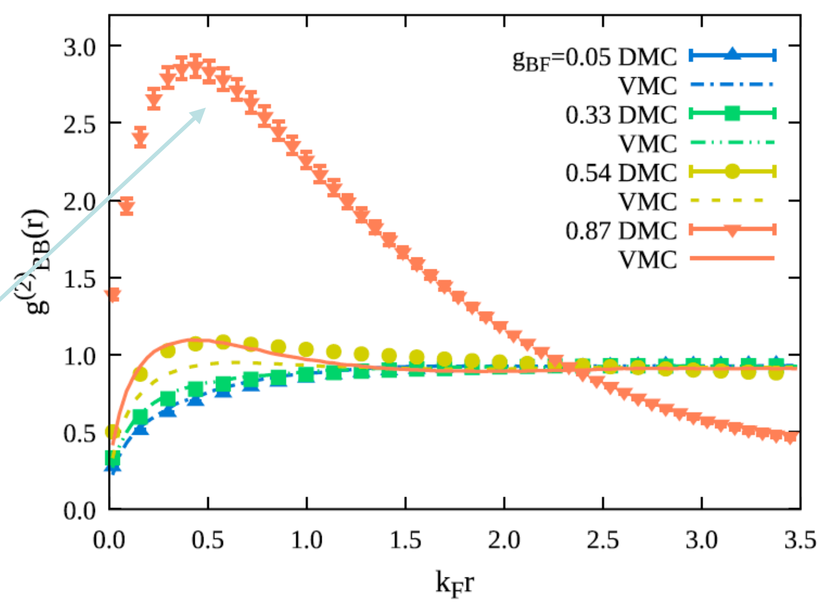
Generic mass ratio $\alpha = m_B/m_F$

$$\mu_B = \frac{4\pi n_0 \eta_B}{m_B} + \frac{\alpha+1}{2\alpha}\frac{E_F}{g_{BF}}\left[1 + \frac{1}{g_{BF}}\ln\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{\sqrt{e}(\alpha+1)}\right]$$

$$\mu_F = E_F + \frac{\alpha+1}{2\alpha}x\frac{E_F}{g_{BF}}\left[1 + \frac{1}{g_{BF}}\ln\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{\alpha+1}\right]$$

Second-order perturbative calculation valid for both attractive and repulsive interaction. QMC performed for repulsive BF mixture.

Perturbative curve works well till line of instability (and even slightly above it).



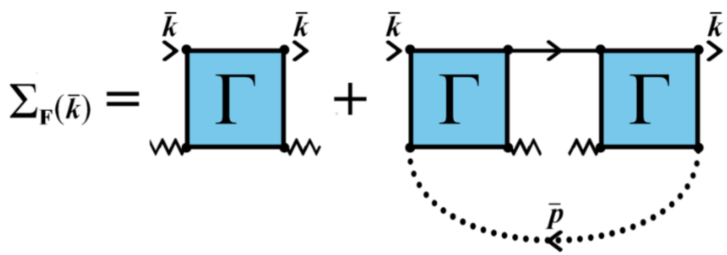
Evidence of boson clustering past the instability line from the BB pair distribution function.

More on perturbative expansion

Fermion self-energy

$$\Sigma_{\mathrm{F}}(k,\omega) = \frac{\pi n_{\mathrm{B}} \tilde{g}_{\mathrm{BF}}}{m_r} + \frac{\pi n_{\mathrm{B}} \tilde{g}_{\mathrm{BF}}^2}{2m_r} \left\{ \ln\left(\frac{(\alpha+1)^2 + A(\alpha+1) - 2\kappa^2}{(\alpha+1)^2} - i0^+ + \sqrt{1 + \frac{2A}{\alpha+1} - \frac{4\kappa^2 - A^2}{(\alpha+1)^2} - i0^+} \right) - \ln 2 \right.$$
$$\left. + \frac{\alpha+1}{\alpha-1} \ln \frac{B(\alpha-1) - (\alpha-1)^2 + 2\kappa^2 - (\alpha-1)\sqrt{(\alpha-1-B)^2 - 4\kappa^2 - i0^+}}{2\kappa^2} \right\}$$

$\tilde{g}_{\mathrm{BF}} = 1/g_{\mathrm{BF}} = -1/\ln(k_{\mathrm{F}} a_{\mathrm{BF}})$ $\quad A \equiv \kappa^2 - \nu\alpha,\ B \equiv \kappa^2 + \nu\alpha$ $\quad \nu \equiv \omega/\varepsilon_{\mathrm{F}}$ $\quad \kappa \equiv k/k_{\mathrm{F}}$



Fermion quasiparticle residue at k_F

$$Z(k_{\mathrm{F}}) = \left[1 - \frac{\partial}{\partial\omega} \mathrm{Re}\Sigma_{\mathrm{F}}(k_{\mathrm{F}},\omega) \right]^{-1}_{\omega=\mu_{\mathrm{F}}}$$

$$Z(k_{\mathrm{F}}) = 1 - \frac{\sqrt{2}}{8} \frac{(\alpha+1)^{3/2}}{\alpha} |\tilde{g}_{\mathrm{BF}}|^{3/2} \sqrt{x} + \frac{1}{4} \left(\frac{\alpha+1}{\alpha} \right)^2 \tilde{g}_{\mathrm{BF}}^2 x$$

Fermion effective mass

$$\frac{m_{\mathrm{F}}}{m^*} = \left[1 + \frac{m_{\mathrm{F}}}{k_{\mathrm{F}}} \frac{\partial \mathrm{Re}\Sigma_{\mathrm{F}}(k,\varepsilon_{\mathrm{F}})}{\partial k} \right]_{k=k_{\mathrm{F}}} Z(k_{\mathrm{F}})$$

$$\frac{m_{\mathrm{F}}}{m_{\mathrm{F}}^*} = 1 - \frac{\sqrt{2}}{8} \frac{(\alpha+1)^{3/2}}{w} |g_{\mathrm{BF}}|^{3/2} \sqrt{x} + \frac{\alpha+1}{4\alpha} g_{\mathrm{BF}}^2 x$$

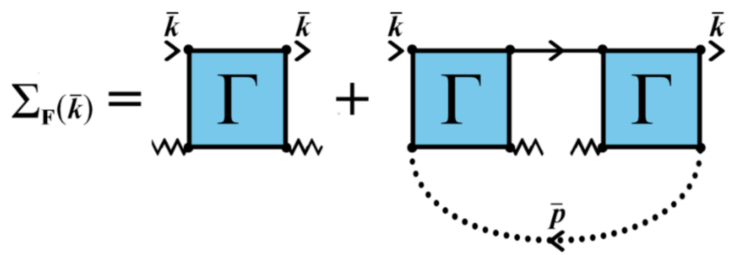
More on perturbative expansion

Fermion self-energy

$$\Sigma_{\rm F}(k,\omega) = \frac{\pi n_{\rm B}\tilde{g}_{\rm BF}}{m_r} + \frac{\pi n_{\rm B}\tilde{g}_{\rm BF}^2}{2m_r}\left\{ \ln\left(\frac{(\alpha+1)^2 + A(\alpha+1) - 2\kappa^2}{(\alpha+1)^2} - i0^+ + \sqrt{1 + \frac{2A}{\alpha+1} - \frac{4\kappa^2 - A^2}{(\alpha+1)^2} - i0^+}\right) - \ln 2 \right.$$

$$\left. + \frac{\alpha+1}{\alpha-1}\ln\frac{B(\alpha-1) - (\alpha-1)^2 + 2\kappa^2 - (\alpha-1)\sqrt{(\alpha-1-B)^2 - 4\kappa^2 - i0^+}}{2\kappa^2}\right\}$$

$\tilde{g}_{\rm BF} = 1/g_{\rm BF} = -1/\ln(k_{\rm F}a_{\rm BF})$ $\quad A \equiv \kappa^2 - \nu\alpha,\ B \equiv \kappa^2 + \nu\alpha$ $\quad \nu \equiv \omega/\varepsilon_{\rm F}$ $\quad \kappa \equiv k/k_{\rm F}$



Fermion quasiparticle residue at k_F

$$Z(k_{\rm F}) = \left[1 - \frac{\partial}{\partial\omega}{\rm Re}\Sigma_{\rm F}(k_{\rm F},\omega)\right]^{-1}_{\omega=\mu_{\rm F}}$$

$$Z(k_{\rm F}) = 1 - \frac{\sqrt{2}}{8}\frac{(\alpha+1)^{3/2}}{\alpha}|\tilde{g}_{\rm BF}|^{3/2}\sqrt{x} + \frac{1}{4}\left(\frac{\alpha+1}{\alpha}\right)^2\tilde{g}_{\rm BF}^2 x$$

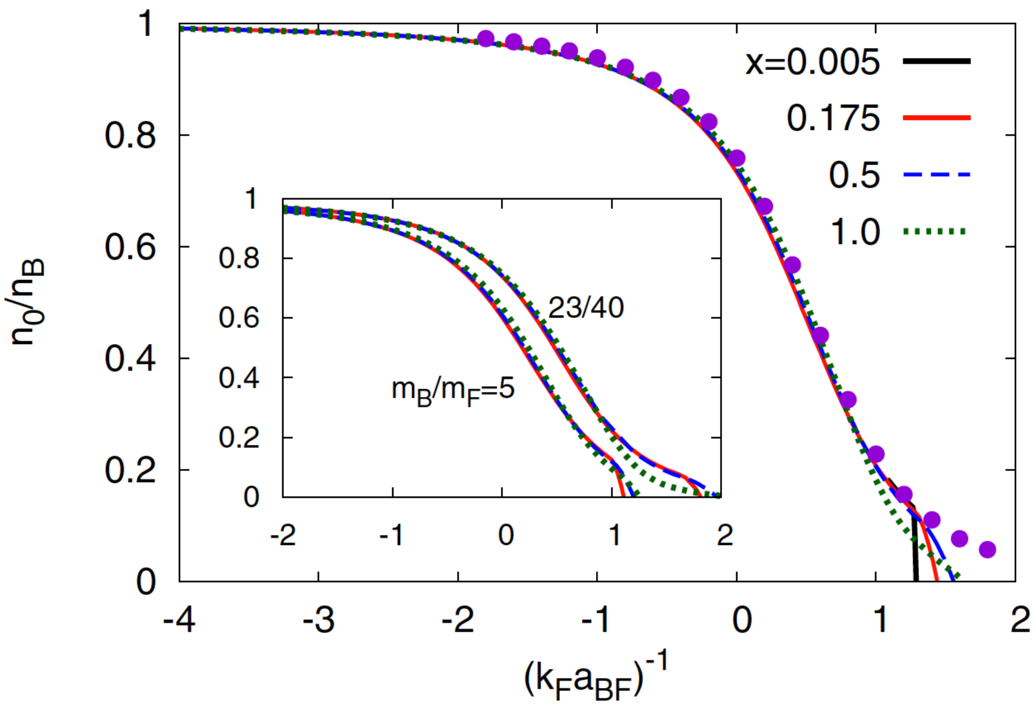
Fermion effective mass

$$\frac{m_{\rm F}}{m^*} = \left[1 + \frac{m_{\rm F}}{k_{\rm F}}\frac{\partial{\rm Re}\Sigma_{\rm F}(k,\varepsilon_{\rm F})}{\partial k}\right]_{k=k_{\rm F}} Z(k_{\rm F})$$

$$\frac{m_{\rm F}}{m_{\rm F}^*} = 1 - \frac{\sqrt{2}}{8}\frac{(\alpha+1)^{3/2}}{w}|g_{\rm BF}|^{3/2}\sqrt{x} + \frac{\alpha+1}{4\alpha}g_{\rm BF}^2 x$$

Non-analytic dependence due to interplay between Fermi step and condensed bosons.

Comparison with (Fermi) polaron quasiparticle residue Z

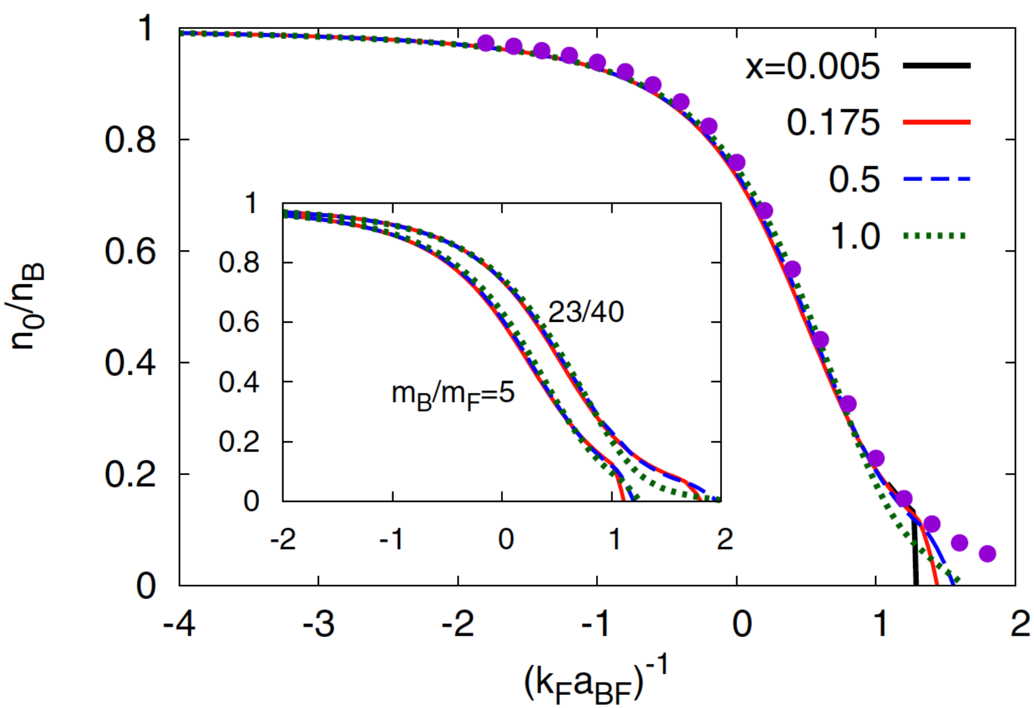


Lines: our calculations at four different concentrations for **zero Bose repulsion**.

Circles: Diagrammatic MC results for *Z* [J. Vlietinck, J. Ryckebusch, K. Van Houcke, PRB **87**, 115133 (2013)]

Surprising agreement between 'universal condensate fraction' and Fermi polaron quasiparticle residue. 'Explanation':

Comparison with (Fermi) polaron quasiparticle residue Z



Lines: our calculations at four different concentrations for **zero Bose repulsion**.

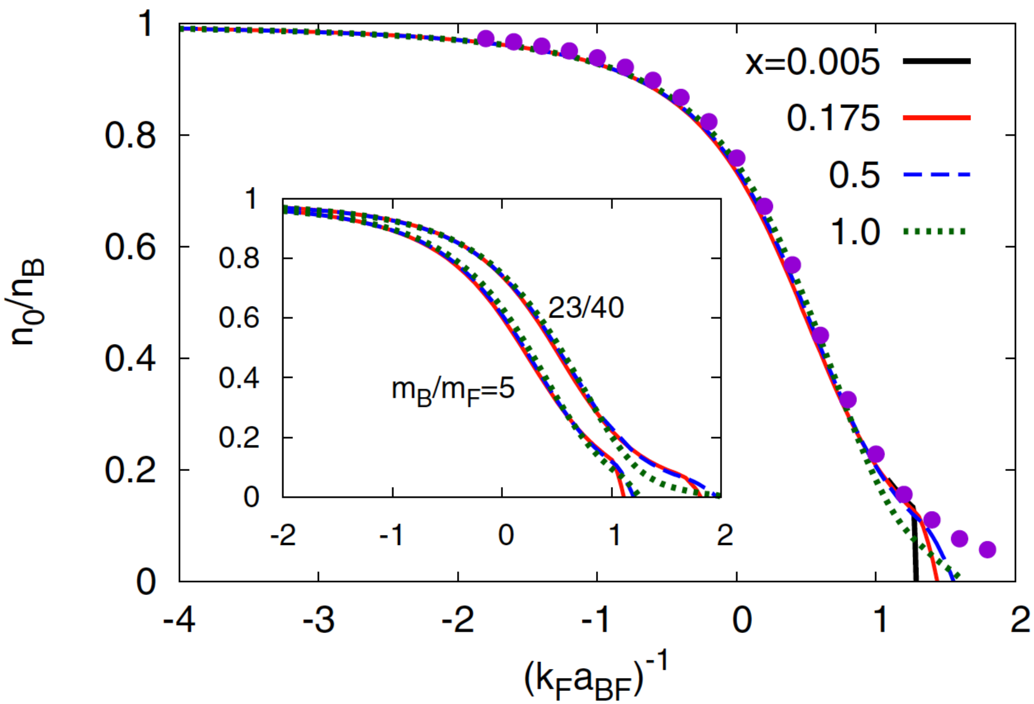
Circles: Diagrammatic MC results for Z [J. Vlietinck, J. Ryckebusch, K. Van Houcke, PRB **87**, 115133 (2013)]

Surprising agreement between 'universal condensate fraction' and Fermi polaron quasiparticle residue. 'Explanation':

$$\frac{n_0}{n_B} = \frac{n_B(k=0)}{N_B} \rightarrow \frac{n_{pol}(k=0)}{1} = n_{pol}(k=0) - n_{pol}(k=0^+)$$

$n_{pol}(k \neq 0) \propto \frac{1}{V} \rightarrow 0$ in thermodynamic limit

Comparison with (Fermi) polaron quasiparticle residue Z



Lines: our calculations at four different concentrations for **zero Bose repulsion**.

Circles: Diagrammatic MC results for Z [J. Vlietinck, J. Ryckebusch, K. Van Houcke, PRB **87**, 115133 (2013)]

Surprising agreement between 'universal condensate fraction' and Fermi polaron quasiparticle residue. 'Explanation':

$$\frac{n_0}{n_B} = \frac{n_B(k=0)}{N_B} \rightarrow \frac{n_{pol}(k=0)}{1} = n_{pol}(k=0) - n_{pol}(k=0^+)$$

$$= \lim_{k_{F\downarrow}\rightarrow 0} n_\downarrow(k_{F\downarrow}^-) - n_\downarrow(k_{F\downarrow}^+) = Z$$

$$n_{pol}(k \neq 0) \propto \frac{1}{V} \rightarrow 0$$

in thermodynamic limit

Back to the explanation for the connection between n_0/n_B and *Z*

$$\frac{n_0}{n_B} = \frac{n_B(k=0)}{N_B} \rightarrow \frac{n_{pol}(k=0)}{1} = n_{pol}(k=0) - n_{pol}(k=0^+) \quad = \lim_{k_{F\downarrow}\to 0} n_\downarrow(k_{F\downarrow}^-) - n_\downarrow(k_{F\downarrow}^+) = Z$$

Back to the explanation for the connection between n_0/n_B and *Z*

$$\frac{n_0}{n_B} = \frac{n_B(k=0)}{N_B} \rightarrow \frac{n_{pol}(k=0)}{1} = n_{pol}(k=0) - n_{pol}(k=0^+) = \lim_{k_{F\downarrow}\to 0} n_\downarrow(k_{F\downarrow}^-) - n_\downarrow(k_{F\downarrow}^+) = Z$$

This is the weak step.

Back to the explanation for the connection between n_0/n_B and Z

$$\frac{n_0}{n_B} = \frac{n_B(k=0)}{N_B} \to \frac{n_{pol}(k=0)}{1} = n_{pol}(k=0) - n_{pol}(k=0^+) \quad = \lim_{k_{F\downarrow}\to 0} n_\downarrow(k_{F\downarrow}^-) - n_\downarrow(k_{F\downarrow}^+) = Z$$

This is the weak step.

n_0/n_B is defined in the thermodynamic limit: $V \to \infty$ with n_B/n_F fixed (no matter how small we take it). First take $V \to \infty$ and then $n_B/n_F \to 0$, if interested in the polaron limit.

So $N_B = n_B V$ is always infinite.

For a single impurity instead $N_B = 1$ from the outset and only then $V \to \infty$.

Back to the explanation for the connection between n_0/n_B and Z

$$\frac{n_0}{n_B} = \frac{n_B(k=0)}{N_B} \to \frac{n_{pol}(k=0)}{1} = n_{pol}(k=0) - n_{pol}(k=0^+) = \lim_{k_{F\downarrow}\to 0} n_\downarrow(k_{F\downarrow}^-) - n_\downarrow(k_{F\downarrow}^+) = Z$$

This is the weak step.

n_0/n_B is defined in the thermodynamic limit: $V \to \infty$ with n_B/n_F fixed (no matter how small we take it). First take $V \to \infty$ and then $n_B/n_F \to 0$, if interested in the polaron limit.

So $N_B = n_B V$ is always infinite.

For a single impurity instead $N_B = 1$ from the outset and only then $V \to \infty$.

$$\Rightarrow \quad \lim_{n_B/n_F \to 0} \frac{n_B(k=0)}{N_B} \neq \frac{n_{pol}(k=0)}{1}$$