Bose mixtures at finite temperature: magnetism and condensation phenomena

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BOSE EINSTEIN CONDENSATION

CON IL SOSTEGNO DELLA

PROVINCIA AUTONOMA DI TRENTO

RPMBT22
- Tsukuba 23rd-27th September 2024

CNR – Istituto Nazionale di Ottica
Pitaevskii Center on Bose-Einstein Condensation
Dipartimento di Fisica – Università di Trento

Mixtures of Bose gases: two independent condensates

First realization in 1997 and 2002

homonuclear (^{87}Rb - ^{87}Rb)

VOLUME 78, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JANUARY 1997

Production of Two Overlapping Bose-Einstein Condensates by Sympathetic Cooling

C. J. Myatt, E. A. Burt, R. W. Ghrist, E. A. Cornell, and C. E. Wieman
JILA and Department of Physics, University of Colorado and NIST, Boulder, Colorado 80309
(Received 20 September 1996)

A new apparatus featuring a double magneto-optic trap and an Ioffe-type magnetic trap was used to create condensates of 2×10^6 atoms in either of the $|F = 2, m = 2\rangle$ or $|F = 1, m = -1\rangle$ spin states of ^{87}Rb. Overlapping condensates of the two states were also created using nearly lossless sympathetic cooling of one state via thermal contact with the other evaporatively cooled state. We observed that (i) the scattering length of the $|1, -1\rangle$ state is positive, (ii) the rate constant for binary inelastic collisions between the two states is $2.2(9) \times 10^{-14}$ cm^3/s, and (iii) there is a repulsive interaction between the two condensates. Similarities and differences between the behaviors of the two spin states are observed.
[S0031-9007(96)02208-9]

heteronuclear (^{87}Rb - ^{41}K)

VOLUME 89, NUMBER 19 PHYSICAL REVIEW LETTERS 4 NOVEMBER 2002

Two Atomic Species Superfluid

G. Modugno, M. Modugno, F. Riboli, G. Roati, and M. Inguscio
LENS, Università di Firenze and INFM, Via Nello Carrara 1, 50019 Sesto Fiorentino, Italy
(Received 23 May 2002; published 21 October 2002)

We produce a quantum degenerate mixture composed by two Bose-Einstein condensates of different atomic species, ^{41}K and ^{87}Rb. We study the dynamics of the superfluid system in an elongated magnetic trap, where off-axis collisions between the two interacting condensates induce scissorlike oscillations.

- **Peculiar platform realized with ultracold atoms. Analogies with multiband superconductors (MgB2 - iron-based SC's - layered cuprates).**
  In Bose mixtures U(1)xU(1) is broken while in SC's only overall U(1)
- **Main new features**
  - **Two Goldstone modes (density and spin waves). Mass and spin superfluidity. Dissipationless superfluid drag**
  - **Repulsive mixtures =====> magnetic phenomena**
  - **Attractive mixtures ======> gas-liquid condensation**

Simple mean-field theory (T=0)

$$\frac{E}{V} = \frac{1}{2} g_{11} n_1^2 + \frac{1}{2} g_{22} n_2^2 + g_{12} n_1 n_2$$

$g_{11} > 0 \quad g_{22} > 0 \quad g_{12}$ either positive or negative

Miscible mixture: compressibility matrix must be positive definite

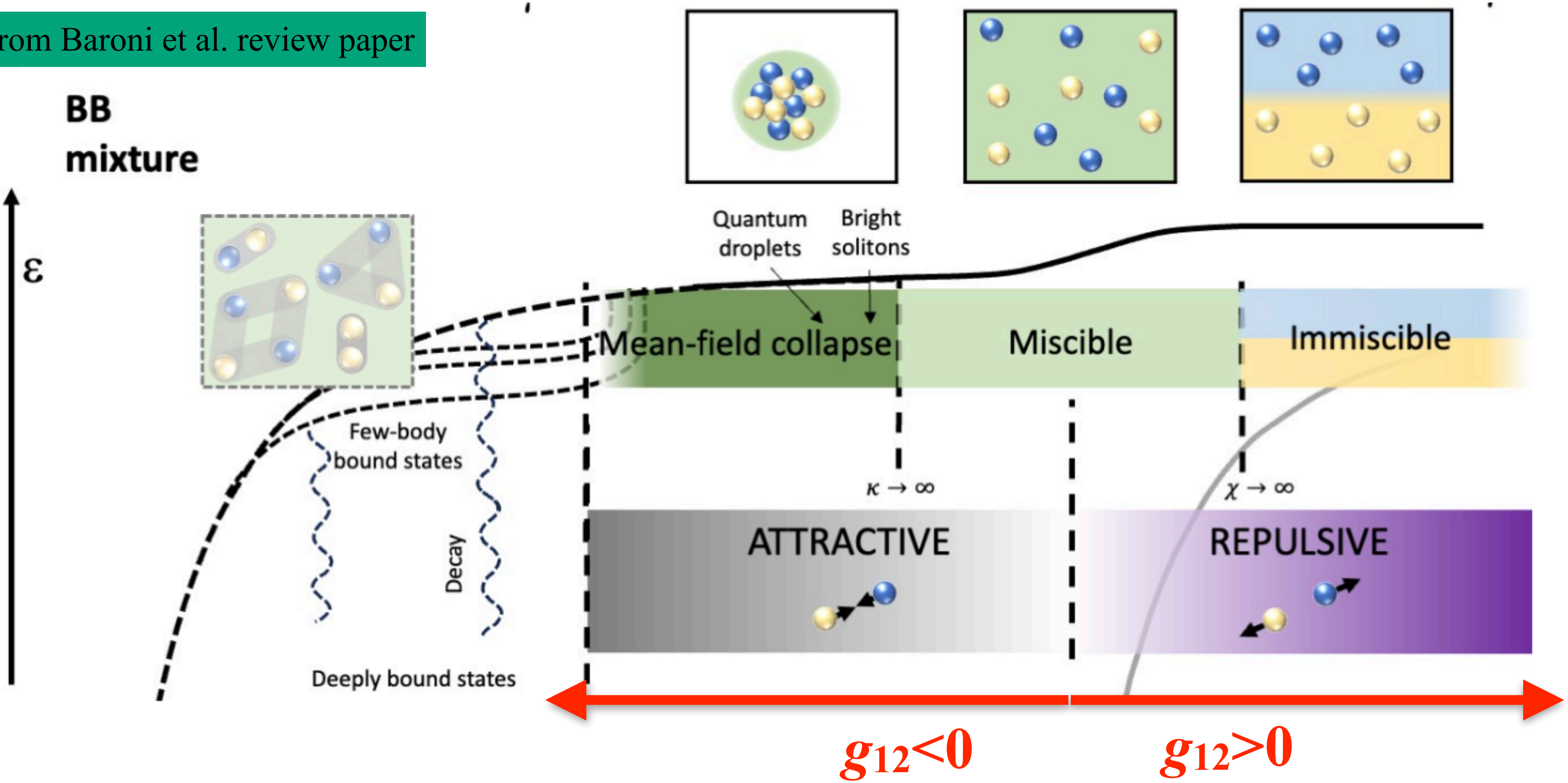
$$\det \begin{pmatrix} \partial^2 (E/V)/\partial n_1^2 & \partial^2 (E/V)/\partial n_1 \partial n_2 \\ \partial^2 (E/V)/\partial n_2 \partial n_1 & \partial^2 (E/V)/\partial n_2^2 \end{pmatrix} = g_{11} g_{22} - g_{12}^2 > 0$$

if $g_{11} g_{22} - g_{12}^2 < 0$

a) **$g_{12}>0$ ====> phase separation: ferromagnetism with finite polarization**

b) **$g_{12}<0$ ====> mean-field collapse (beyond mean-field effects play role)**

From Baroni et al. review paper



Repulsive branch: magnetic analogy

$$E = n\,\delta g Z^2/2 - \hbar\Omega_R\sqrt{1-Z^2}\cos\phi - \hbar\delta_{\rm eff} Z$$

$$E(Z,\phi) \propto -B_3 Z - \frac{|\alpha| n}{2} Z^2 - B_1\sqrt{1-Z^2}\cos\phi.$$

$Z = (n_\uparrow - n_\downarrow)/n = S_3/n$

$\phi = \arctan(S_2/S_1)$ in plane angle

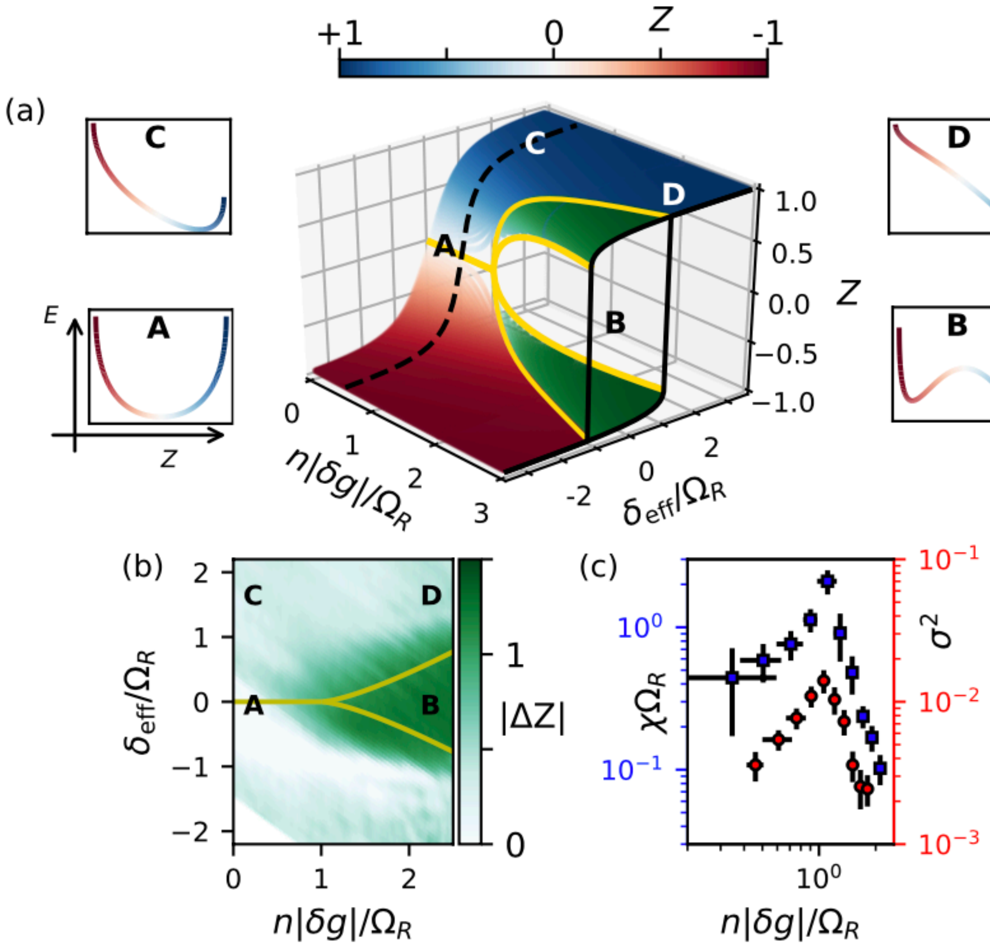
B_3 axial field - detuning

B_1 trans. field - coherent coupling

$|\alpha| = g - g_{12}$ ferro-interaction

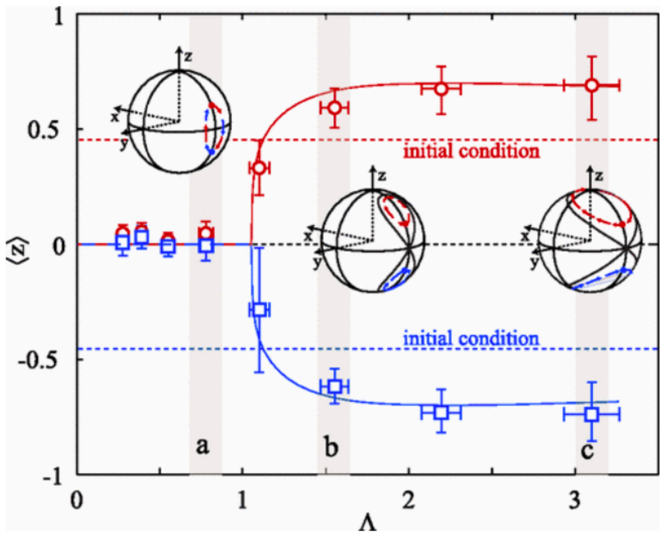
Experiments at very low temperatures ($T \simeq 0$)

Quantum phase transition

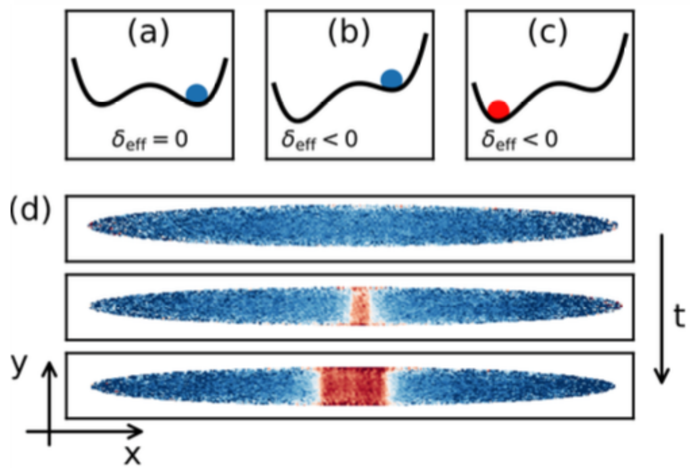


Ferrari's group Trento 2023

Oberthaler's group Heidelberg 2010



Also false vacuum decay (Nature Physics 2024)

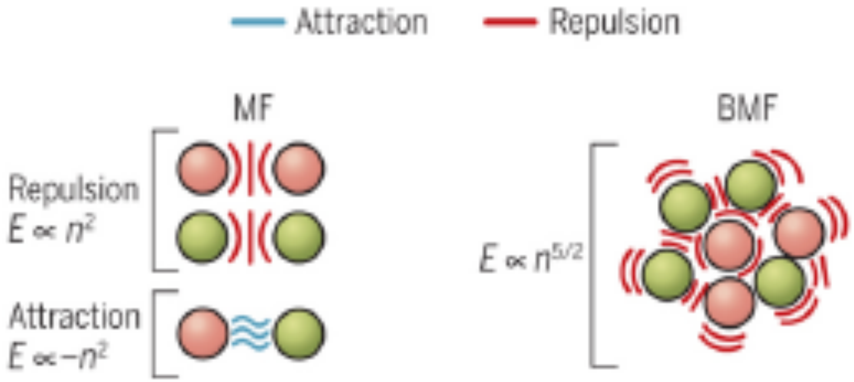


Attractive Bose mixtures and formation of quantum droplets

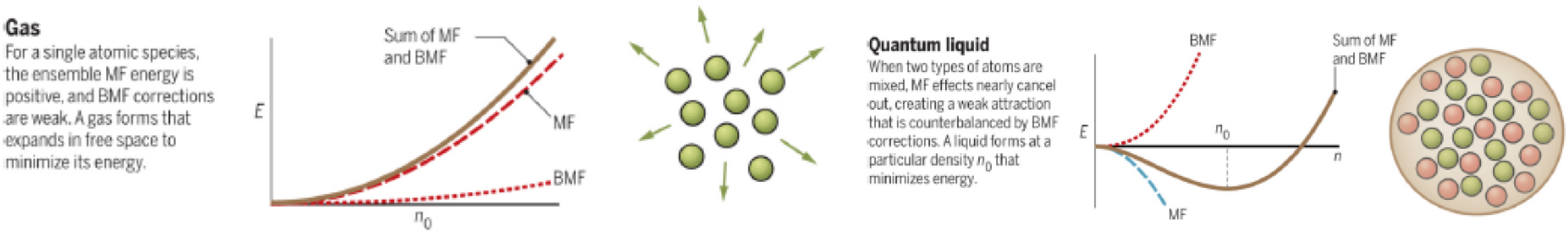
two hyperfine states of ^{39}K (LENS + ICFO) + theory (Petrov 2015)

- **Mean-field term would lead to collapse**

- **Repulsive beyond mean-field quantum fluctuations stabilise system**



GRAPHIC: K. SUTLIFF/*SCIENCE*"



Two experimental realisations with ^{39}K in 2017

REPORT

QUANTUM FLUIDS

Quantum liquid droplets in a mixture of Bose-Einstein condensates

C. R. Cabrera,* L. Tanzi,* J. Sanz, B. Naylor, P. Thomas, P. Cheiney, L. Tarruell†

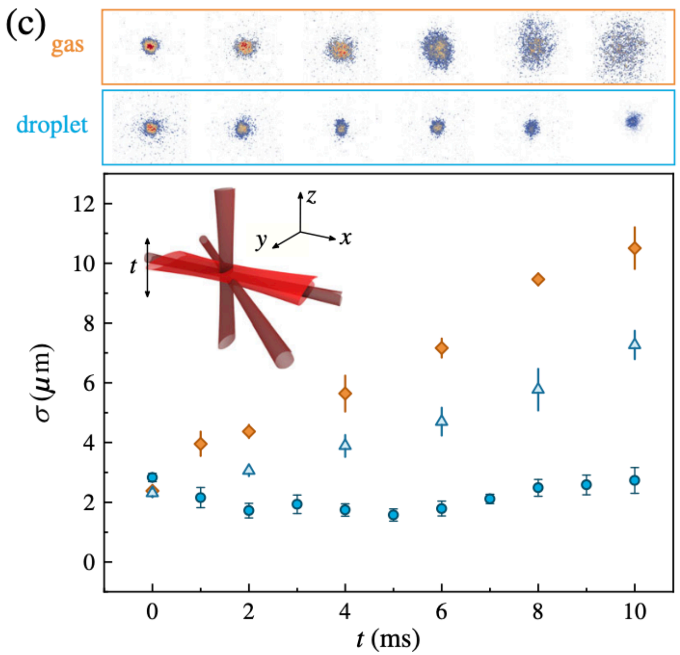
PHYSICAL REVIEW LETTERS **120,** 235301 (2018)

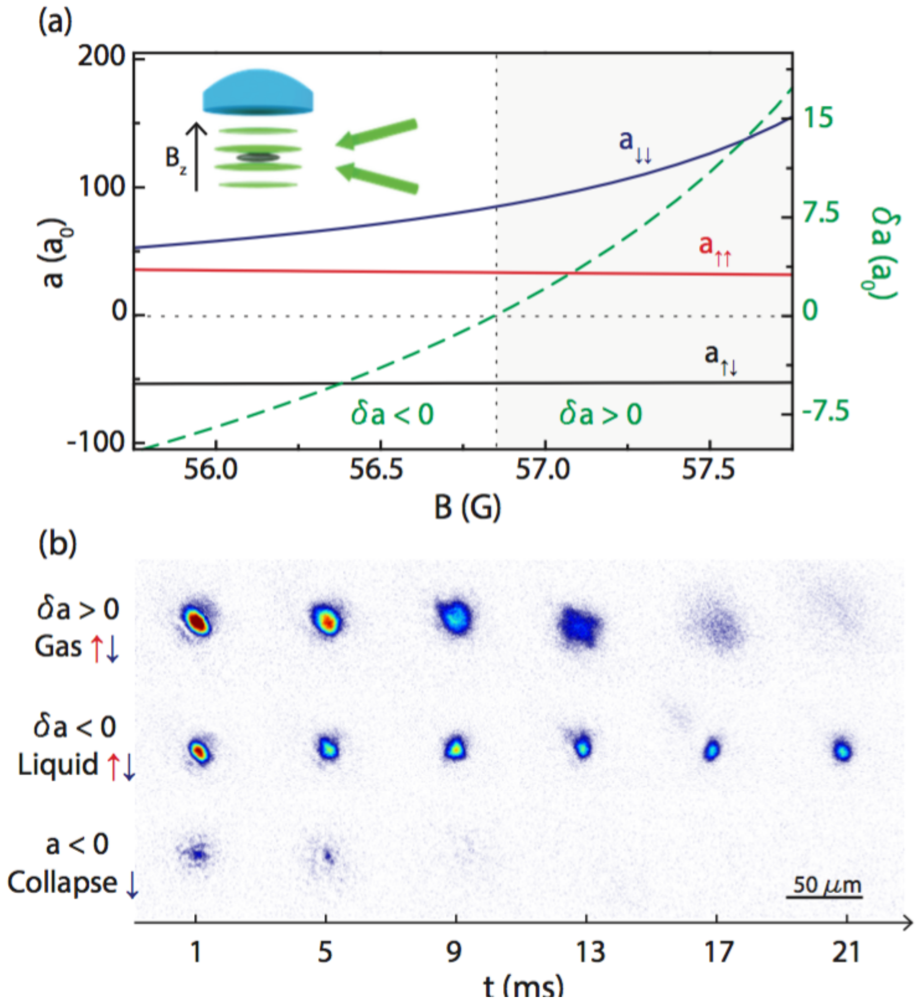
Self-Bound Quantum Droplets of Atomic Mixtures in Free Space

G. Semeghini,[1,2,*] G. Ferioli,[1,2] L. Masi,[1,2] C. Mazzinghi,[1] L. Wolswijk,[1] F. Minardi,[2,1,3] M. Modugno,[4,5] G. Modugno,[1,2] M. Inguscio,[2,1] and M. Fattori[1,2]

After being released the cloud

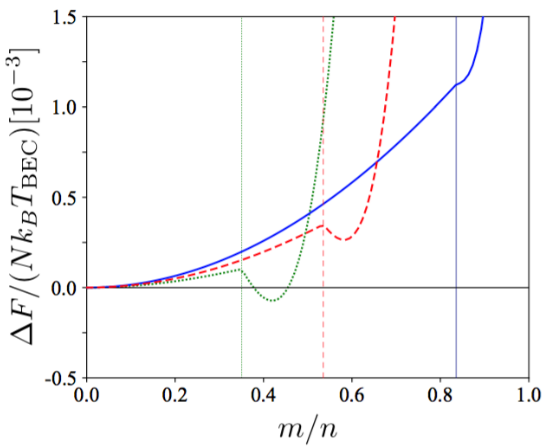
- expands (gas)
- shrinks (collapse)
- remains stable (liquid)





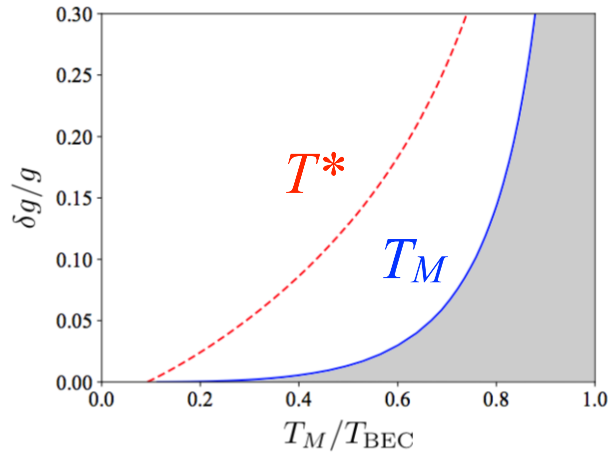
Thermal effects on phase separation and gas-liquid transition?

Repulsive mixtures:
perturbation methods predict FM transition if $g_{12} < g$ (T=0 miscible region)



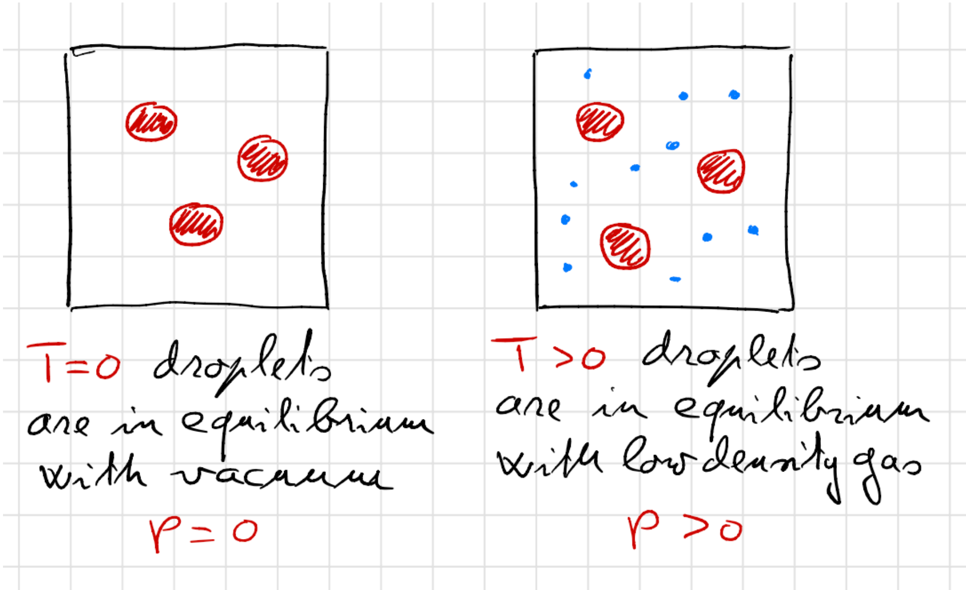
Finite-*T* Beliaev theory
(Ota et al. PRL 2019)
T<T* m=0 (mixed state)
T>T* m>0 (FM state)

$T>T_M$ minimum at m>0



Attractive mixtures:
Perturbation methods fail to describe gas-liquid equilibrium

At *T=0* theory (Petrov PRL 2015)
Energy functional: mean field + LHY
But density modes have imaginary speed of sound —-> no TD!



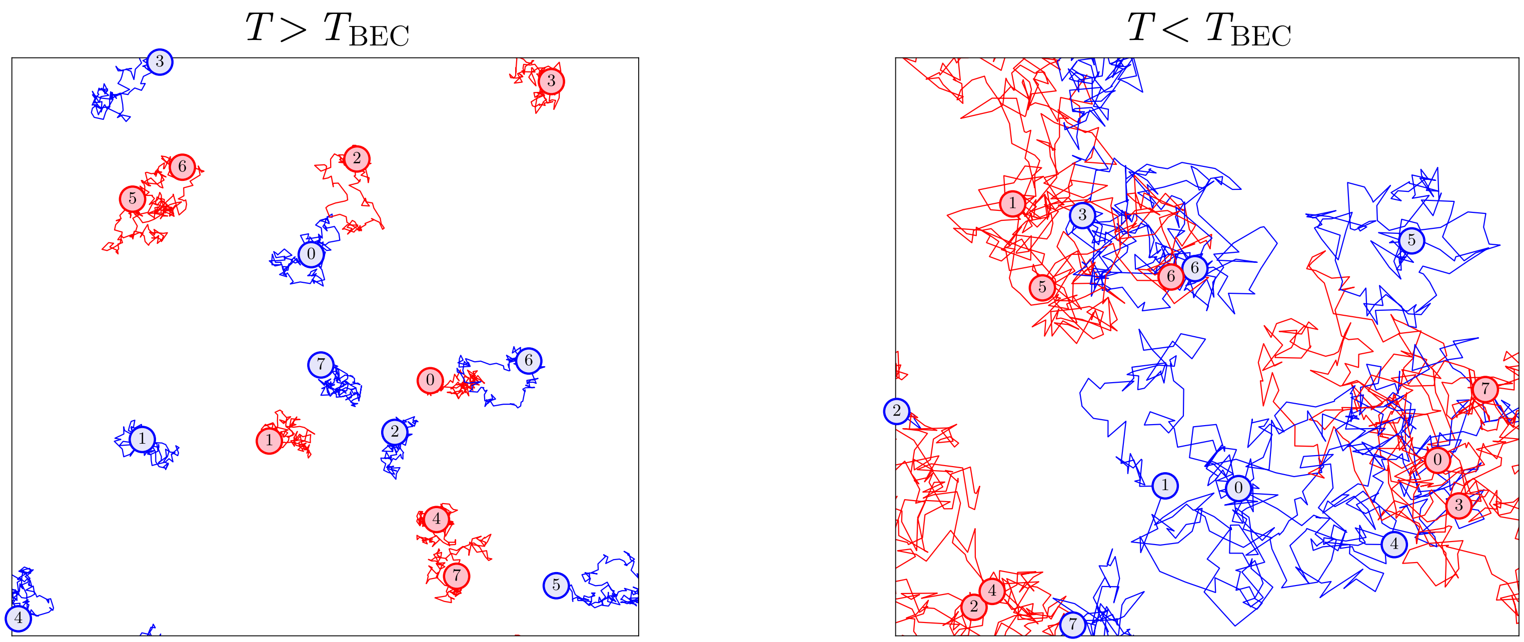
Use Path-Integral Monte-Carlo: Microscopic Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N_1}\nabla_i^2 - \frac{\hbar^2}{2m}\sum_{i'=1}^{N_2}\nabla_{i'}^2 + \sum_{i<j}^{N_1} v(|\mathbf{r}_i - \mathbf{r}_j|) + \sum_{i'<j'}^{N_2} v(|\mathbf{r}_{i'} - \mathbf{r}_{j'}|) + \sum_{i,i'}^{N_1,N_2} v_{12}(|\mathbf{r}_i - \mathbf{r}_{i'}|)\ .$$

$N_1 = N_2$ balanced mixture
$v(r)$ HS potential
$v_{12}(r)$ contact potential $a_{12}<0$

PIMC calculates exact partition function using convolution and high-temperature expansion + worm algorithm to sample permutations

$$Z = \frac{1}{N!}\sum_P \int d\mathbf{R}\,\rho(\mathbf{R}, P\mathbf{R}, \beta) = \frac{1}{N!}\sum_P \int d\mathbf{R} \int d\mathbf{R}_2 \cdots \int d\mathbf{R}_M\, \rho(\mathbf{R}, \mathbf{R}_2, \delta_\tau) \cdots \rho(\mathbf{R}_M, P\mathbf{R}, \delta_\tau)$$

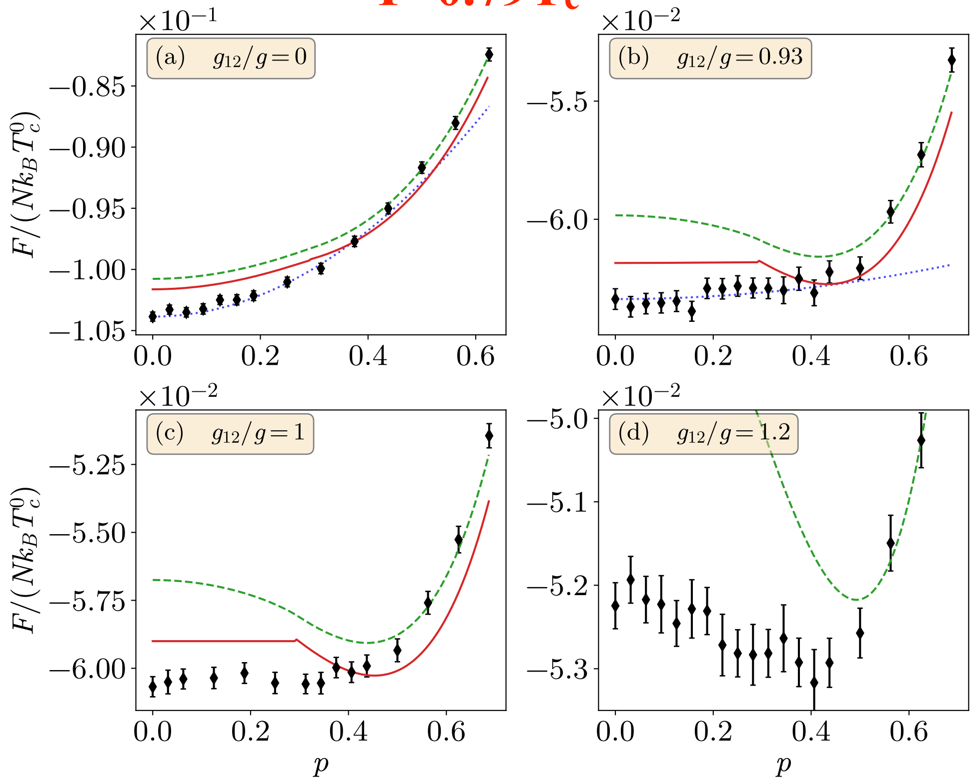


Repulsive mixtures: free energy in PIMC

$$F = -PV + \mu N$$

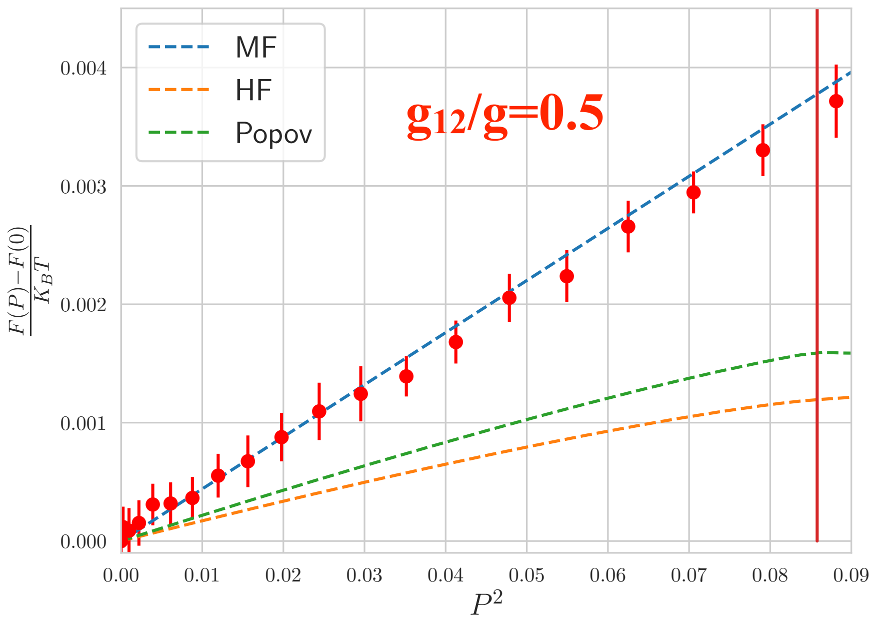
If $g_{12} < g$ minimum of F is at p=0

T=0.79T$_c$



Magnetic susceptibility

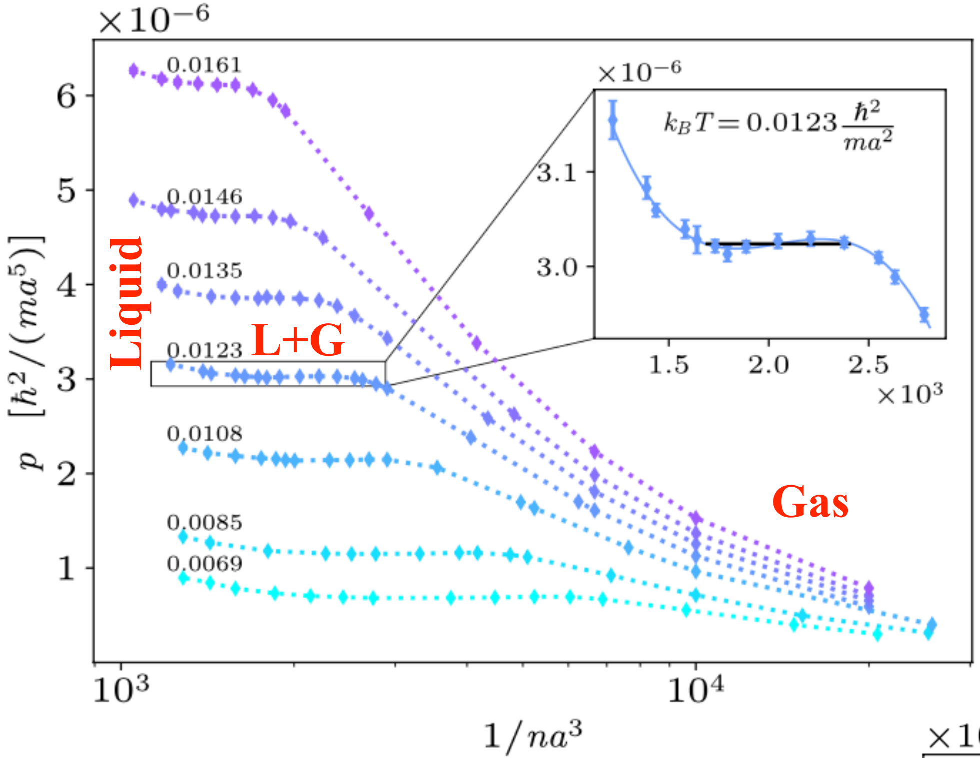
$$F(p) = F(0) + \frac{N^2}{2V}\frac{p^2}{\chi}$$



$$\chi^{-1} = \frac{g - g_{12}}{2}$$

independent of temperature
phase separation occurs as at T=0

Attractive mixtures: gas-liquid coexistence

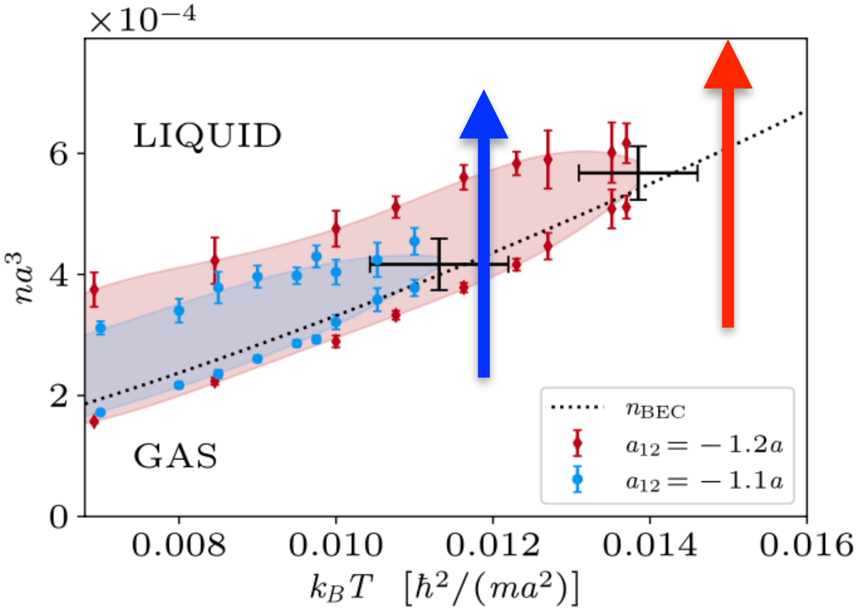


Isothermal lines

At the tricritical point coexistence region vanishes

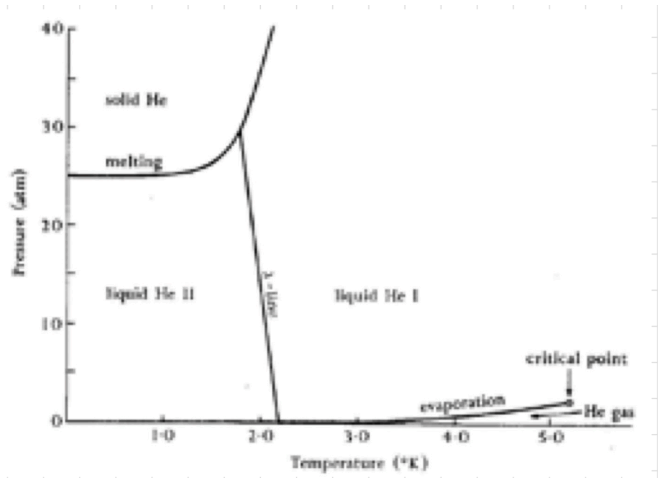
- **BEC transition is first order**
- **BEC transition is second order**

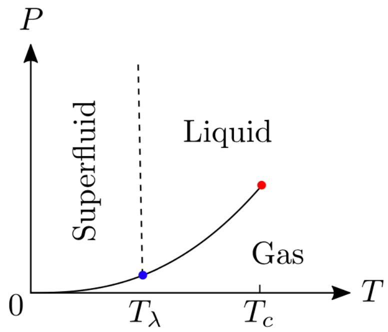
First-order transition line terminates at tricritical point



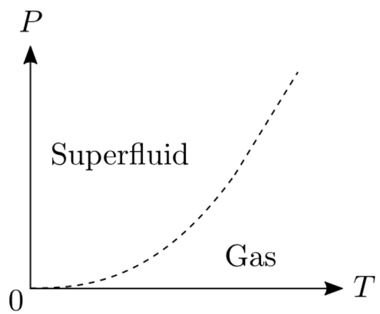
Topology of phase diagram (Son, Stephanov, Yee (2021) for single component)

Phase diagram of ^{4}He



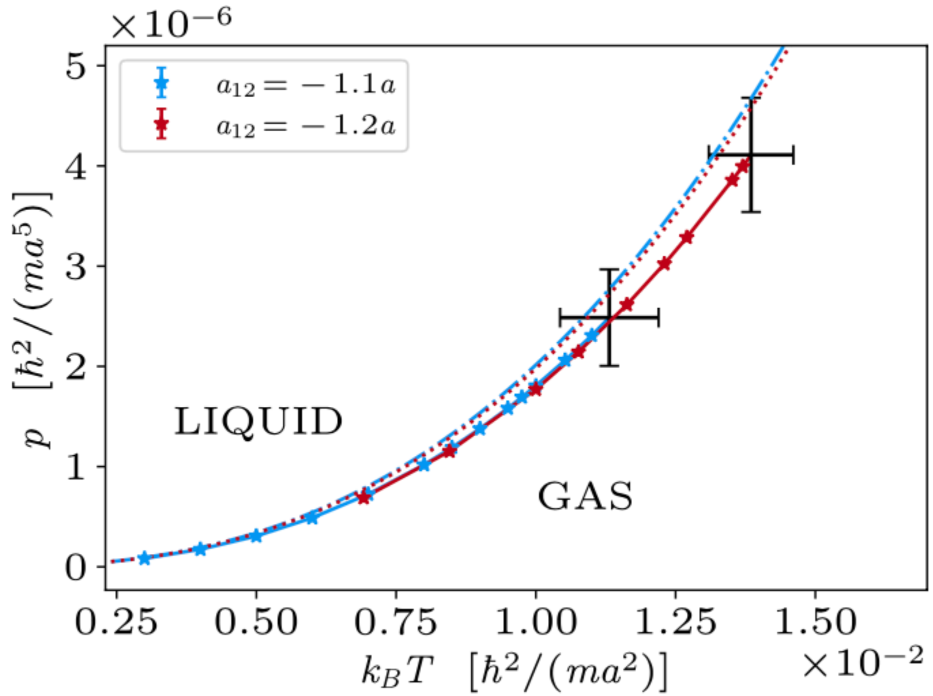


superfluid helium

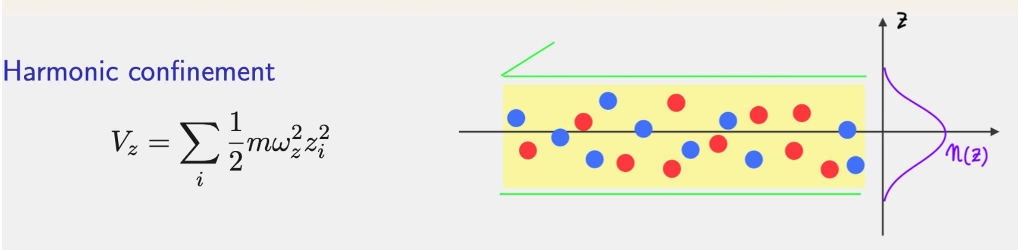


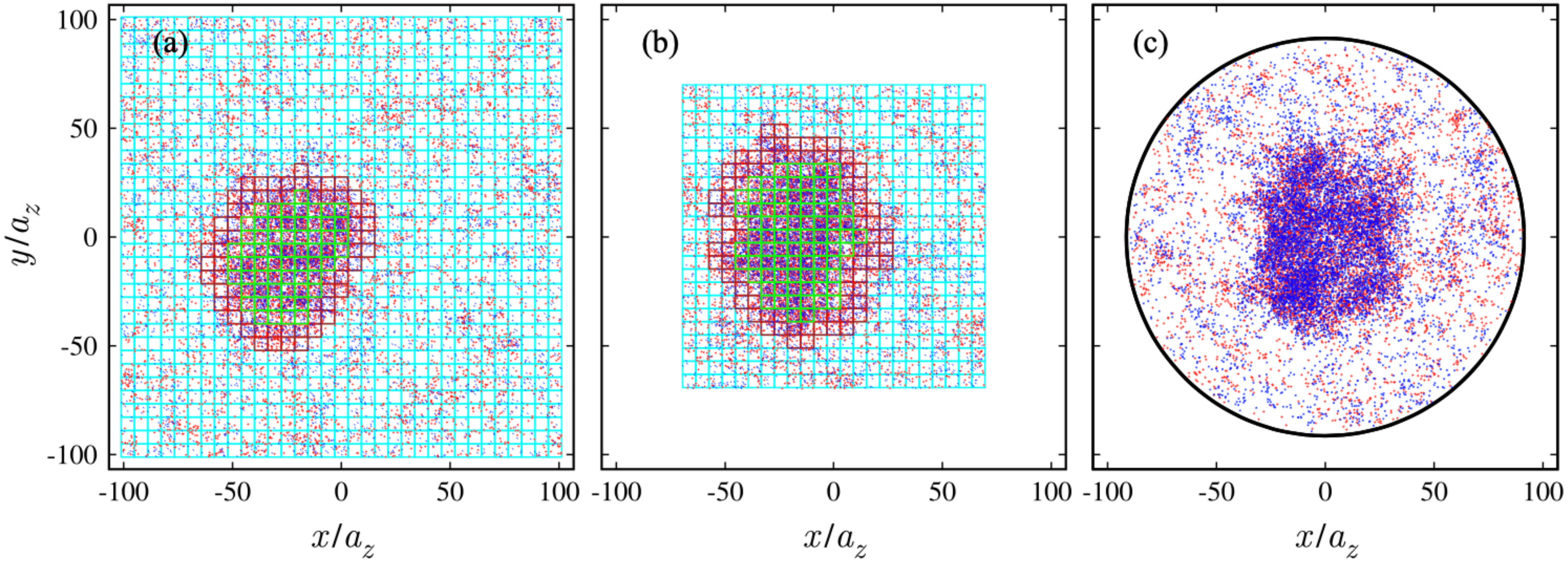
standard ultracold gase

Attractive mixtures



Quantum droplets in 2D



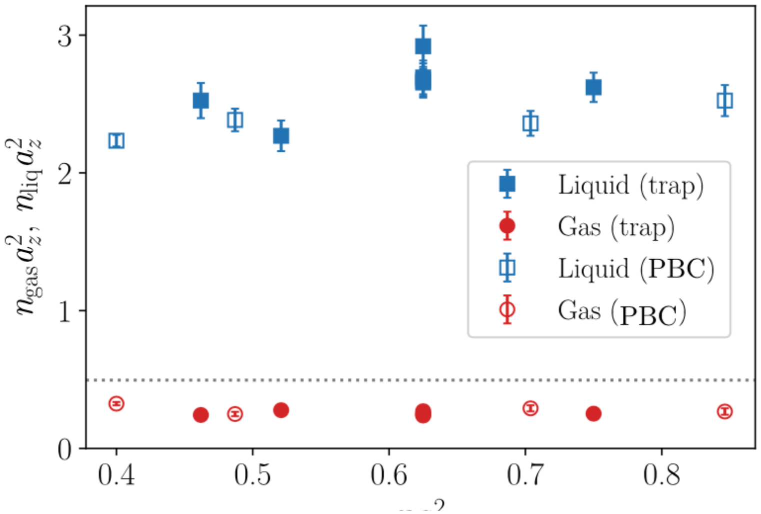


$N \sim 16000$

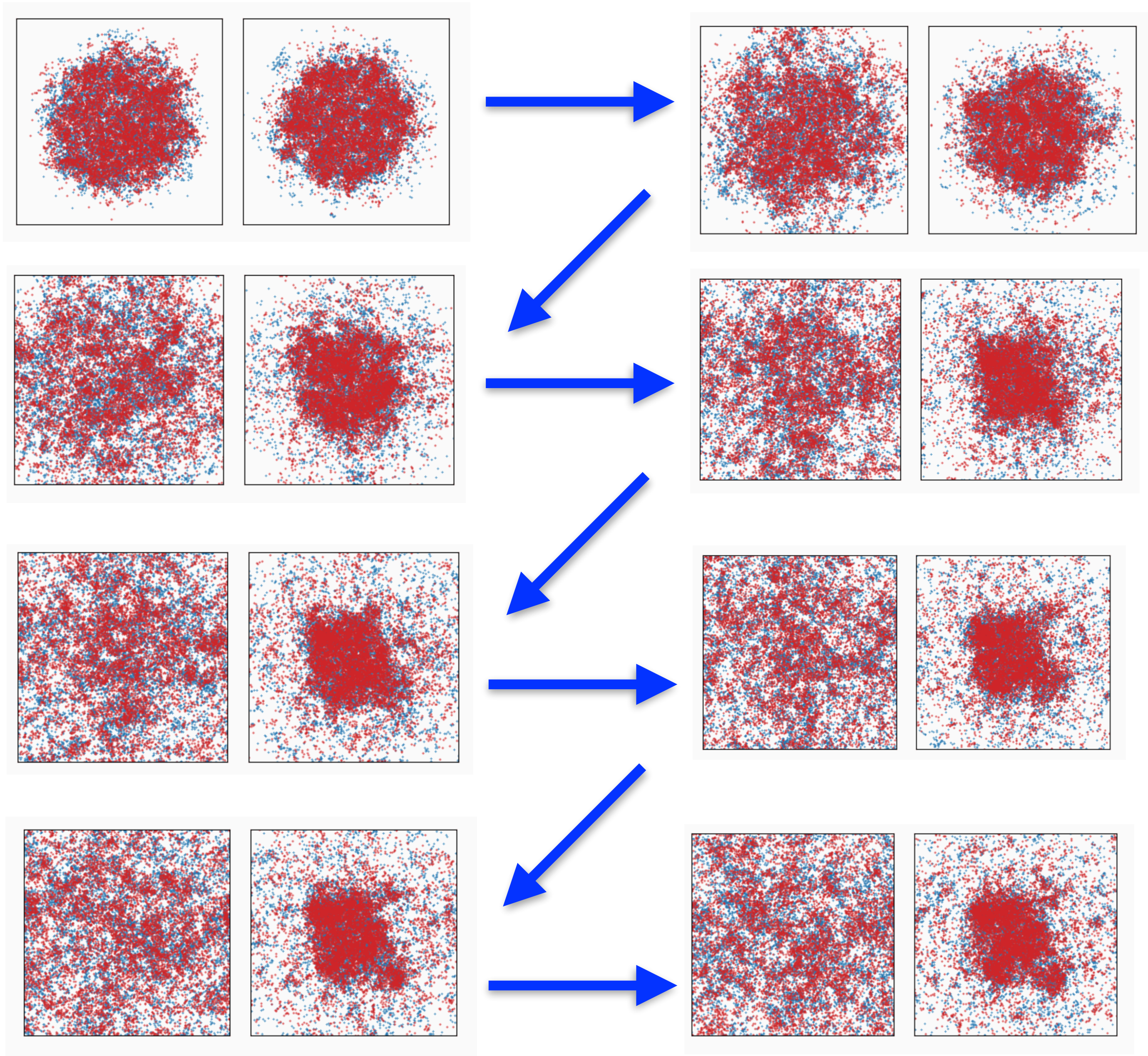
Droplet size grows or shrinks but density remains constant (saturation density of liquid)

Densities can be easily achieved in experiments

(see Dalibard's group 2018-21)



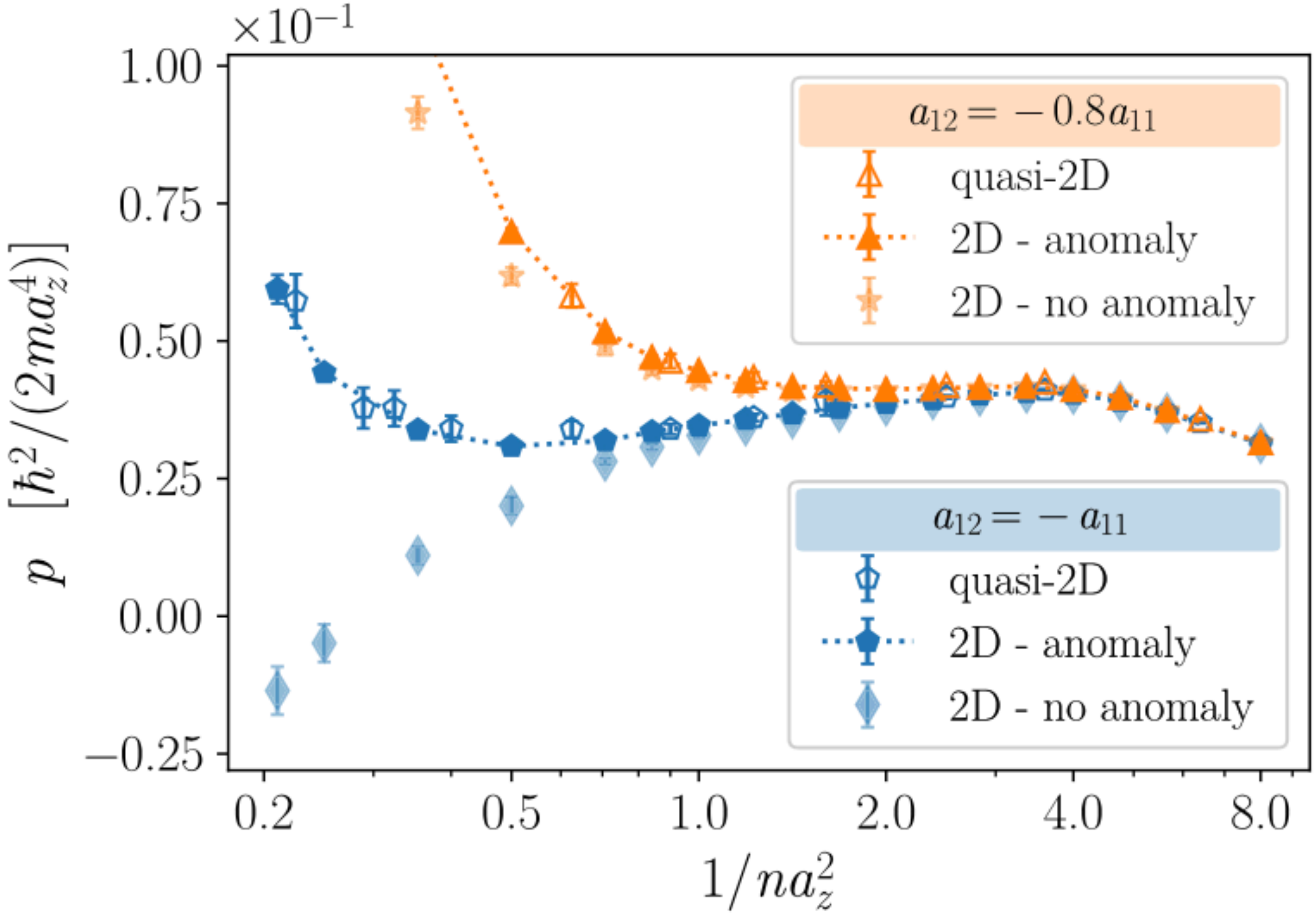
Comparison of equilibrium states: gas vs gas-liquid coexistence



In the gas phase cluster evaporates

In the coexistence region liquid droplet in equilibrium with gas

Comparison with pure 2D model



2D coupling constant
(Petrov, Holzmann, Shlyapnikov PRL 2000)

Logarithmic term breaks scaling invariance (quantum anomaly)

$$g = \frac{g_0}{1 + \frac{g_0}{4\pi}\log(A/na_z^2)}$$

$$g_0 = \sqrt{8\pi}a_{3D}/a_z, \;\; A \approx 0.2284$$

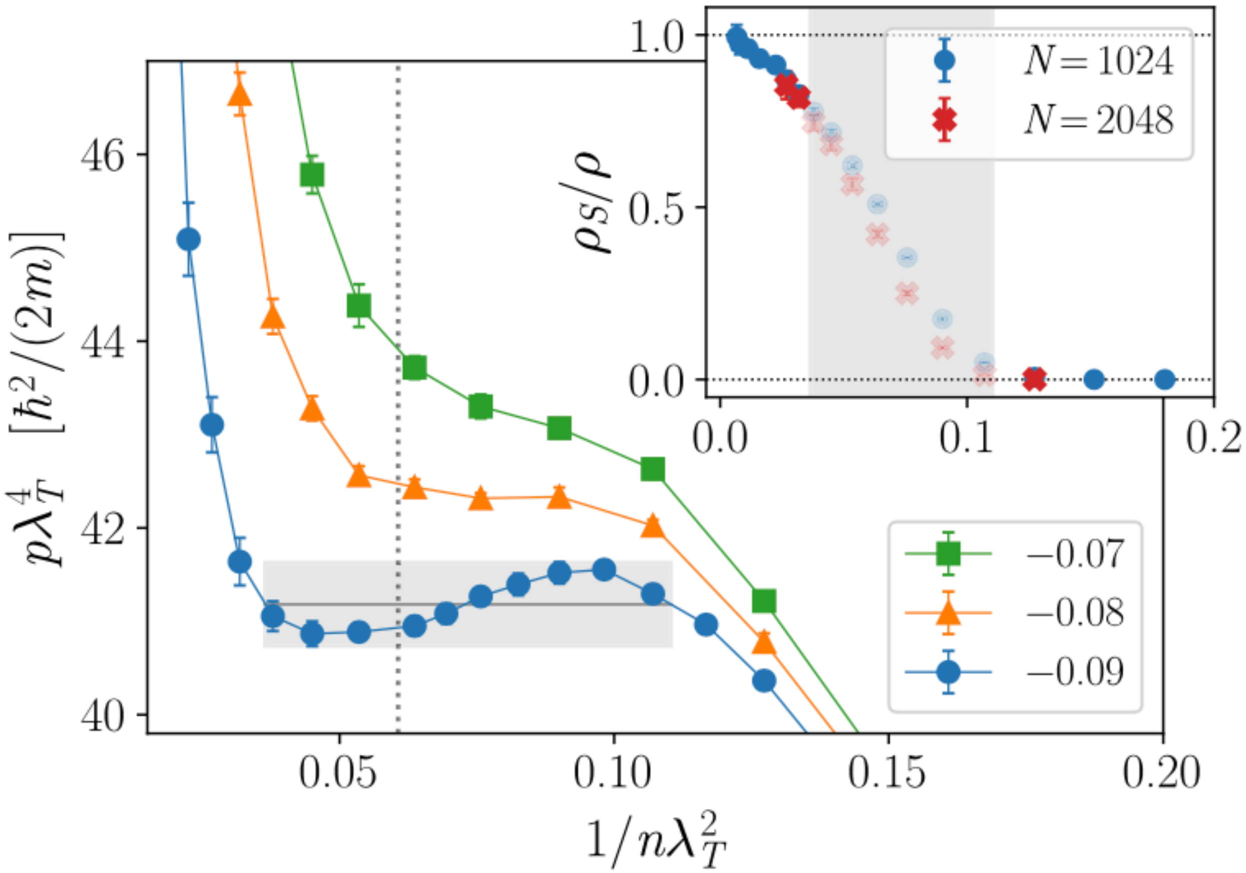
Superfluid transition is discontinuous

<u>BKT transition</u>
(Prokof'ev, Ruebenacker & Svistunov 2001)

$$n_{BKT} = \frac{mk_BT}{2\pi\hbar^2} \log \frac{\xi}{g_0}\,, \qquad \xi \approx 380$$

$\rho_S = 0$ in the gas phase and
$\rho_S \neq 0$ in the liquid phase

Large finite size effects
in the coexistence region



Conclusions

- **Bose mixture: rich platform to study the interplay between quantum degeneracy, magnetism and interactions**

- **Attractive mixtures: gas-liquid coexistence, novel phase diagram with discontinuous superfluid transition**

- **2D quantum droplets do not require high densities and are easier to observe**

Collaborations

- **UNITN+BEC Center** G. Spada 

- **UPC Barcelona**

  J. Boronat  G. Pascual 

- **U. Camerino**

  S. Pilati 

- **U. Newcastle**

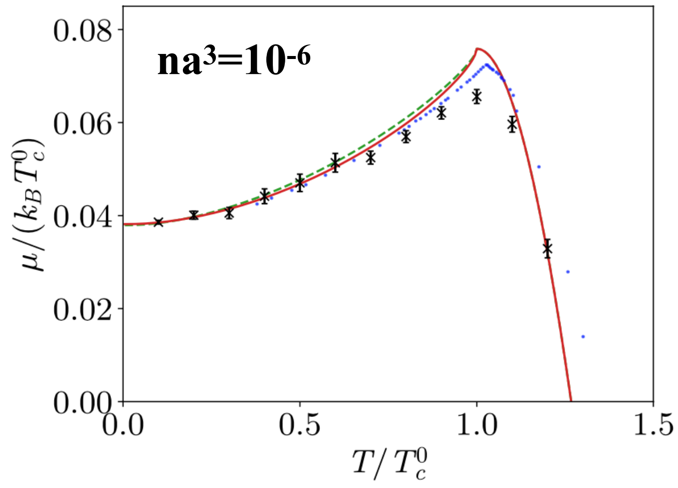
  L. Parisi 

Thank you for your attention

- **Implement calculation of chemical potential in PIMC**

(Herdman et al. PRB 2014)

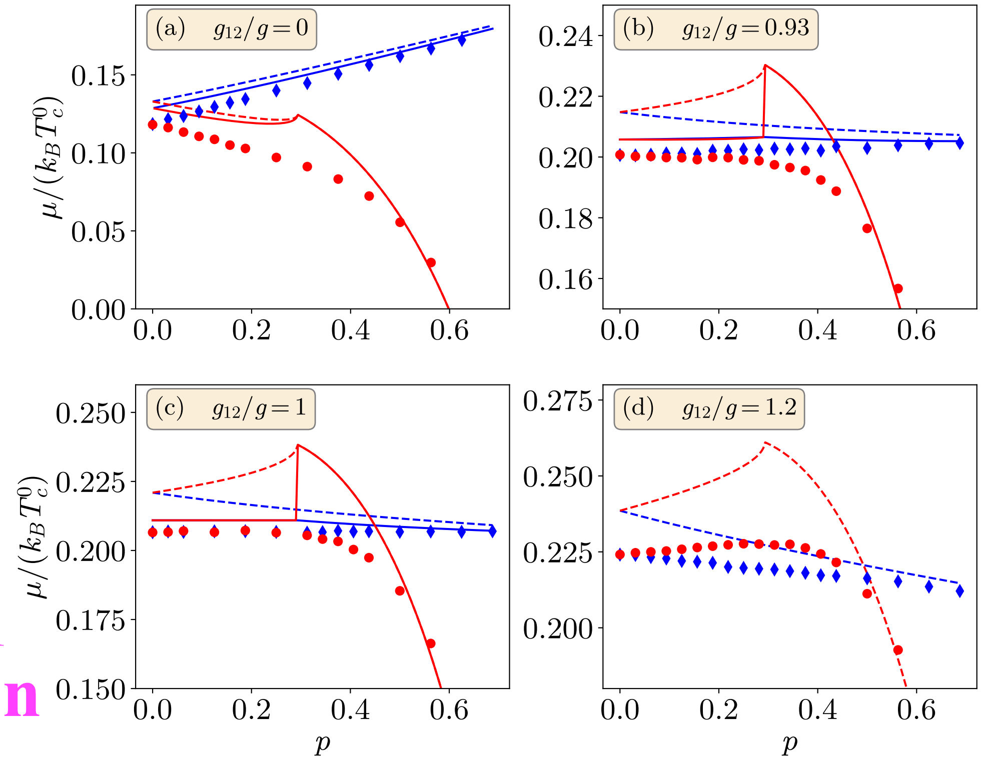
$$\mu(N,T) = -k_B T \log \frac{Z_{N+1}}{Z_N}$$



- **Develop PIMC for mixtures**

Two-component chemical potentials

- Dependence on polarization at fixed T
  p=(N1-N2)/(N1+N2)
- Comparison with HF and Popov theory



If $g_{12} < g$ no crossing of μ_1 and μ_2 at finite polarization