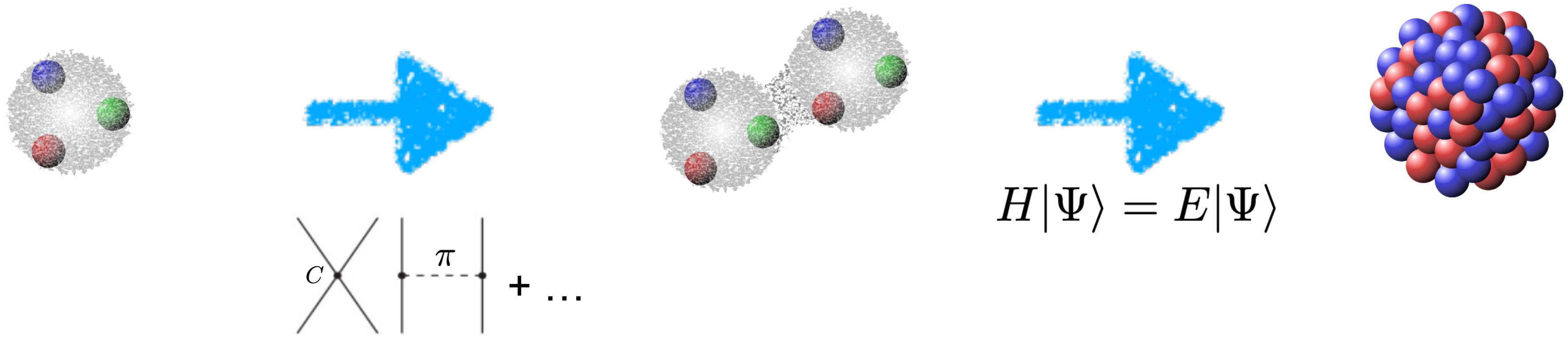
筑波大学
計算科学研究センター
Center for Computational Sciences

Recent advances in ab initio calculations of heavy nuclei



Takayuki Miyagi

Supported by
ERATO
TOMOE

Collaborators

Chalmers University of Technology: **A. Ekström, C. Forssen**

Oak Ridge National Laboratory: **G. Hagen, Baishan Hu**

TRIUMF: **J. D. Holt, P. Navratil**

TU Darmstadt: **K. Hebeler**

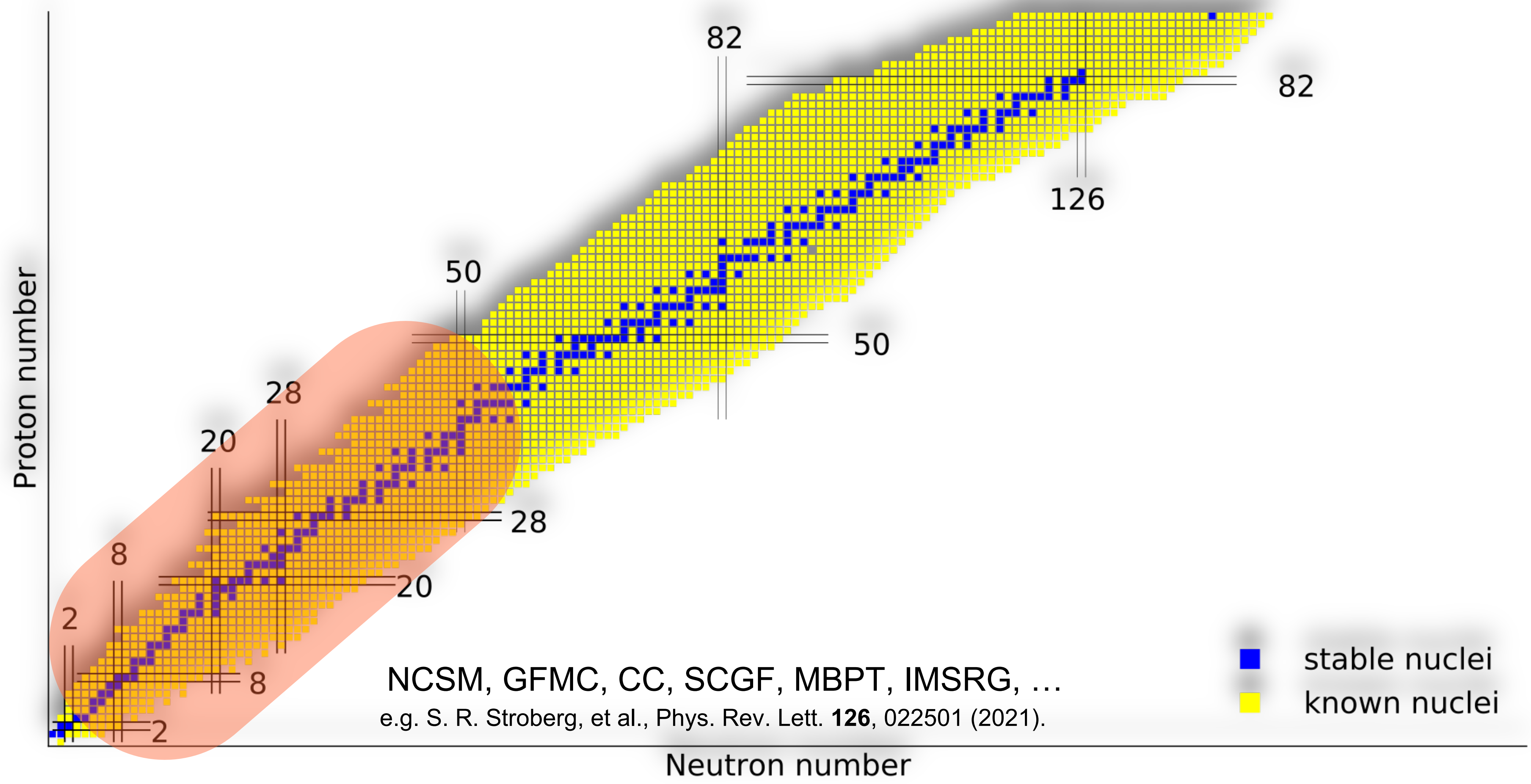
University of Notre Dame: **S. R. Stroberg**

University of Mainz: **W. Jiang**

University of Tennessee: **T. Papenbrock, Z. Sun**

University of Durham: **I. Vernon**

Why heavy nuclei?

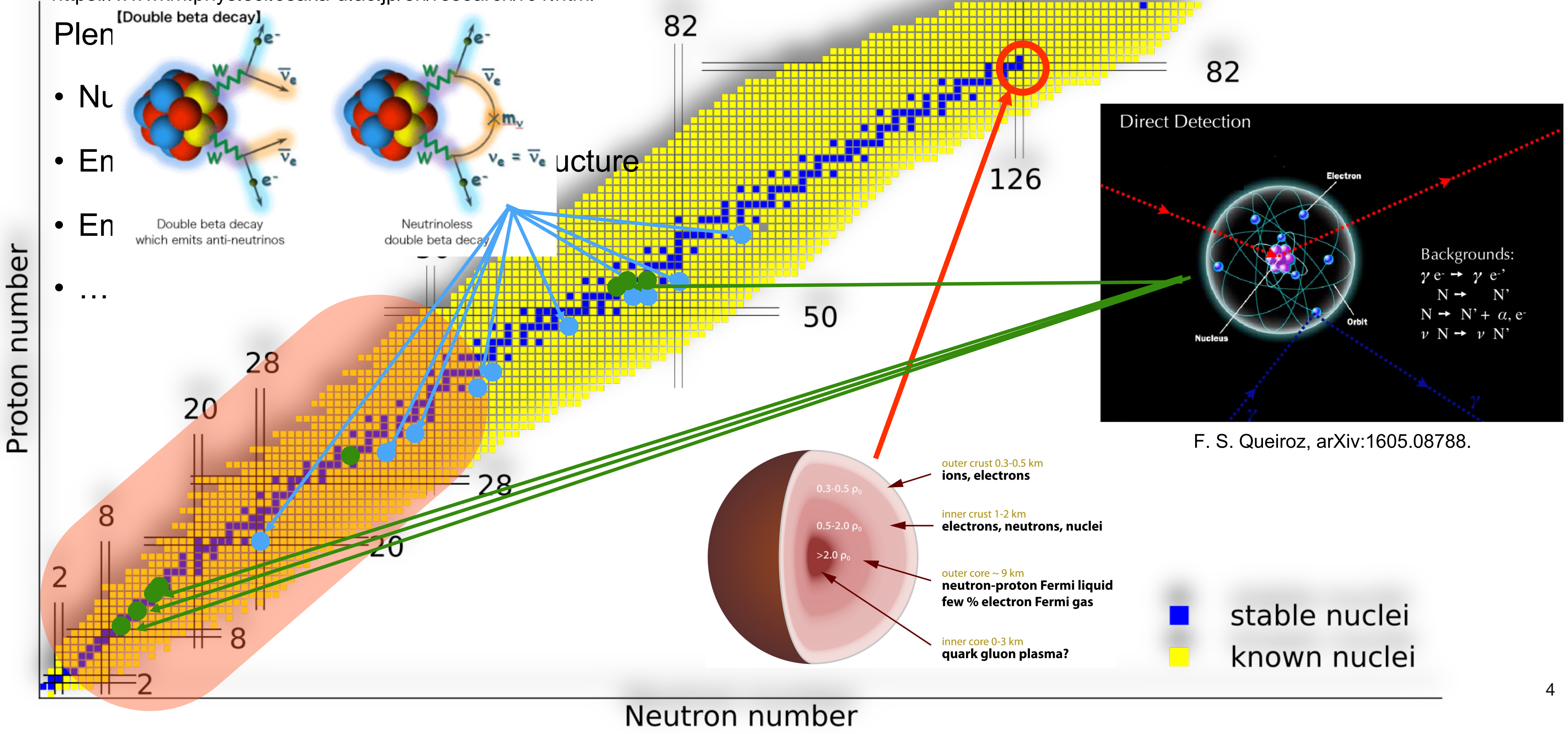


NCSM, GFMC, CC, SCGF, MBPT, IMSRG, …

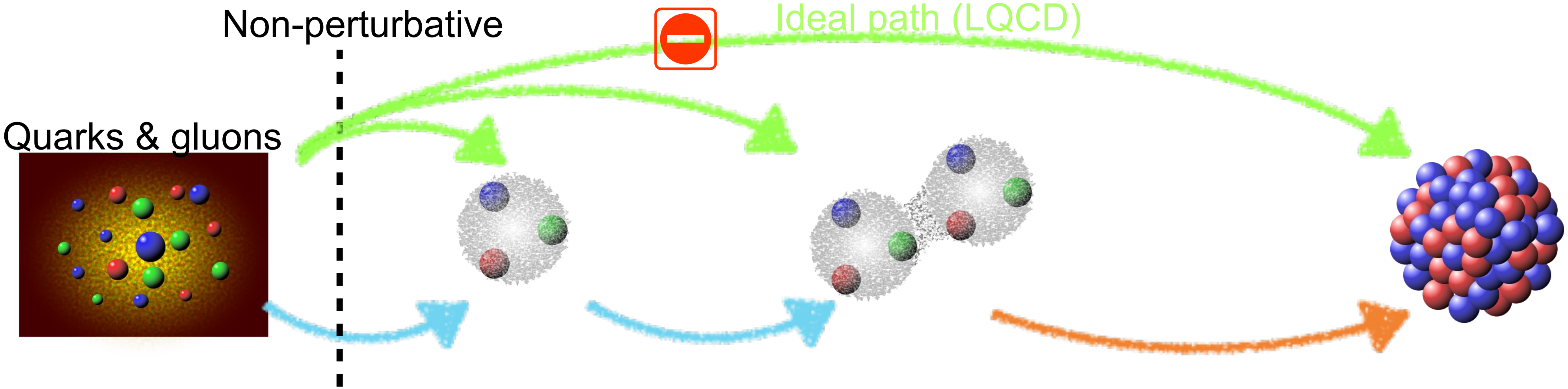
e.g. S. R. Stroberg, et al., Phys. Rev. Lett. **126**, 022501 (2021).

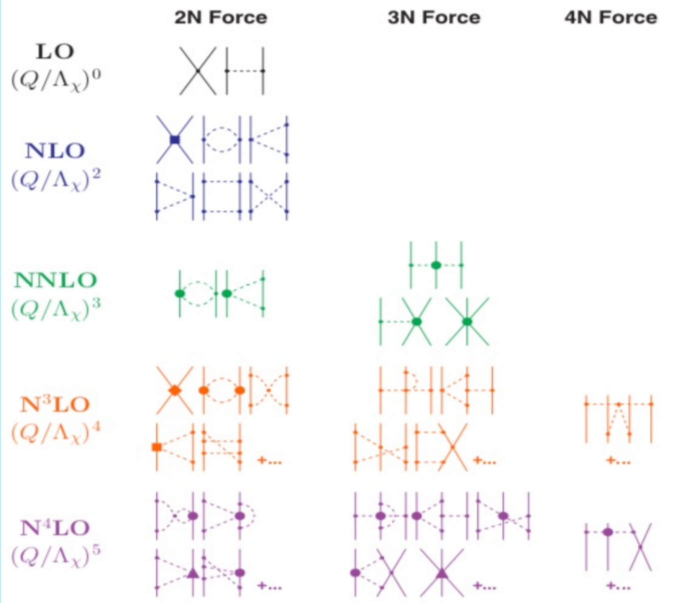
Why heavy nuclei?

https://wwwkm.phys.sci.osaka-u.ac.jp/en/research/r01.html



Nuclear ab initio calculation





Nuclear many-body problem

- Green's function Monte Carlo
- No-core shell model
- Nuclear lattice effective field theory
- Self-consistent Green's function
- Coupled-cluster
- In-medium similarity renormalization group
- Many-body perturbation theory
- …

Nuclear interaction from chiral EFT

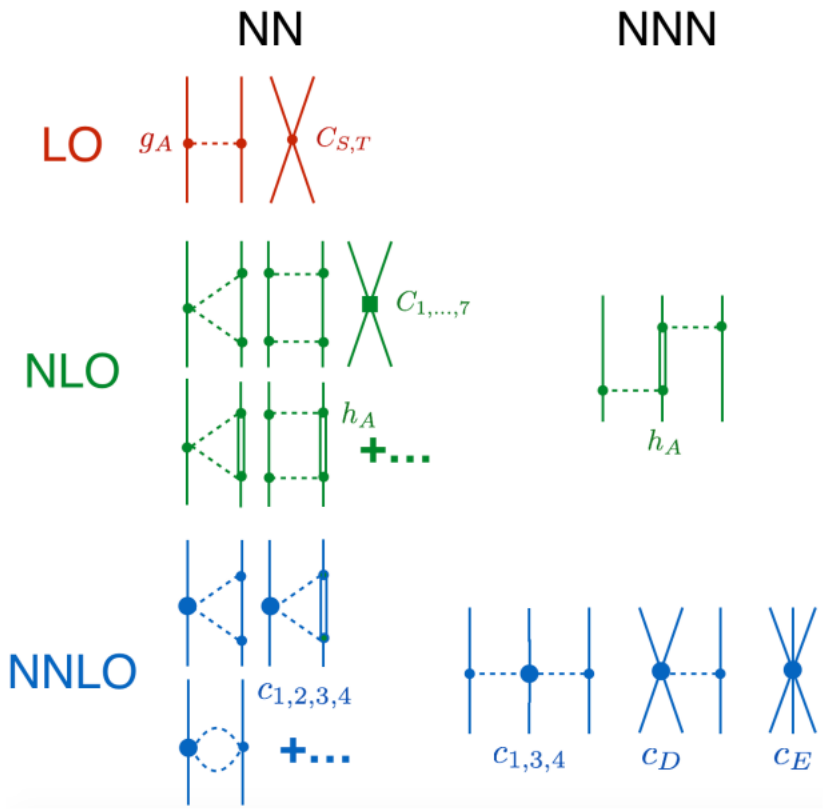
Weinberg, van Kolck, Kaiser, Epelbaum, Glöckle, Meißner, Entem, Machleidt, …

Lagrangian construction

- Chiral symmetry
- Power counting

Systematic expansion

- Unknown LECs
- Many-body interactions
- Estimation of truncation error



Taken from A. Ekström et al., Phys. Rev. C 97, 024332 (2018).

Many-body problem: similarity transformation methods

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Similarity transformation

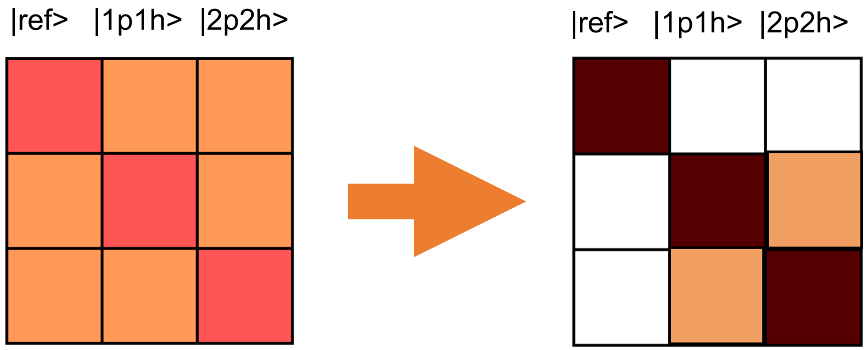
$$H|\Psi\rangle = E_{\mathrm{g.s.}}|\Psi\rangle$$

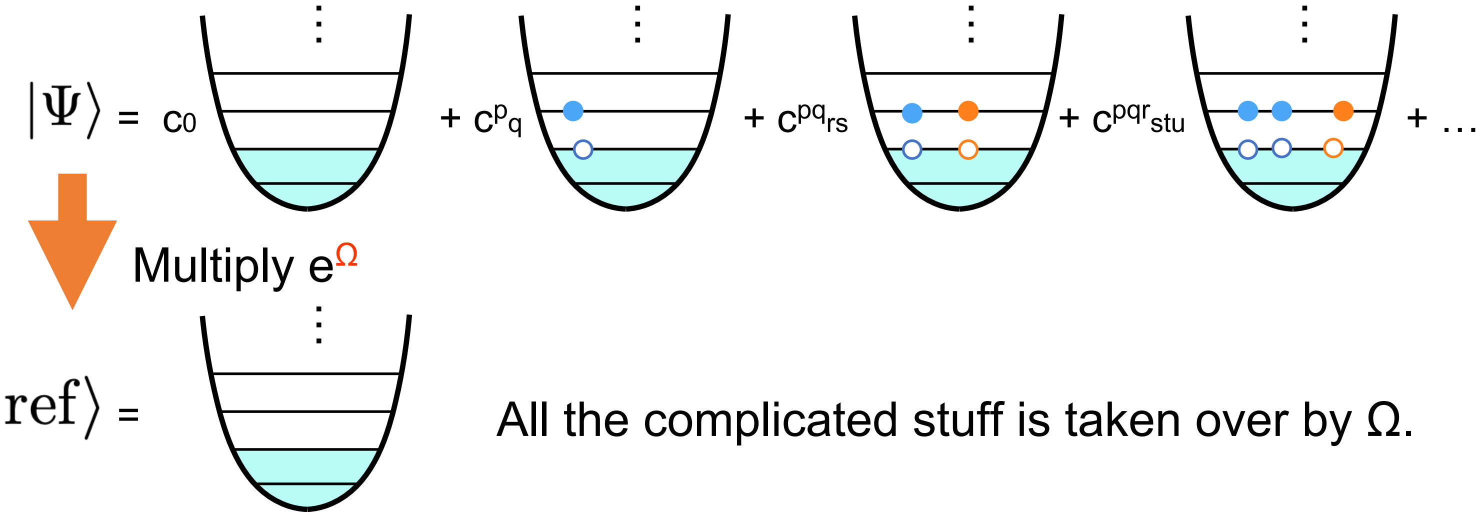
$$e^{\Omega} H e^{-\Omega} e^{\Omega}|\Psi\rangle = E_{\mathrm{g.s.}} e^{\Omega}|\Psi\rangle$$

Multiply e^{Ω} to both side

$$\tilde{H}|\mathrm{ref}\rangle = E_{\mathrm{g.s.}}|\mathrm{ref}\rangle$$

Similarity transformation





All the complicated stuff is taken over by Ω.

How can we find Ω operator?

- Coupled-cluster method (CCM), in-medium similarity renormalization group (IMSRG), …

In-medium similarity renormalization group approach

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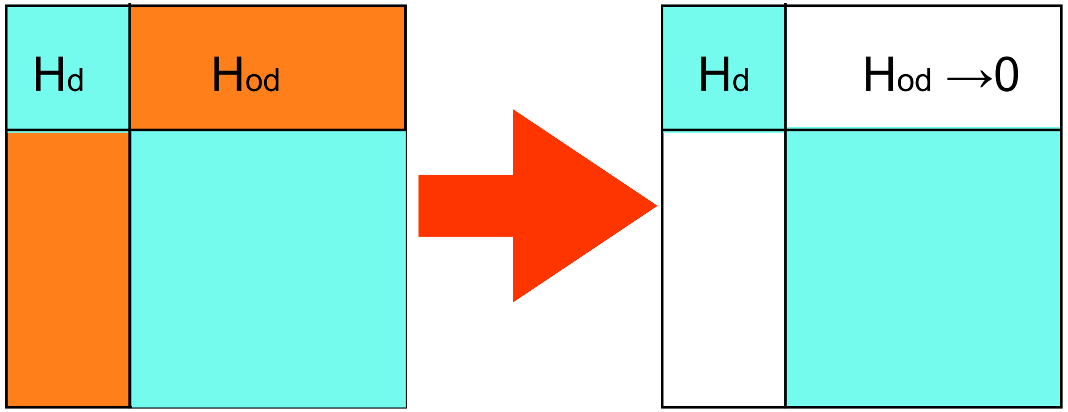
Similarity renormalization group

H. Hergert et al., Phys. Rep. 621, 165 (2016).
S. R. Stroberg et al., Annu. Rev. Nucl. Part. Sci. 69, 307 (2019).

$$H(s) = U^\dagger(s) H(s=0) U(s)$$

$$\frac{dU(s)}{ds} = -\eta(s) U(s)$$

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$



The anti-Hermitian generator η(s) is arbitrary.

How can we choose the functional form to suppress the off-diagonal MEs?

In-medium similarity renormalization group approach

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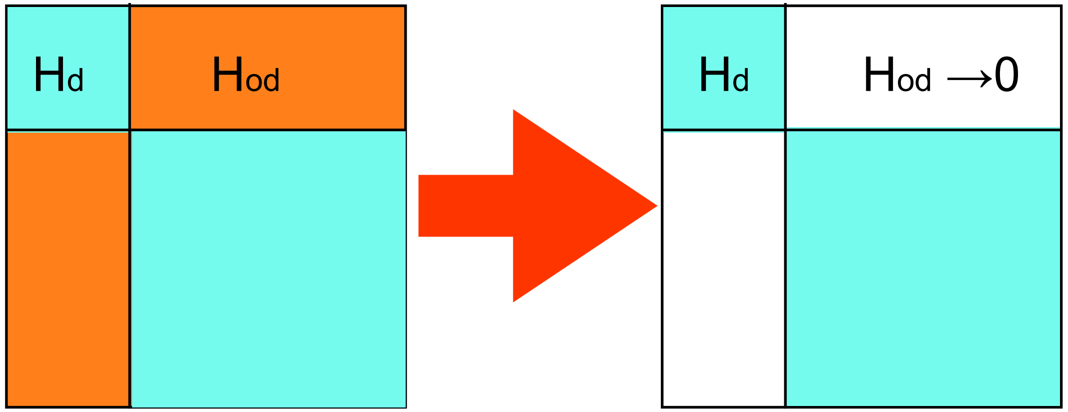

A simple example:

- 2 x 2 Hamiltonian

$$H(s) = \begin{pmatrix} c+z & x \\ x & c-z \end{pmatrix} = cI + z(s)\sigma_3 + x(s)\sigma_1 \qquad \eta(s) = \frac{i}{2}\frac{x(s)}{z(s)}\sigma_2$$

$$\frac{dx(s)}{ds} = -x(s) \rightarrow x(s) = x(0)\exp(-s)$$ Exponential decay of the off-diagonal ME.

$$\text{note} : [\sigma_2, \sigma_3] = 2i\sigma_1$$

In-medium similarity renormalization group approach

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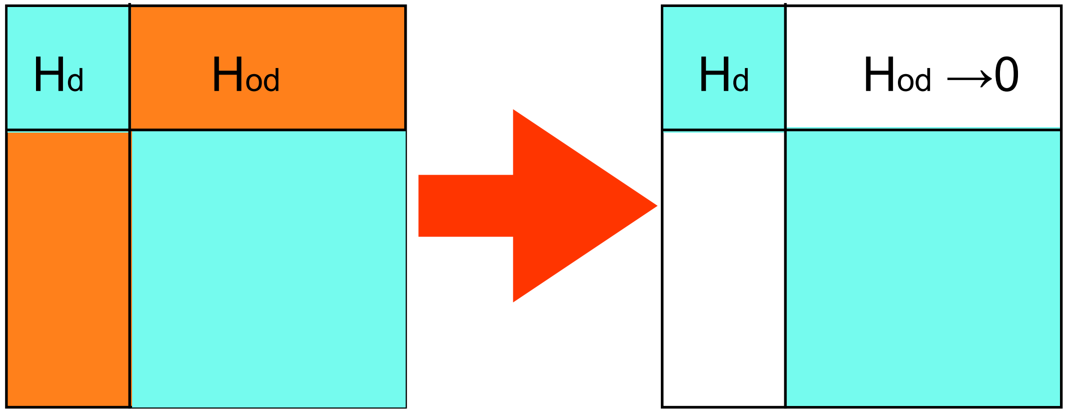

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An expect

$$\eta(s) = \frac{i}{2}\frac{x(s)\sigma_2}{z(s)}$$ → $\dfrac{\text{Off-diagonal MEs need to be suppressed}}{\text{Energy gap from the diagonal MEs}}$ (Anti-Hermitian)

In-medium similarity renormalization group approach

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$$\frac{d\Omega}{ds} = \eta(s) - \frac{1}{2}[\Omega(s), \eta(s)] + \cdots$$

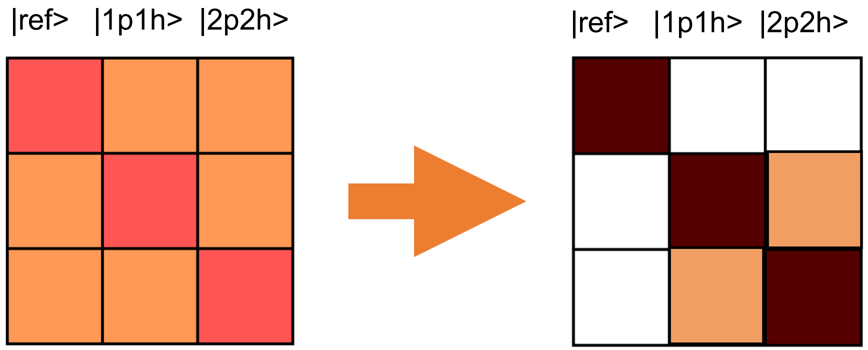
$$H(s) = e^{\Omega(s)} H(s=0) e^{-\Omega(s)} \approx E(s) + \sum_{12} f_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4}\sum_{1234} \Gamma_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

$$\eta(s) = \sum_{12} \eta_{12}(s)\{a_1^\dagger a_2\} + \sum_{1234} \eta_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

$$\eta_{12} = \frac{1}{2}\arctan\left(\frac{2f_{12}}{f_{11} - f_{22} + \Gamma_{1212}}\right)$$

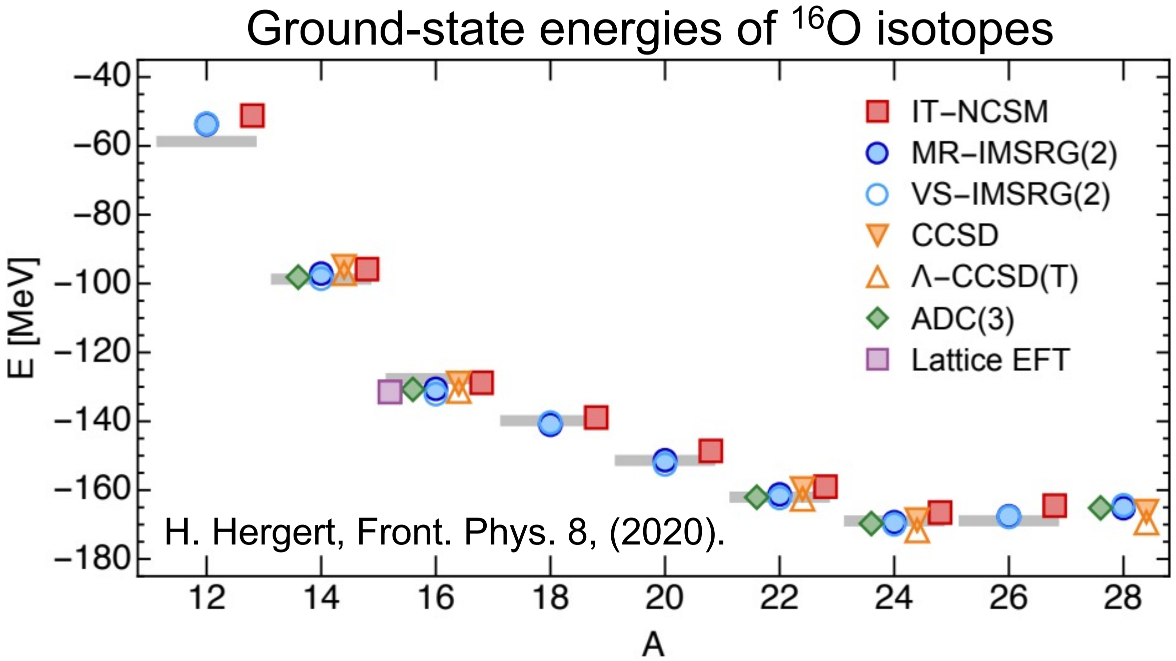
$$\eta_{1234} = \frac{1}{2}\arctan\left(\frac{2\Gamma_{1234}}{f_{11} + f_{22} - f_{33} - f_{44} + A_{1234}}\right)$$

$$A_{1234} = \Gamma_{1212} + \Gamma_{3434} - \Gamma_{1313} - \Gamma_{2424} - \Gamma_{1414} - \Gamma_{2323}$$



Approximation:

- H(s) and η(s) are two-body operators.
- A few % error in the ground-state energy and radius



In-medium similarity renormalization group approach

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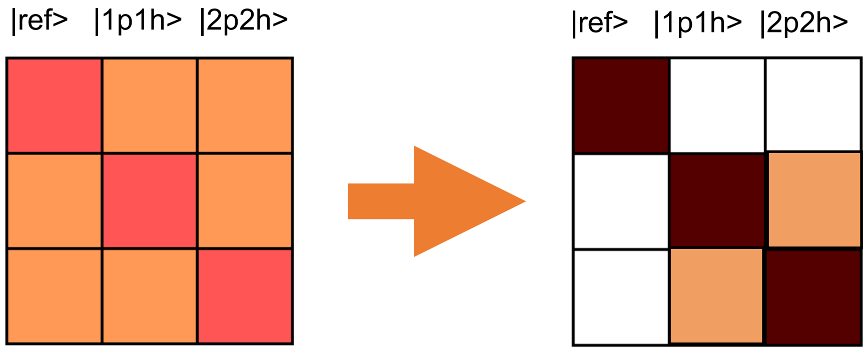
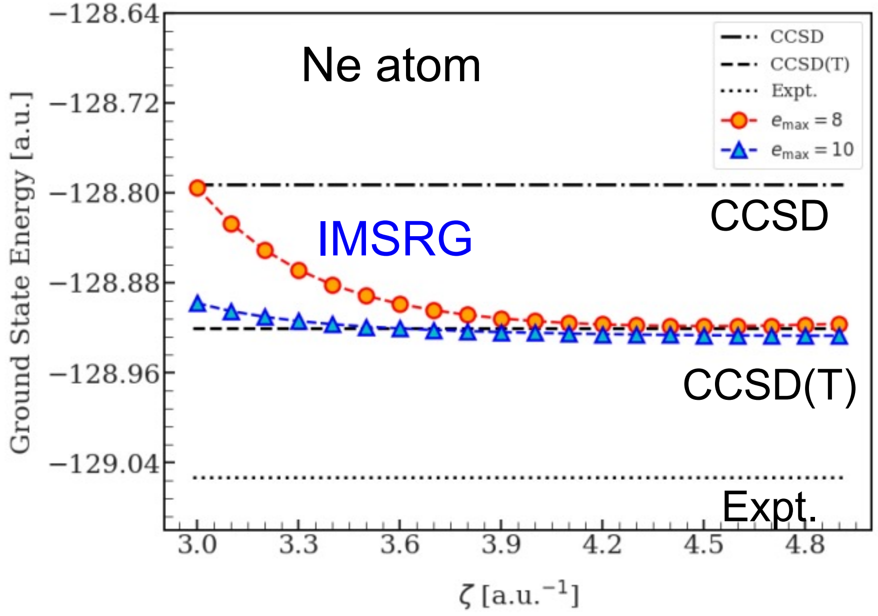


G. Tenkila et al., arXiv:2212.08188



Basis parameter

Approximation:

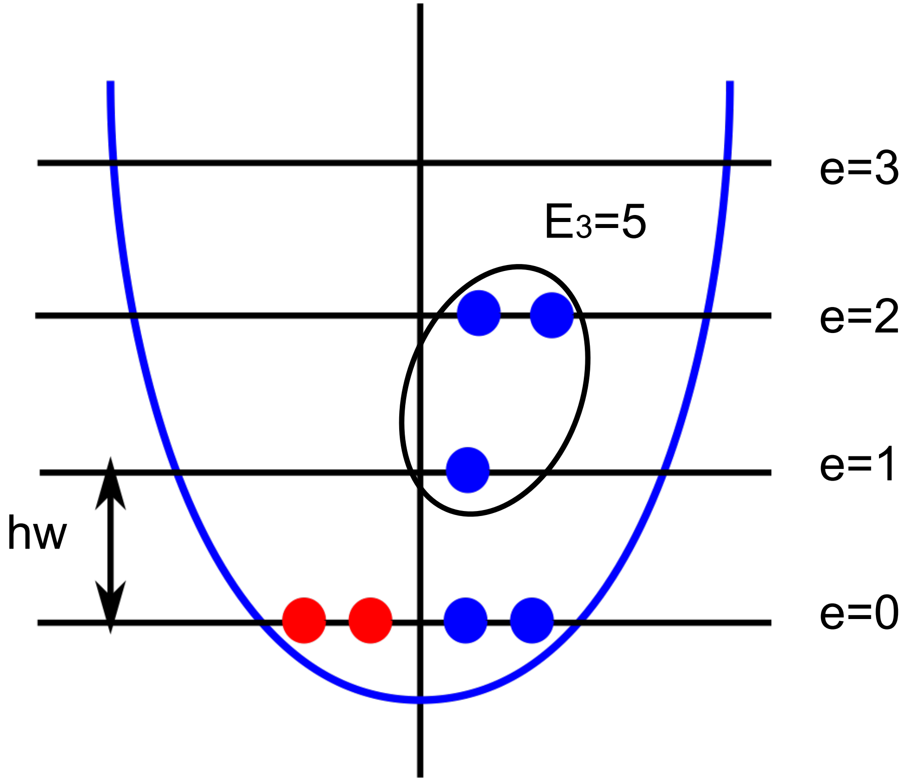
- H(s) and η(s) are two-body operators.
- A few % error in the ground-state energy and radius

Towards heavy nuclei

NN+3N Hamiltonian (harmonic oscillator basis)

Parameters controlling numerical calculations

- Frequency (hw)
- e_{max} (number of major shells)
- E_{3max} (sum of 3B HO quanta)



One has to increase e_{max} and E_{3max} until results converge!

Limited E_{3max} does not allow to access heavy systems.

Towards heavy nuclei

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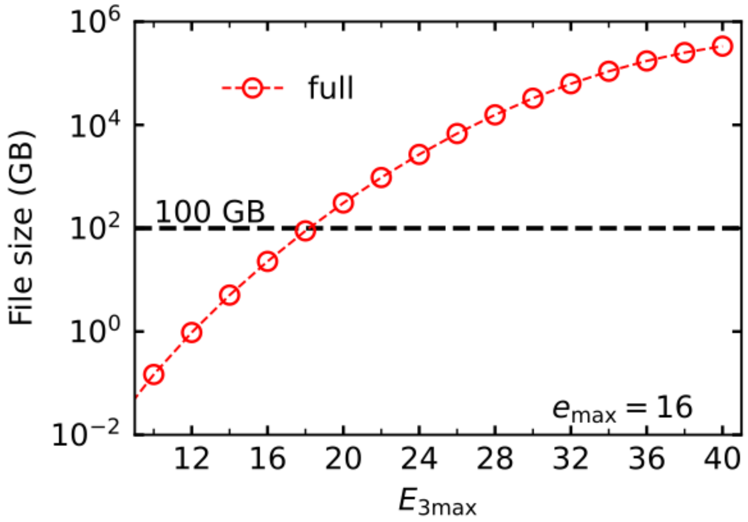

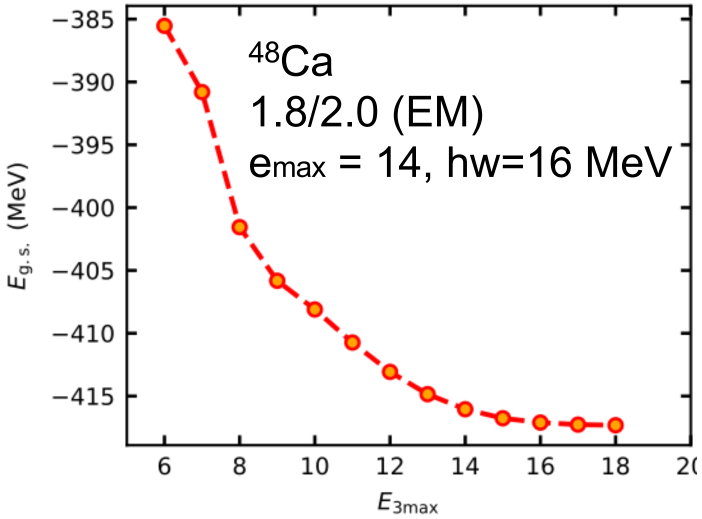

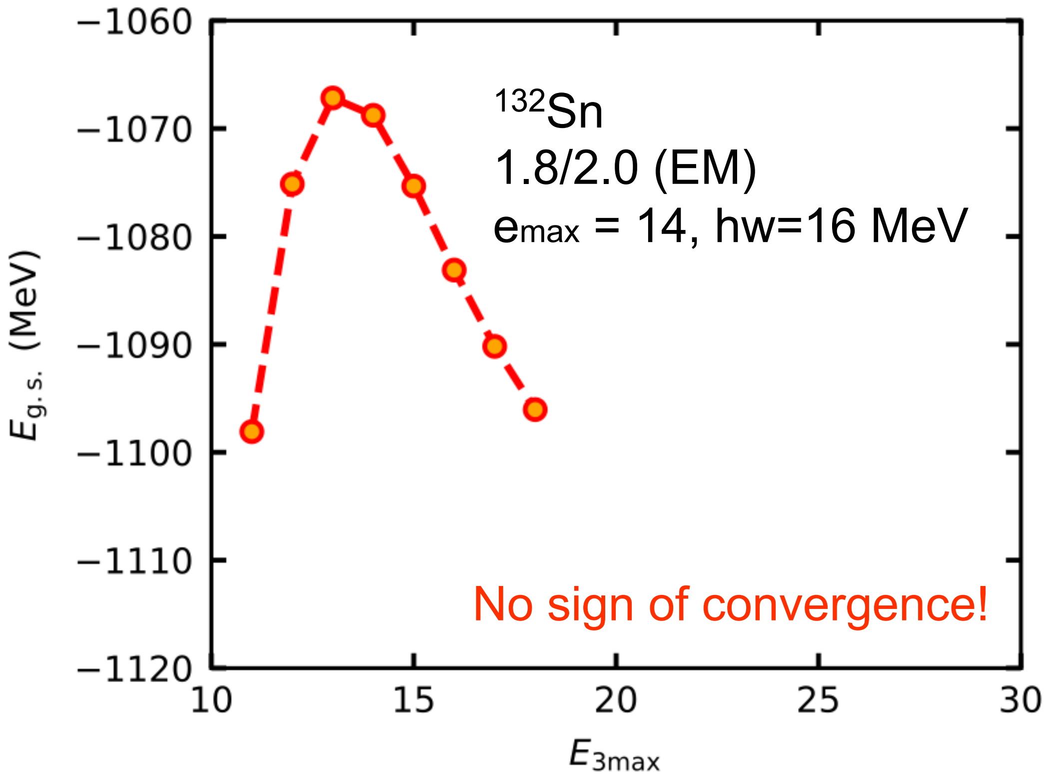

Residual interactions

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Hamiltonian: $$H = \sum_{p'p} t_{p'p} c^\dagger_{p'} c_p + \frac{1}{4} \sum_{p'q'pq} v_{p'q'pq} c^\dagger_{p'} c^\dagger_{q'} c_q c_p + \frac{1}{36} \sum_{p'q'r'pqr} v_{p'q'r'pqr} c^\dagger_{p'} c^\dagger_{q'} c^\dagger_{r'} c_r c_q c_p$$

1N 2N 3N

Normal ordering

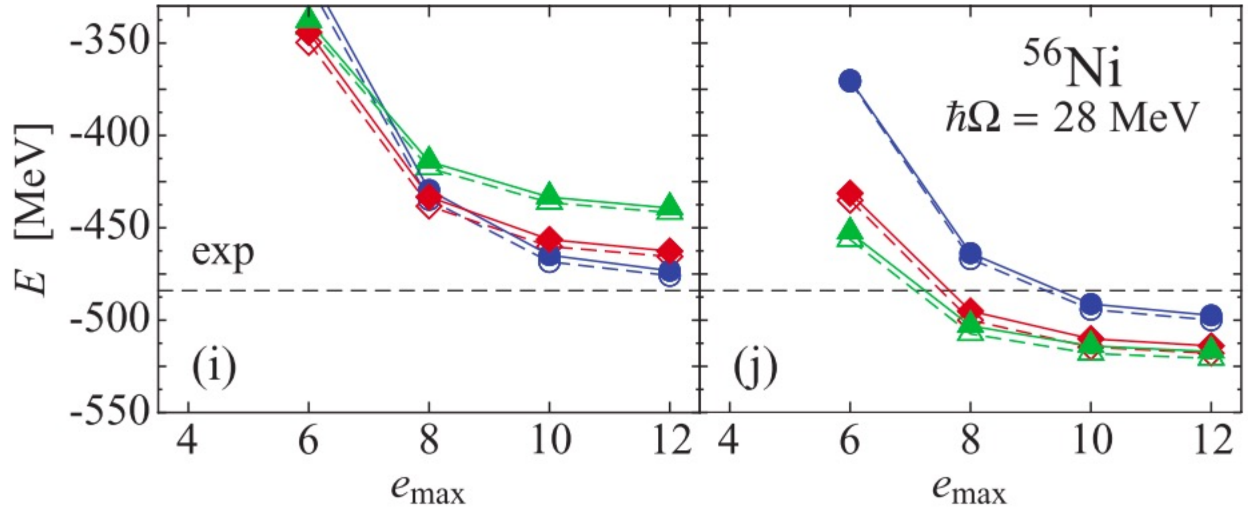
Residual 3N

$$H = E_0 + \sum_{p'p} f_{p'p} : c^\dagger_{p'} c_p : + \frac{1}{4} \sum_{p'q'pq} \Gamma_{p'q'pq} : c^\dagger_{p'} c^\dagger_{q'} c_q c_p : + \frac{1}{36} \sum_{p'q'r'pqr} W_{p'q'r'pqr} : c^\dagger_{p'} c^\dagger_{q'} c^\dagger_{r'} c_r c_q c_p :$$

Input of post mean-field calc. (NO2B)

Effect of residual 3N

CC calculations from S. Binder et al., Phys. Rev. C 87, 021303 (2013).



Solid: with W term
Dashed: without W term

W term: only ~ 1% of the total gs energy!

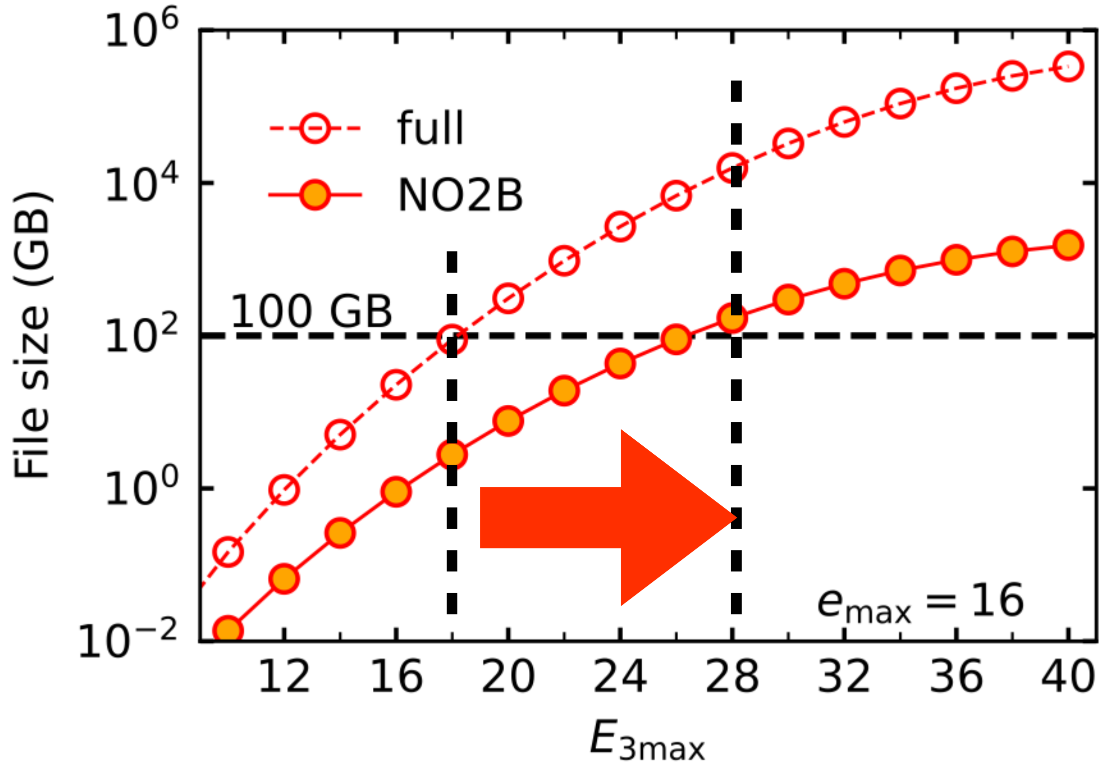
NO2B approximation (neglect W term)

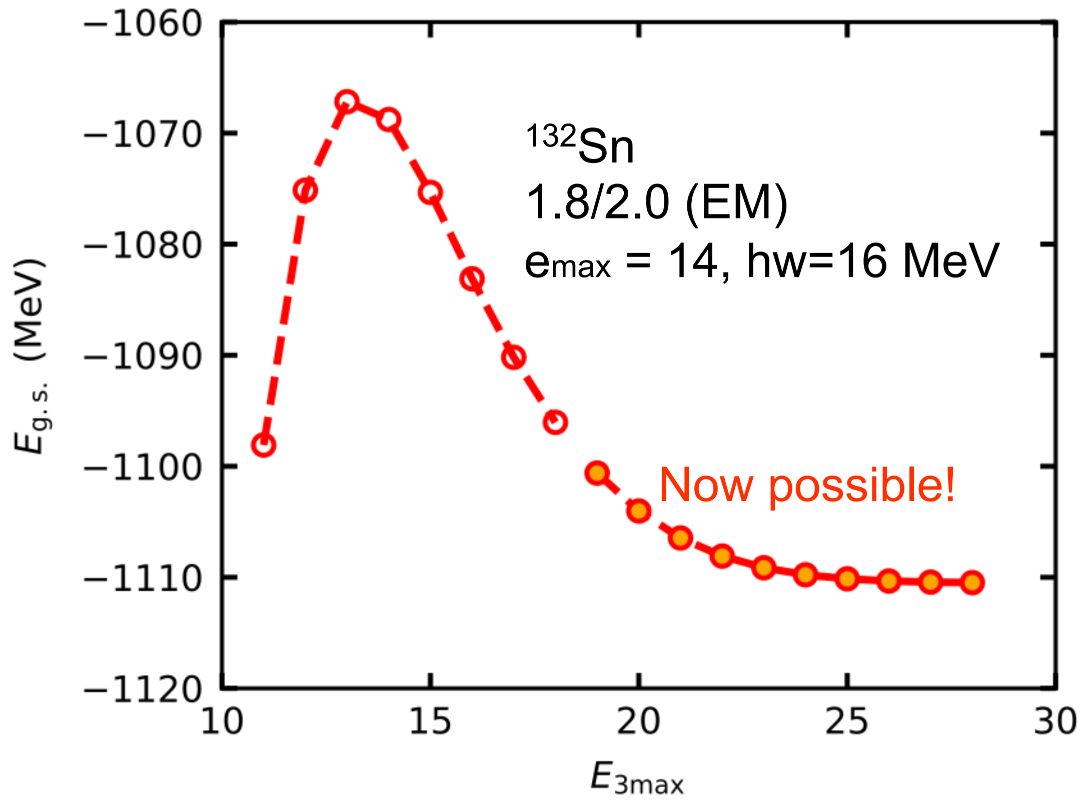
NO2B 3N storage scheme

Store only the matrix elements entering NO2B approximation.

$$V^{\mathrm{NO2B}}_{p'q'r'pqr} = V_{p'q'r'pqr}\tilde{\delta}_{r'r}$$

Determined by symmetry of one-body density matrix
c.f. parity and rotational symmetry for a spherical reference





Extrapolation

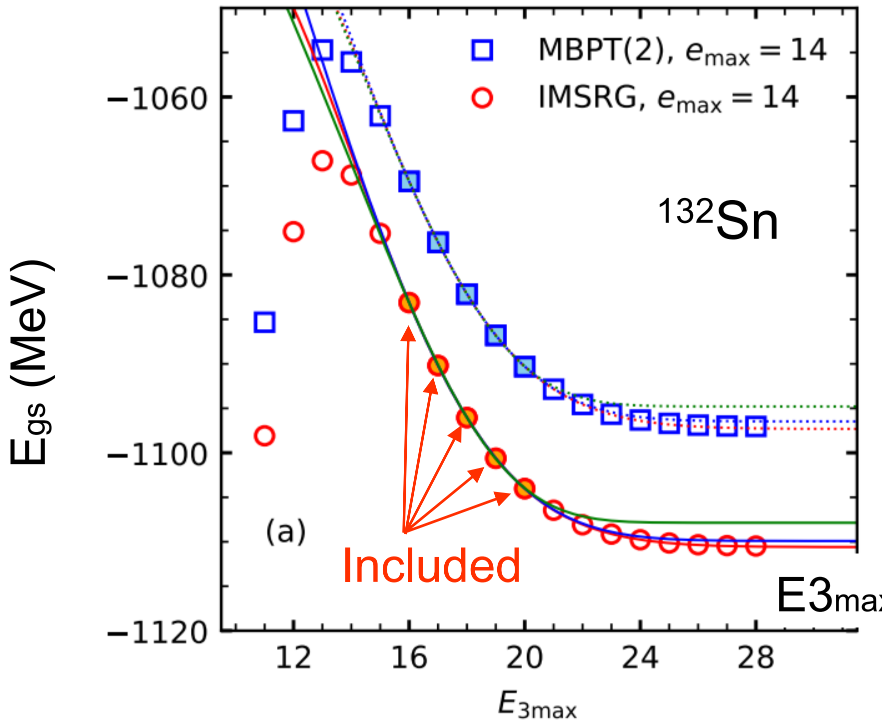
Asymmptotic behavior expected from the 2nd order MBPT.

$$E(E_{3\max}) = A\gamma_{\frac{2}{n}}\left[\left(\frac{E_{3\max}-\mu}{\sigma}\right)^n\right] + C$$

Fitting parameters

$$\mu \approx 3(2n_F + l_F)$$

The same form can be expected for any operators dominated by one-body part, e.g., radius



$E3_{max}$=28 resut is reproduced

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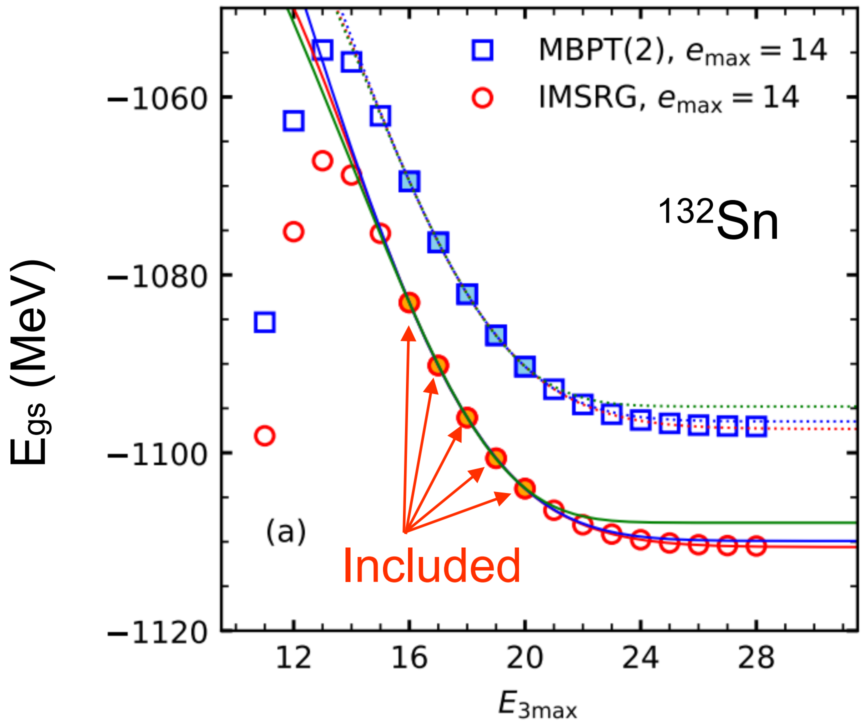
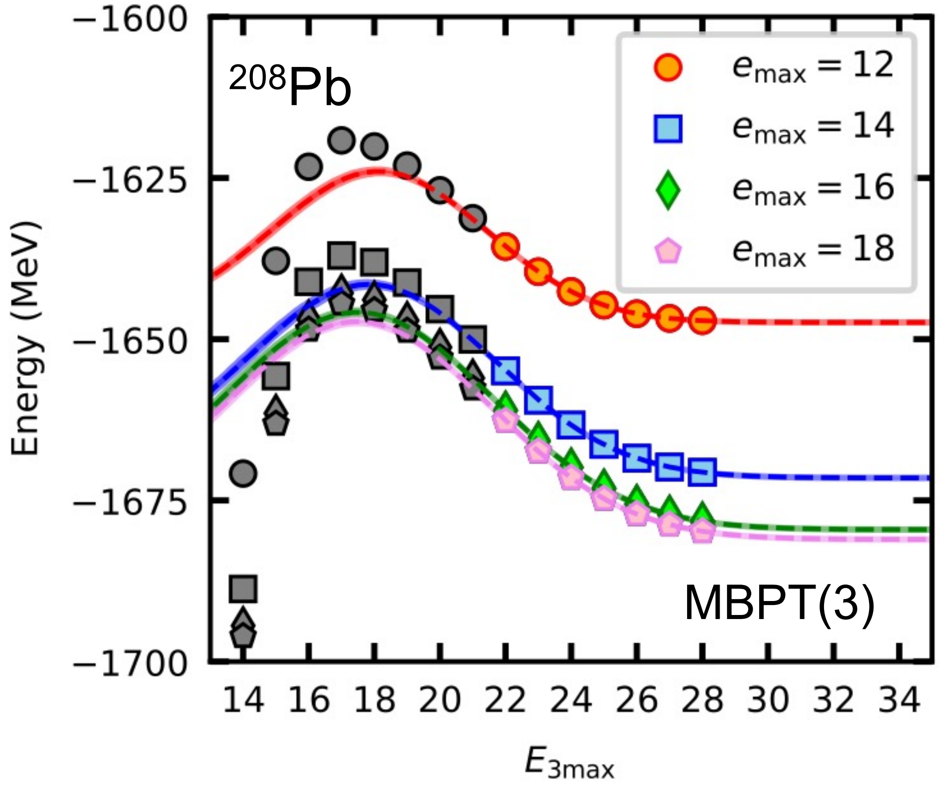
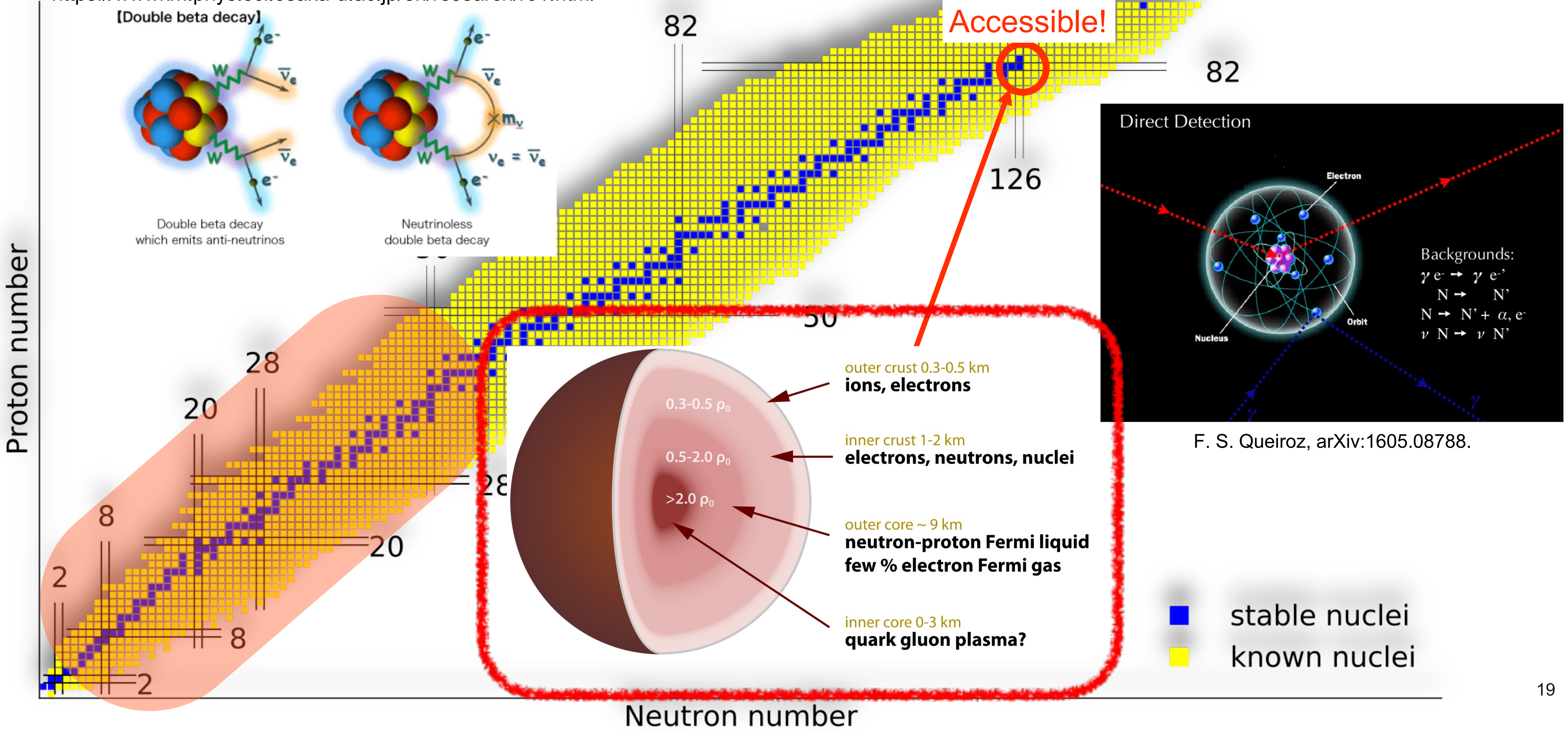




Why heavy nuclei?

https://wwwkm.phys.sci.osaka-u.ac.jp/en/research/r01.html



F. S. Queiroz, arXiv:1605.08788.

Infinite nuclear matter & neutron star

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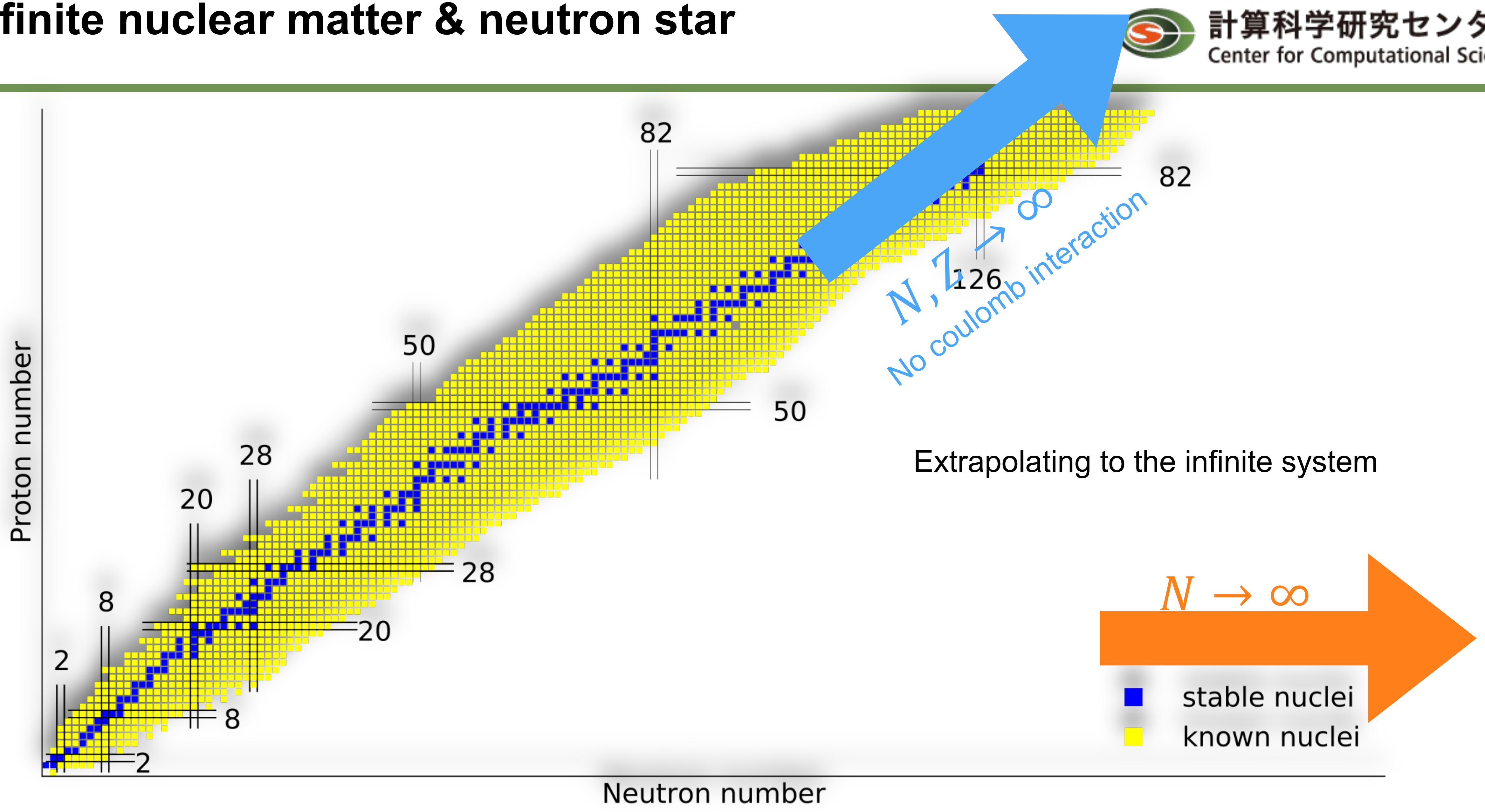
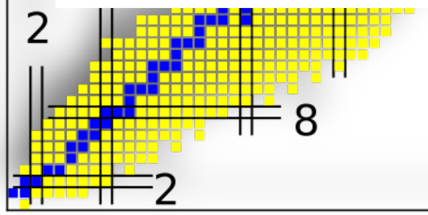


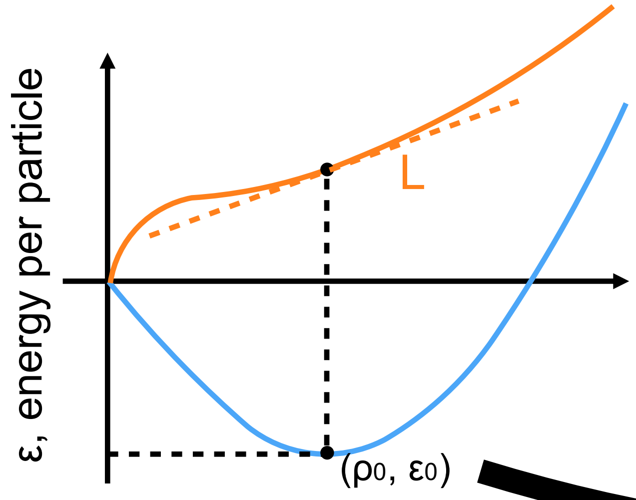
Extrapolating to the infinite system

Infinite nuclear matter & neutron star

Proton number



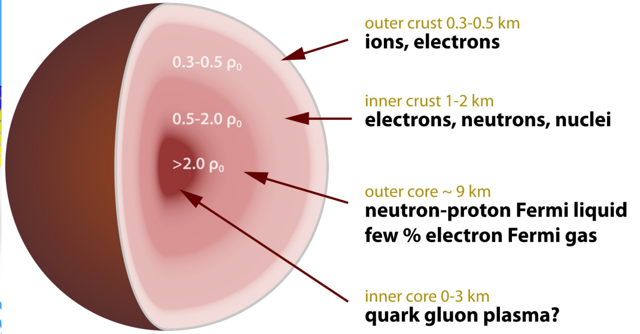
82



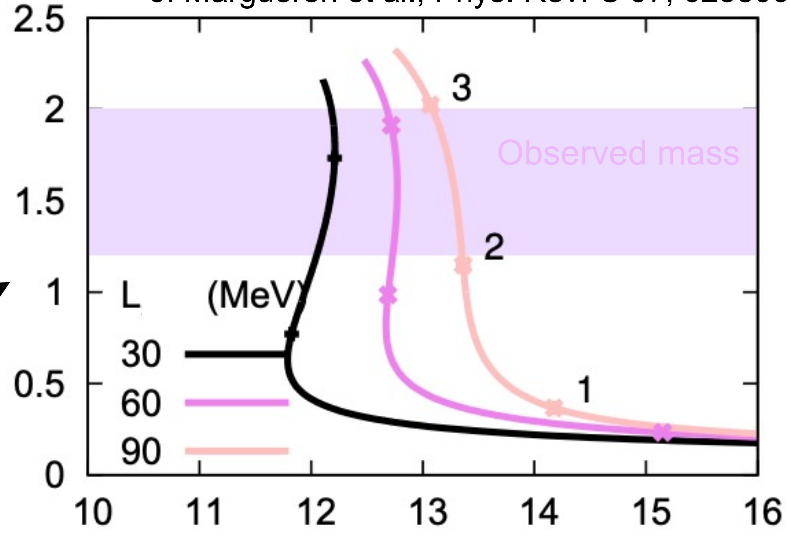
$$\varepsilon = S_\mathrm{n} + Lx + \frac{1}{2}K_\mathrm{n}x^2 + \mathcal{O}(x^3)$$

$$\varepsilon = \varepsilon_0 + \frac{1}{2}Kx^2 + \mathcal{O}(x^3)$$

$$x = \frac{1}{3}\frac{\rho - \rho_0}{\rho_0}$$



J. Margueron et al., Phys. Rev. C 97, 025806 (2018).



known nuclei

Solve TOV equation

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

Neutron number

Correlation connecting finite and infinite systems

Correlation from MF calculations

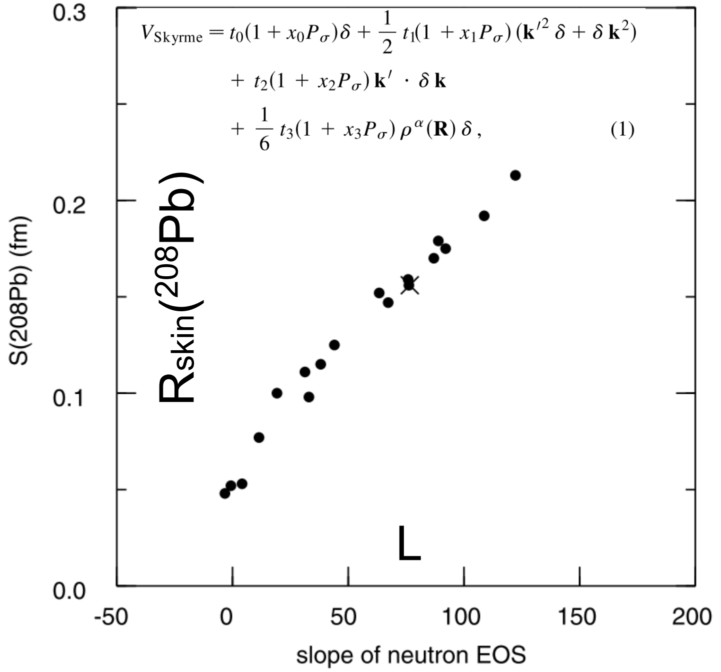
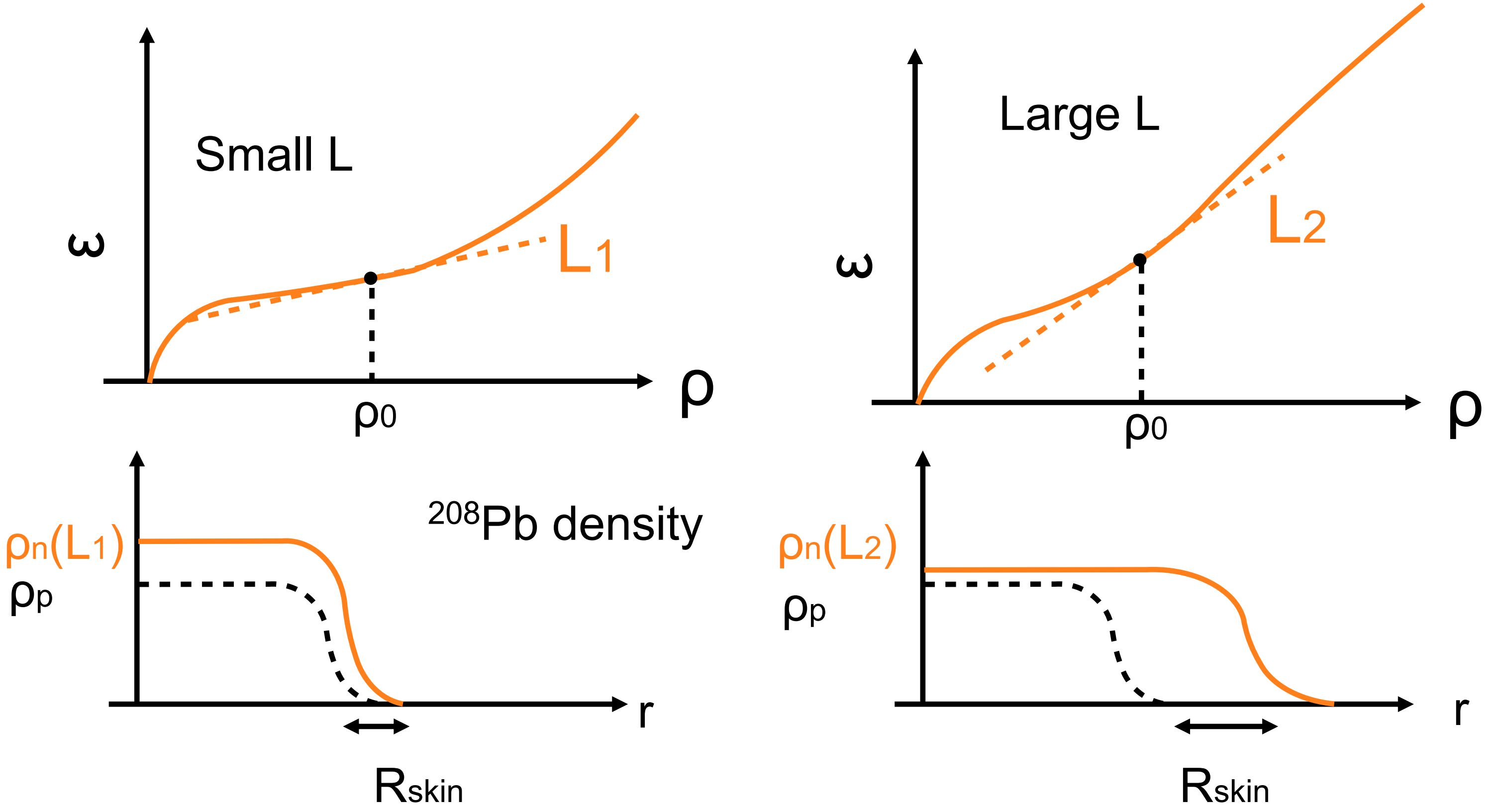


Figure taken from B. Alex Brown, Phys. Rev. Lett. 85, 5296 (2000).



For $L_1 < L_2$, $\rho_n(L_2) < \rho_n(L_1)$

→ $R_n(L_1) < R_n(L_2)$

→ $r_{skin}(L_1) < r_{skin}(L_2)$

*Assumption: proton radius is fitted.

Motivation:

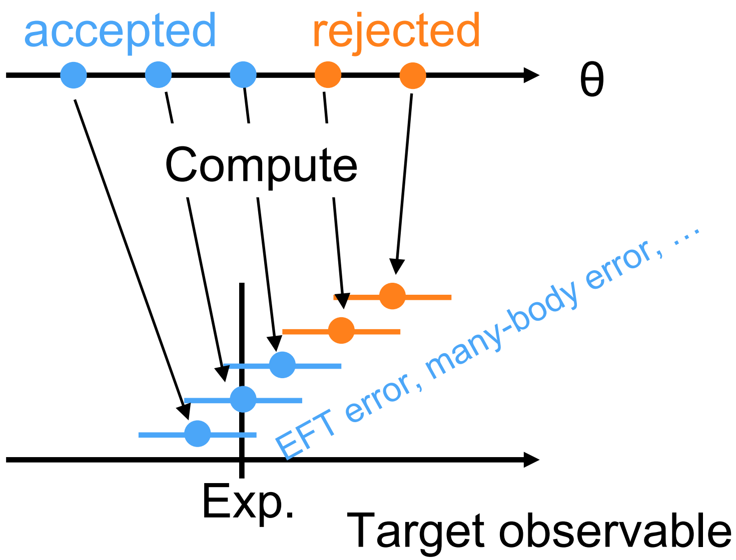
Robustness of the correlation

Narrower prediction of $R_{skin}(^{208}Pb)$

Sampling parameters

Non-implausible (NI) samples

- 17 Unknown LECs @ Delta-full N2LO
  - Constraints:
    - Naturalness: LECs should be O(1)
- Steps:
  - (1) Generate a random 17 dimensional vector θ
  - (2) Evaluate the selected observables
  - (3) Measure how the calculated observables are far from the experiments. If it is too far, θ is implausible and rejected.



Out of ~ 10^9 parameter sets, 34 non-implausible (NI) interactions were found.

Neutron skin thickness of ^{208}Pb

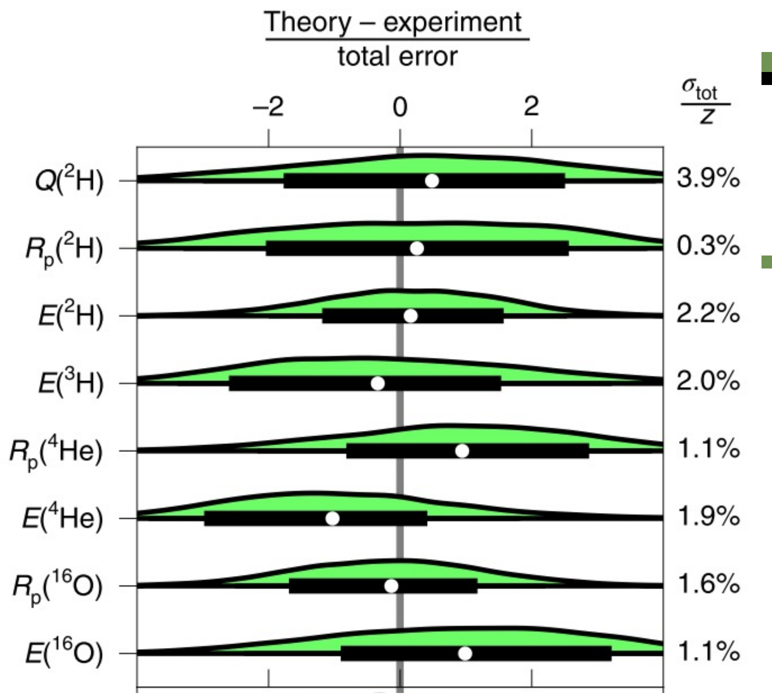
History matching:

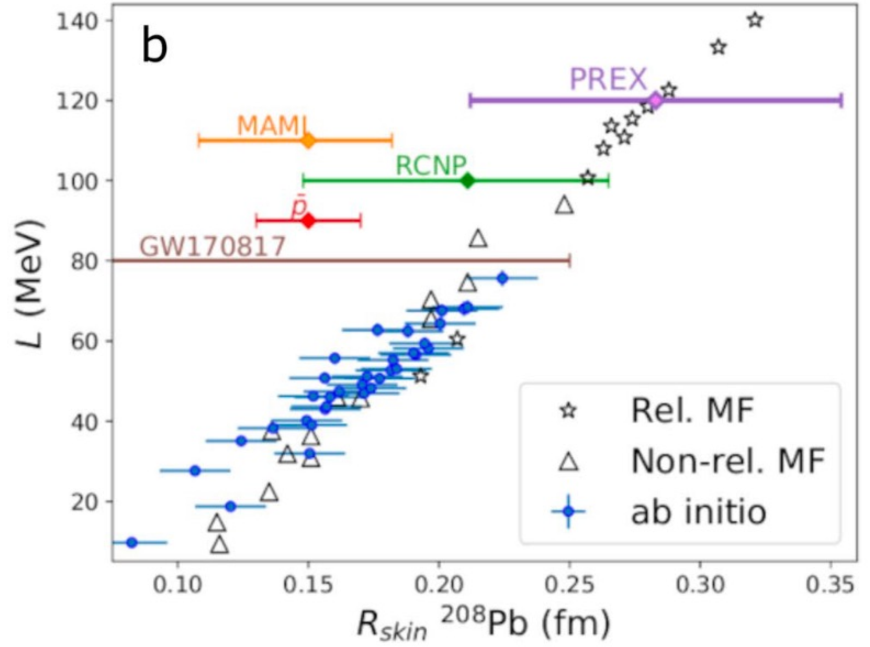
- ✦ Sampling 17 parameters in (delta-full) chiral EFT such that the parameter set is consistent with some selected data.

- ✦ Proton-neutron scattering phase shifts, $E(^2H)$, $R_p(^2H)$, $Q(^2H)$, $E(^3H)$, $E(^4He)$, $R_p(^4He)$, $E(^{16}O)$, and $R_p(^{16}O)$.

~ 10^9 parameter sets → 34 NI parameter sets

History matching





B. Hu, W. Jiang, T. Miyagi, et al., Nat. Phys. 18, 1196 (2022).

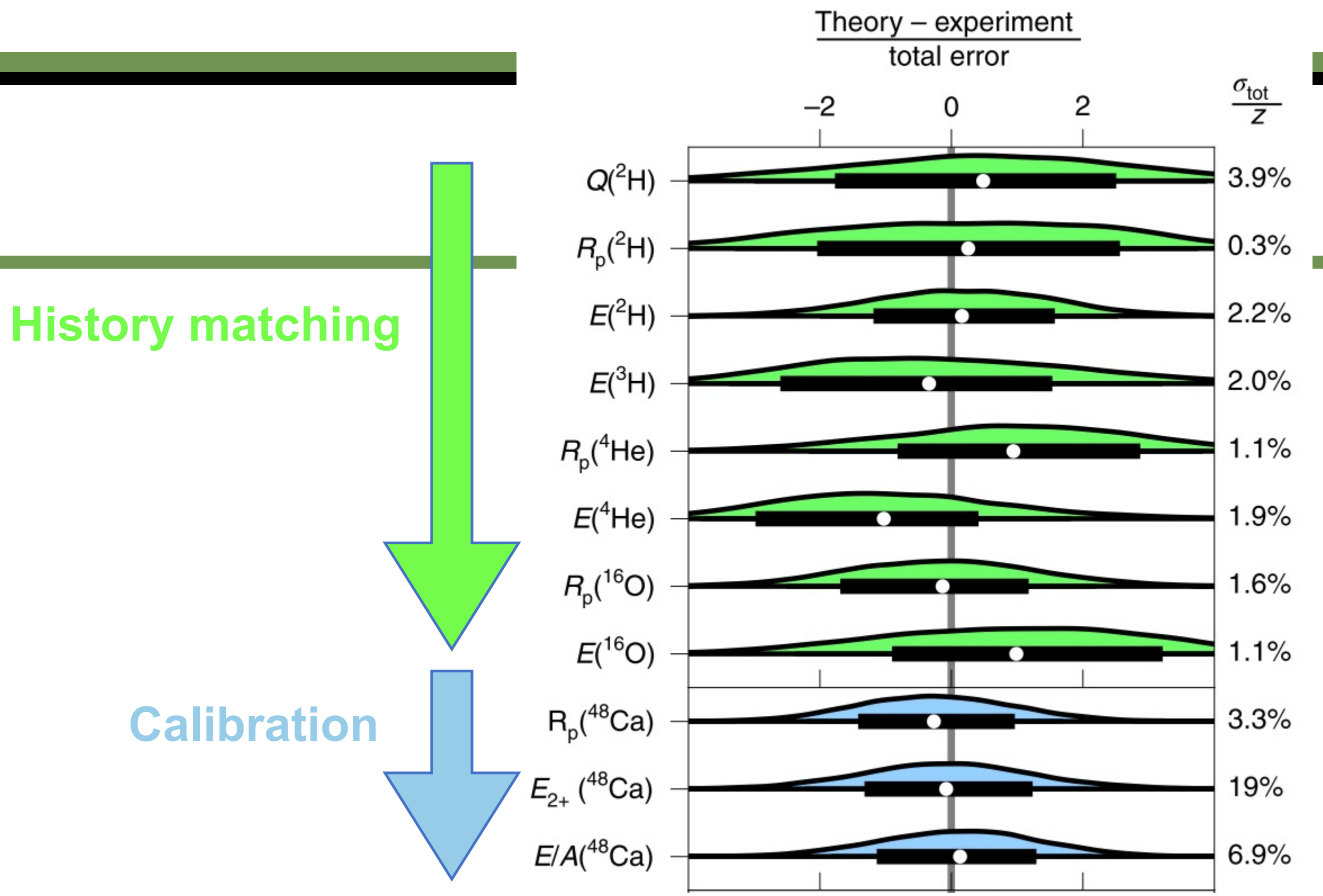
Neutron skin thickness of ^{208}Pb

Calibration:

- Assign weights according to the reproduction of ^{48}Ca data, known as importance resampling method.

$$w_i = \frac{\mathcal{L}(D|\boldsymbol{\theta}_i)}{\sum_{j=1}^{34} \mathcal{L}(D|\boldsymbol{\theta}_j)},$$

$$\mathcal{L}(D|\boldsymbol{\theta}_i) = \mathcal{N}(D, \sigma^2_{\mathrm{exp}} + \sigma^2_{\chi\mathrm{EFT}} + \sigma^2_{\mathrm{MB}})$$



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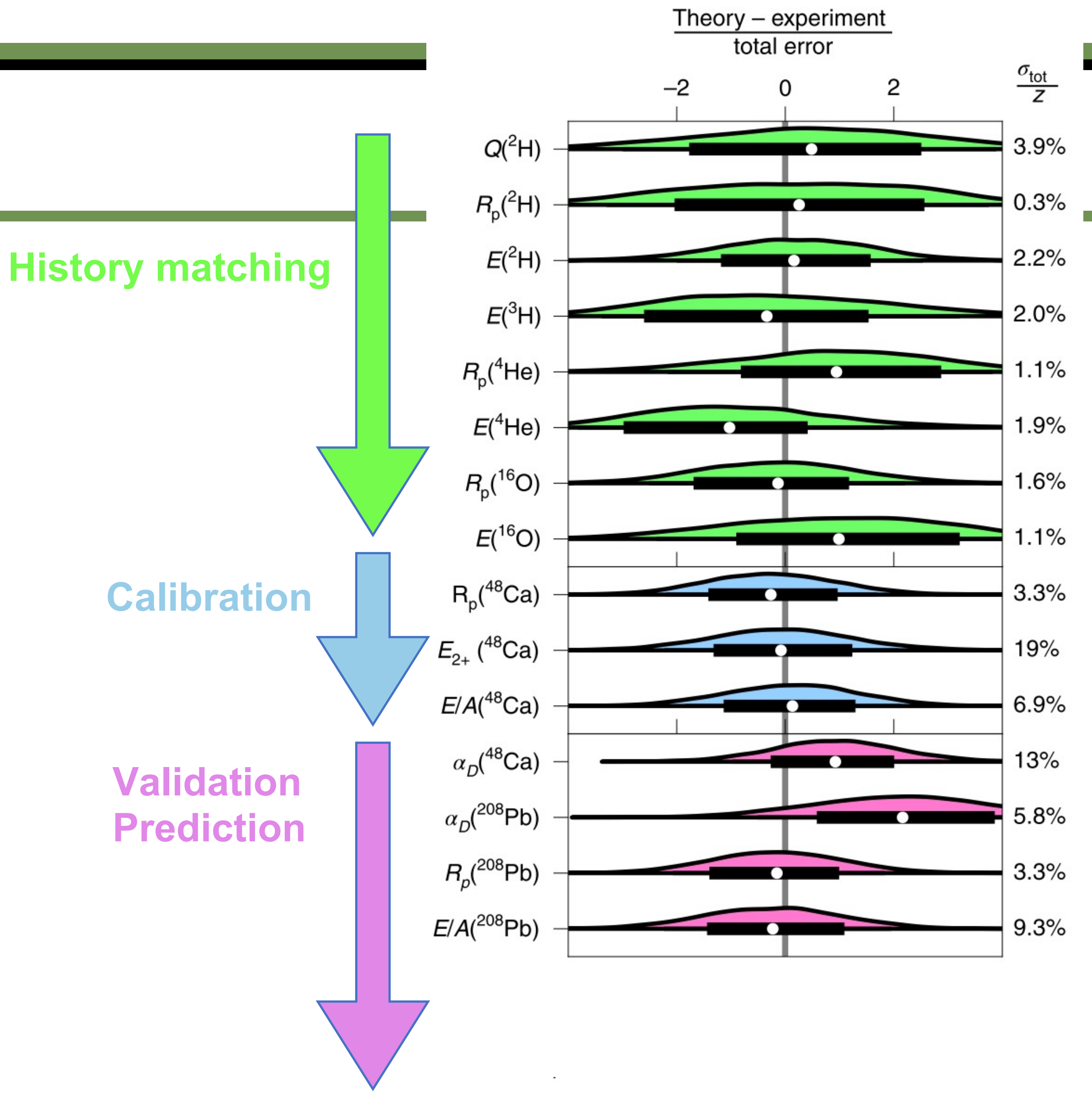
$$\mathcal{L}(D|\boldsymbol{\theta}_i) = \mathcal{N}(D, \sigma_{\rm exp}^2 + \sigma_{\chi \rm EFT}^2 + \sigma_{\rm MB}^2)$$

Validation & prediction:

- The weighted samples are approximately equivalent to the samples extracted from p(θ|D). $\mathrm{PPD} = \{\mathcal{O}_{\rm target}(\theta) : \theta \sim P(\theta|^{48}\mathrm{Ca})\}$



B. Hu, W. Jiang, T. Miyagi, et al., Nat. Phys. 18, 1196 (2022).

Neutron skin thickness of ^{208}Pb

Calibration:

- Assign weights according to the reproduction of ^{48}Ca data, known as importance resampling method.

$$w_i = \frac{\mathcal{L}(D|\boldsymbol{\theta}_i)}{\sum_{j=1}^{34} \mathcal{L}(D|\boldsymbol{\theta}_j)},$$

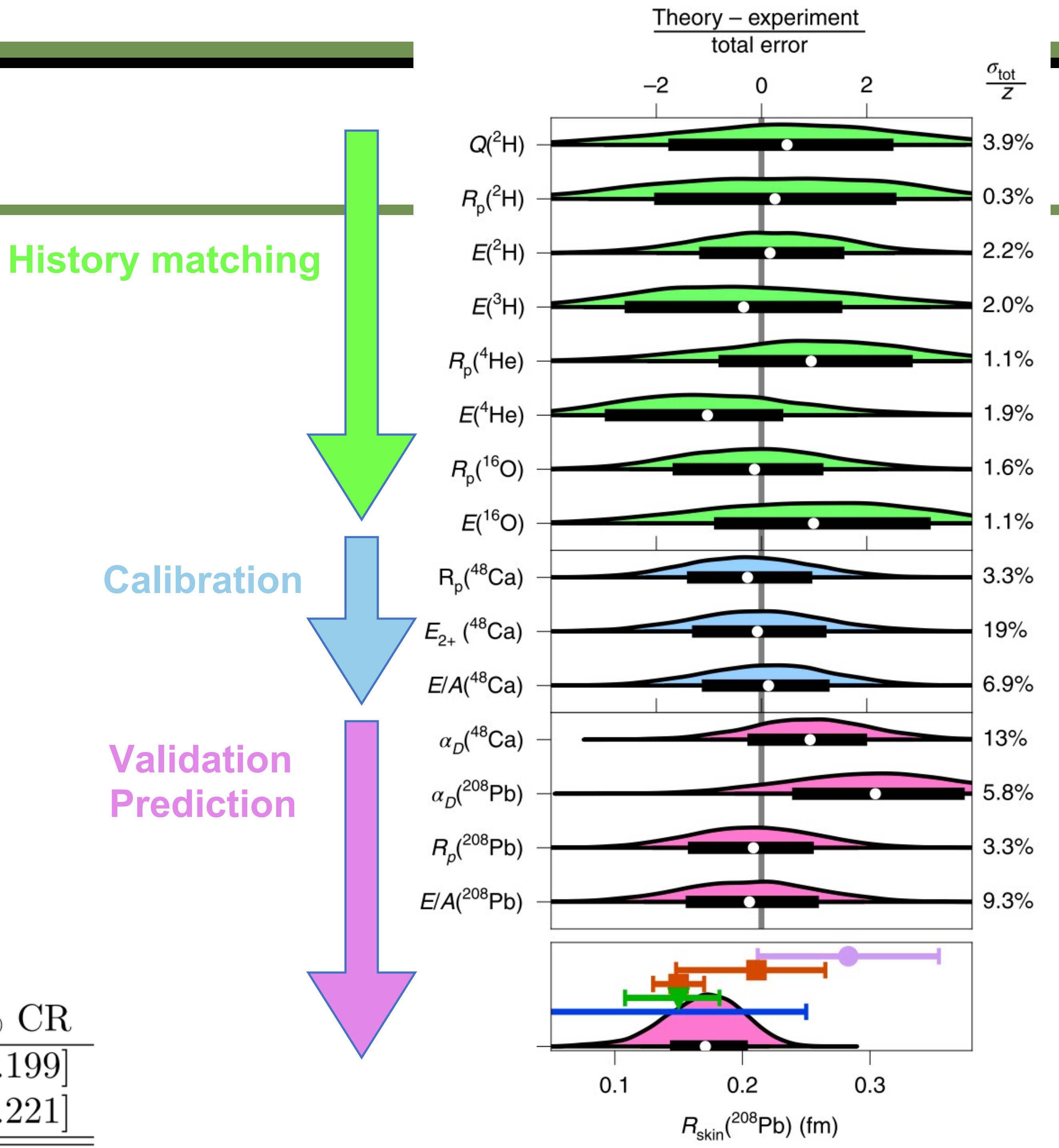
$$\mathcal{L}(D|\boldsymbol{\theta}_i) = \mathcal{N}(D, \sigma^2_{\mathrm{exp}} + \sigma^2_{\chi\mathrm{EFT}} + \sigma^2_{\mathrm{MB}})$$

Validation & prediction:

- The weighted samples are approximately

Neutron skins

Observable	median	68% CR	90% CR
$R_{\mathrm{skin}}(^{48}\mathrm{Ca})$	0.164	[0.141, 0.187]	[0.123, 0.199]
$R_{\mathrm{skin}}(^{208}\mathrm{Pb})$	0.171	[0.139, 0.200]	[0.120, 0.221]

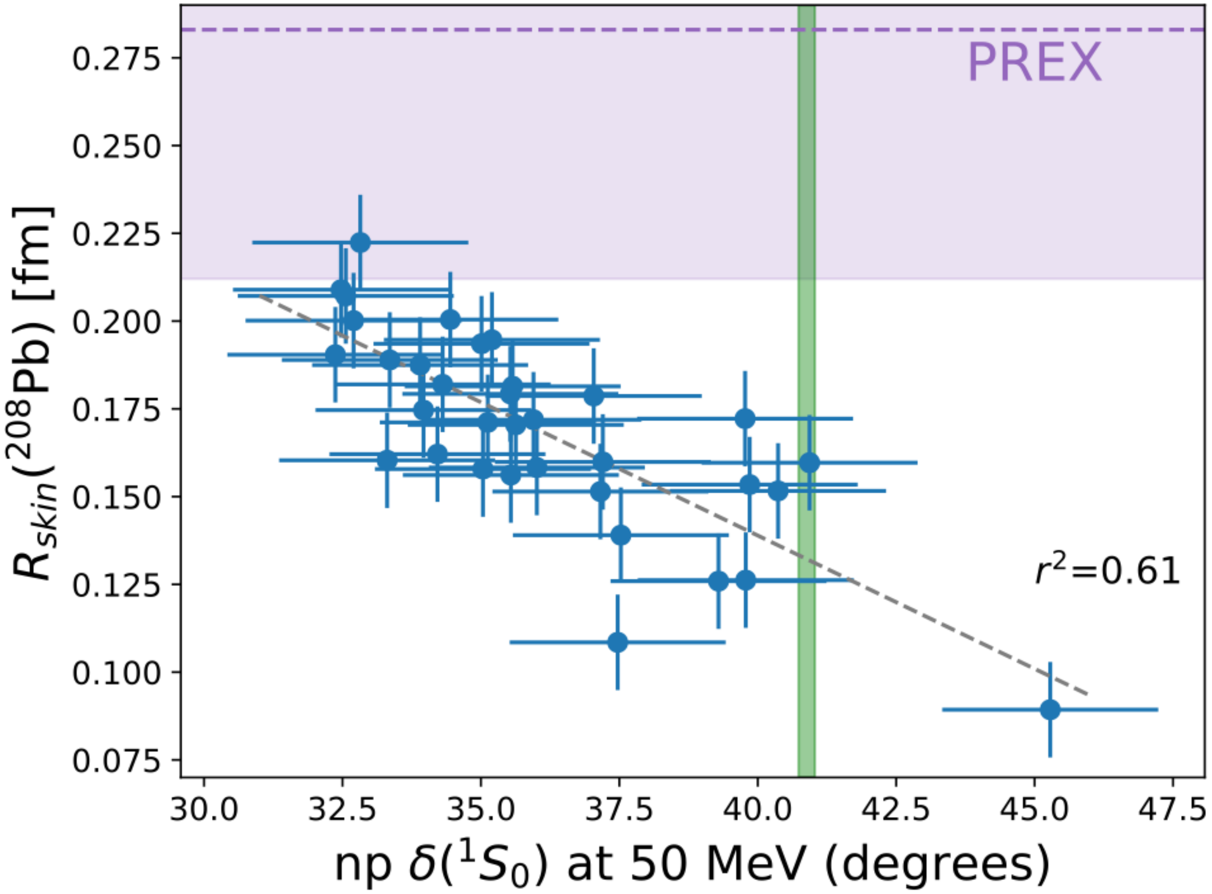


Neutron skin of ^{208}Pb

Ab initio prediction 0.14 < Rskin(^{208}Pb) < 0.20 is relatively narrow.

Constraining on S-wave scattering phase shift rules out thick Rskin(^{208}Pb).

Correlation connecting few- and many-body systems



B. Hu, W. Jiang, T. Miyagi, et al., Nat. Phys. 18, 1196 (2022).

Ongoing development

Hang Yu

Construction of a fast and accurate emulator

- ✦ parametric matrix model P. Cook et al., arXiv: 2401.11694

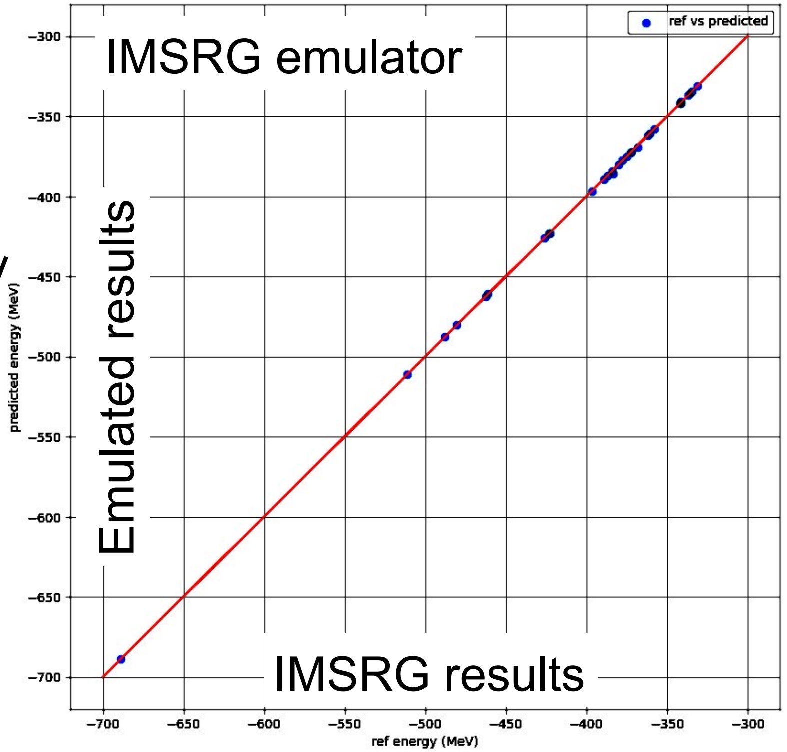
Data-driven eigenvector continuation*-like method

~ $10^{6\text{-}9}$ times speed up!

With the emulator, one can explore

- Impact of 3N interaction in medium-mass nuclei
- Observables to further constrain LECs in ChEFT

Ground-state energy of ^{44}Ca



* See S. Yoshida's talk for the details of the eigenvector continuation

Summary & outlook

The nuclear ab initio calculations of heavy nuclei are becoming feasible.

We combined the state-of-the-art techniques to predict the neutron skin of ^{208}Pb, including the possible uncertainties.

The well-known $R_{skin}(^{208}Pb)$ vs L correlation can be found in ab initio calculations.

NN scattering phase-shift is crucial to constrain $R_{skin}(^{208}Pb)$.

More things need to be done.

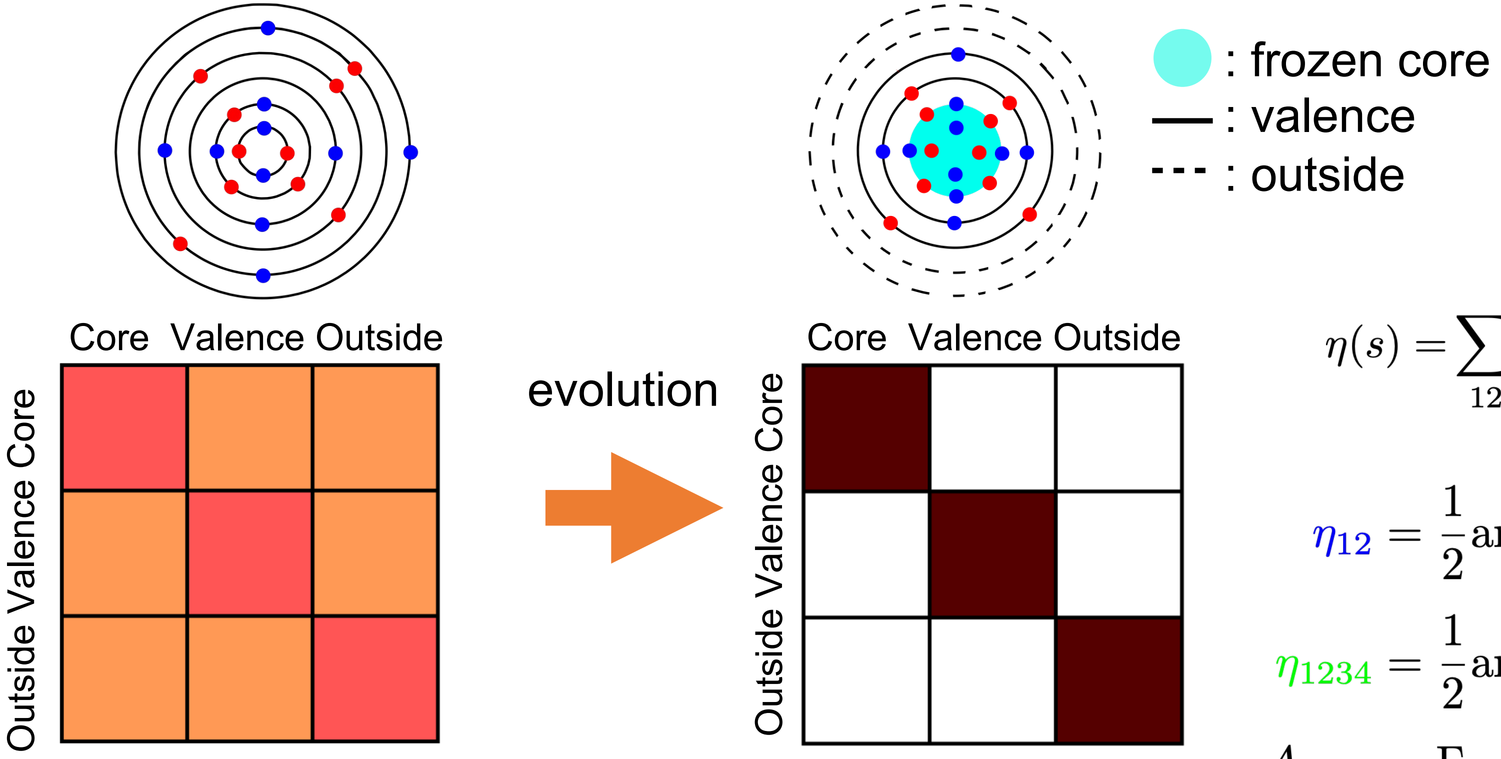
- Better quantified uncertainty, Cutoff independence, CREX vs PREX, …

The same strategy can be applied to other research.

- 0vbb decay, WIMP-nucleus scattering, electric dipole moment, …

Backup slides

Valence-space in-medium similarity renormalization group



Similarity transformation

H $\qquad$ $H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$

$$H(s) \approx E(s) + \sum_{12} f_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4}\sum_{1234} \Gamma_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

s: flow parameter

$$\frac{d\Omega}{ds} = \eta(s) - \frac{1}{2}[\Omega(s), \eta(s)] + \cdots$$

$$\eta(s) = \sum_{12} \eta_{12}(s)\{a_1^\dagger a_2\} + \sum_{1234} \eta_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

$$\eta_{12} = \frac{1}{2}\arctan\left(\frac{2f_{12}}{f_{11} - f_{22} + \Gamma_{1212} + \Delta}\right)$$

$$\eta_{1234} = \frac{1}{2}\arctan\left(\frac{2\Gamma_{1234}}{f_{11} + f_{22} - f_{33} - f_{44} + A_{1234} + \Delta}\right)$$

$$A_{1234} = \Gamma_{1212} + \Gamma_{3434} - \Gamma_{1313} - \Gamma_{2424} - \Gamma_{1414} - \Gamma_{2323}$$

f_{12}, Γ_{1234} : matrix element we want to suppress

$$\mathcal{O}(s) = e^{\Omega(s)}\mathcal{O}e^{-\Omega(s)} \approx \mathcal{O}^{[0]}(s) + \sum_{12}\mathcal{O}^{[1]}_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4}\sum_{1234}\mathcal{O}^{[2]}_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

Normal ordering wrt a single Slater determinant

Initial Hamiltonian is expressed with respect to nucleon vacuum

$$H = \sum_{pq} t_{pq} a_p^\dagger a_q + \frac{1}{4}\sum_{pqrs} V_{pqrs} a_p^\dagger a_q^\dagger a_s a_r + \frac{1}{36} V_{pqrstu} a_p^\dagger a_q^\dagger a_r^\dagger a_u a_t a_s$$

- Hamiltonian normal ordered with respect to a single Slater determinant

$$H = E_0 + \sum_{pq} f_{pq}\{a_p^\dagger a_q\} + \frac{1}{4}\sum_{pqrs} \Gamma_{pqrs}\{a_p^\dagger a_q^\dagger a_s a_r\} + \frac{1}{36} W_{pqrstu}\{a_p^\dagger a_q^\dagger a_r^\dagger a_u a_t a_s\}$$

$$E_0 = \sum_{pq} t_{pq}\rho_{pq} + \frac{1}{2}\sum_{pqrs} V_{pqrs}\rho_{pr}\rho_{qs} + \frac{1}{6}\sum_{pqrstu} V_{pqrstu}\rho_{ps}\rho_{qt}\rho_{ru}, \qquad \Gamma_{pqrs} = V_{pqrs} + \sum_{tu} V_{pqtrsu}\rho_{tu}$$

$$f_{pq} = t_{pq} + \sum_{rs} V_{prqs}\rho_{rs} + \frac{1}{2}\sum_{rstu} V_{prsqtu}\rho_{rt}\rho_{su}, \qquad W_{pqrstu} = V_{pqrstu}$$

- Normal ordered two-body (NO2B) approximation:

$$H \approx E_0 + \sum_{pq} f_{pq}\{a_p^\dagger a_q\} + \frac{1}{4}\sum_{pqrs} \Gamma_{pqrs}\{a_p^\dagger a_q^\dagger a_s a_r\}$$

E3max extrapolation

One has to make sure that HF results are well converged.

Assuming that the employed nuclear interaction is soft enough: $E_{\mathrm{corr}} \approx E^{[2]}_{\mathrm{MBPT}}$

$$\Gamma_{abij} = V^{\mathrm{NN}}_{abij} + \sum_k V^{\mathrm{3N}}_{abkijk} n_k$$

With an optimal frequency, MP(2) energy can be approximated as $E^{[2]}_{\mathrm{MBPT}} \approx \frac{1}{4\hbar\Omega} \sum_{abij} \frac{\Gamma_{ijab}\Gamma_{abij}}{e_i + e_j - e_a - e_b}$

- MP(2) enegy difference between E3max and E3max+1: $\Delta E^{[2]}_{\mathrm{MBPT}} = \frac{1}{2\hbar\Omega} \sum_{ijk} \sum_{ab} \frac{V^{\mathrm{NN}}_{ijab} V^{\mathrm{3N}}_{abkijk}}{e_i + e_j + e_k - e_a - e_b - e_k} \delta_{E_{3\max}, e_a + a_b + e_k}$
- Further assumption: $V^{\mathrm{NN}}_{abij} \approx \bar{V}^{NN} \exp\left\{ -\left[\frac{m\epsilon_0 (e_a + e_b - e_i - e_j)}{\Lambda^2_{\mathrm{NN}}} \right]^n \right\}, \quad V^{\mathrm{3N}}_{abkijk} \approx \bar{V}^{3N} \exp\left\{ -\left[\frac{m\epsilon_0 (e_a + e_b + e_k - e_i - e_j - e_k)}{\Lambda^2_{\mathrm{3N}}} \right] \right.$

One finds:

$$\Delta E^{[2]}_{\mathrm{MBPT}} \approx AX \exp\left[-\frac{X^n}{\sigma^n} \right], \quad X = E_{3\max} - \mu, \quad \frac{1}{\sigma^n} = m^n \epsilon_0^n \left(\frac{1}{\Lambda^{2n}_{\mathrm{NN}}} + \frac{1}{\Lambda^{2n}_{\mathrm{3N}}} \right)$$

After integrating the above, one obtains:

$$E(E_{3\max}) = A\gamma_{\frac{2}{n}} \left[\left(\frac{E_{3\max} - \mu}{\sigma} \right)^n \right] + C$$

E_{3max} convergence in heavy nuclei

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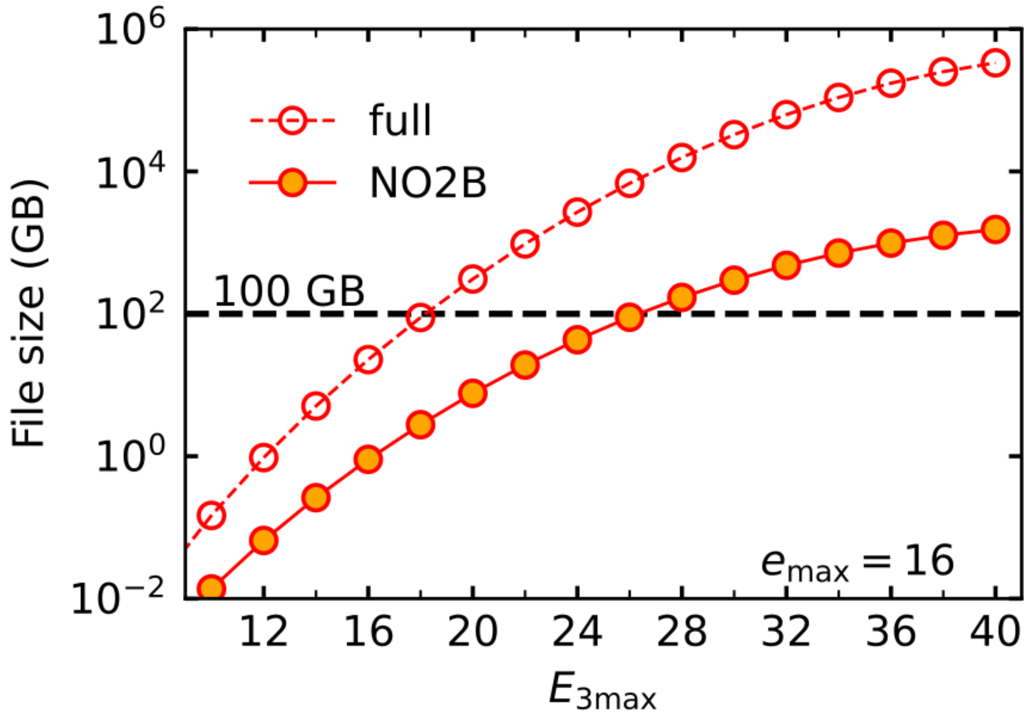
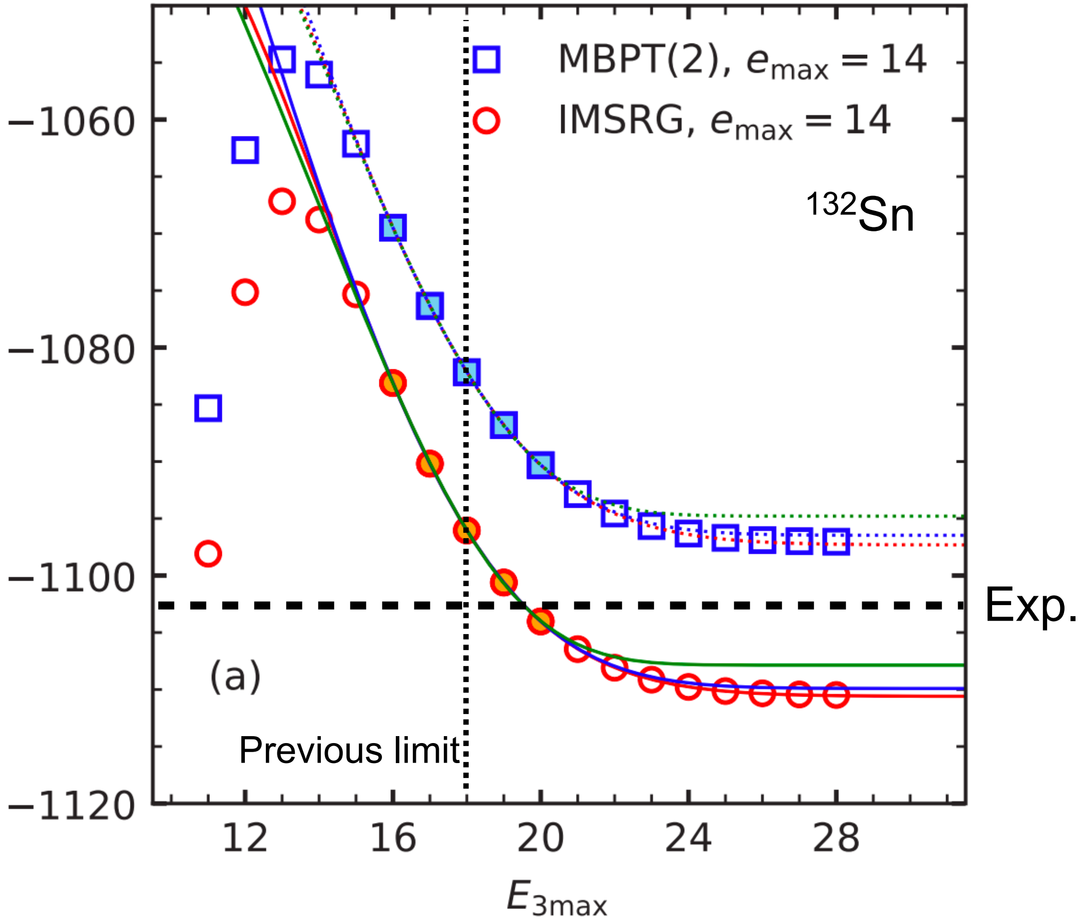
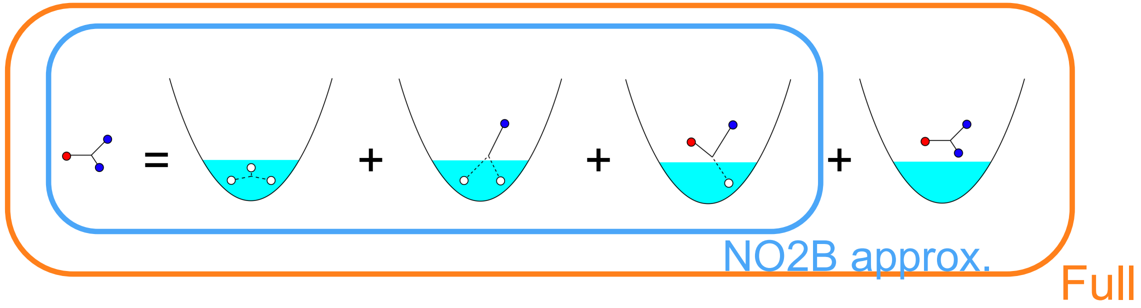


TM, S. R. Stroberg, P. Navrátil, K. Hebeler, and J. D. Holt, Phys. Rev. C 105, 014302 (2022).



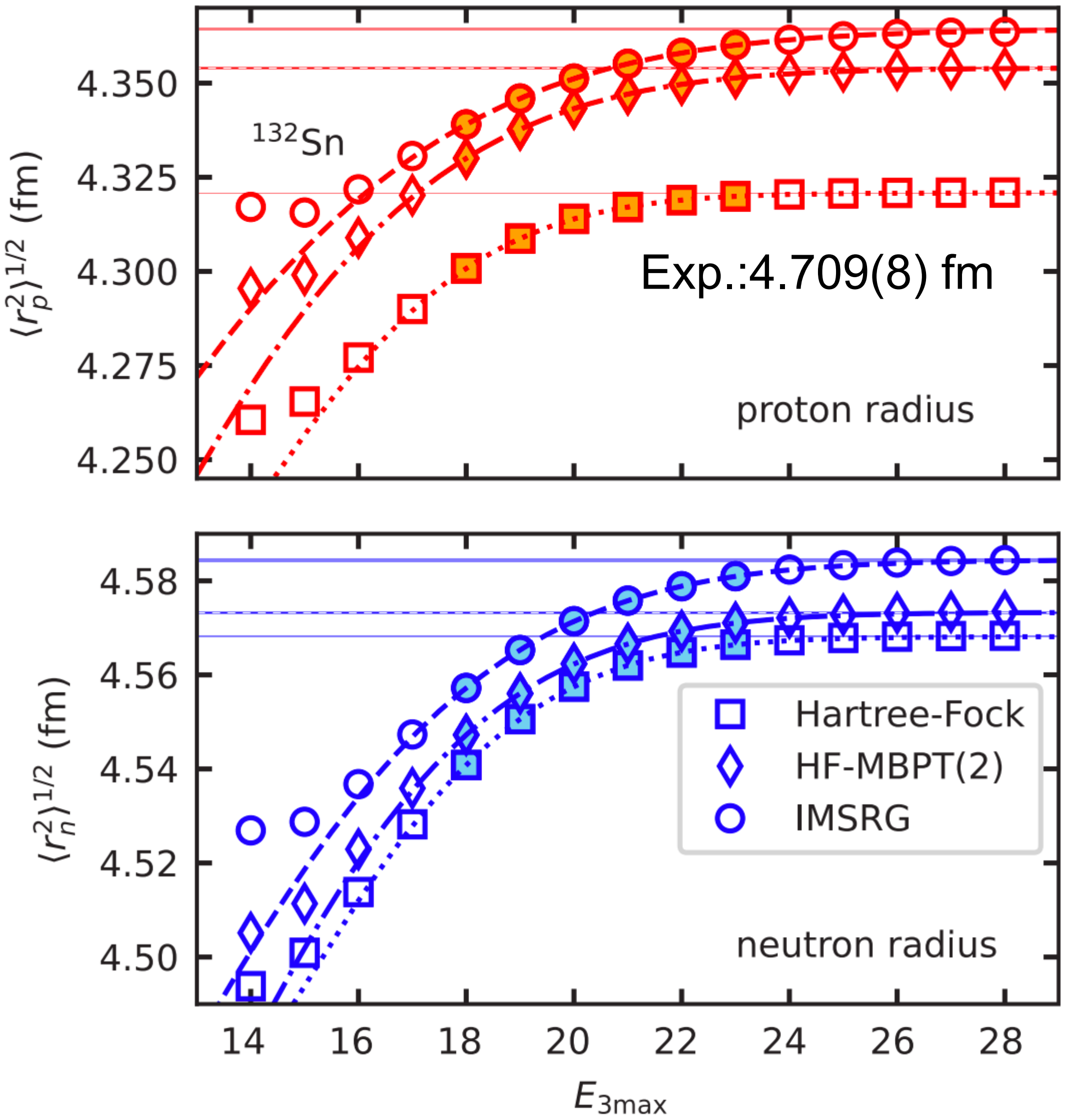


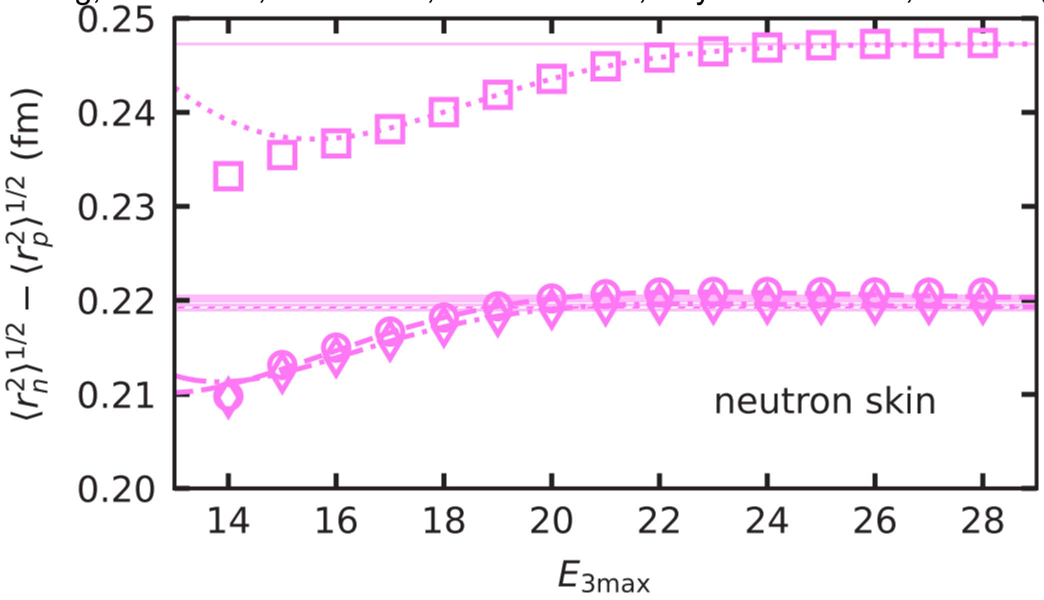
NO2B approximation error ~ a few %
[S. Binder et al., Phys. Rev. C 87, 021303 (2013).]

Asymptotic form: $E \approx A\gamma_{\frac{2}{n}}\left[\left(\frac{E_{3\max}-\mu}{\sigma}\right)^{n}\right] + E_{\infty}$

Radii

TM, S. R. Stroberg, P. Navrátil, K. Hebeler, and J. D. Holt, Phys. Rev. C 105, 014302 (2022).





Asymptotic form: $\langle r^2 \rangle \approx A\gamma_{\frac{2}{n}}\left[\left(\frac{E_{3\max}-\mu}{\sigma}\right)^n\right] + \langle r^2 \rangle_\infty$

Radii

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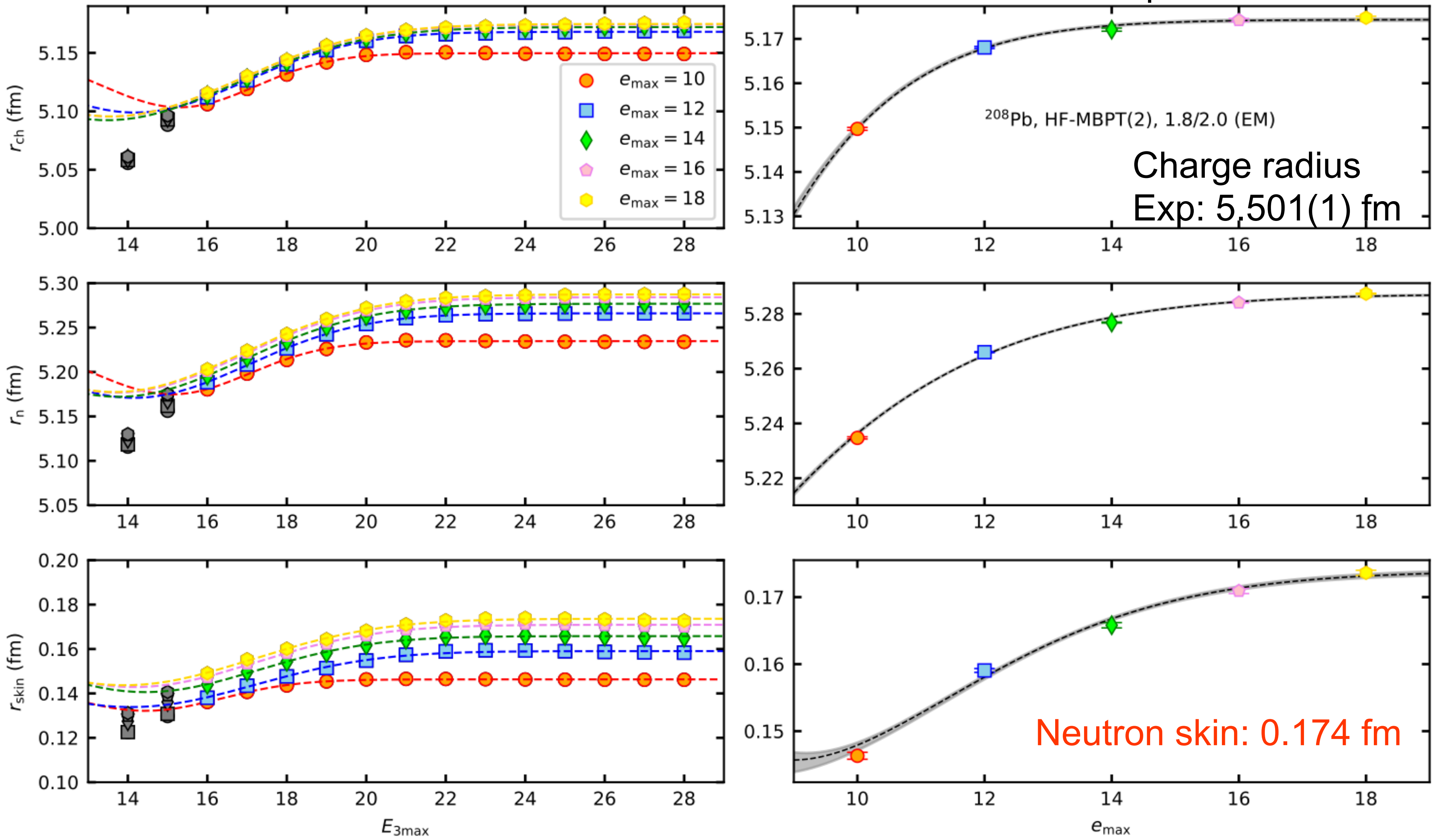
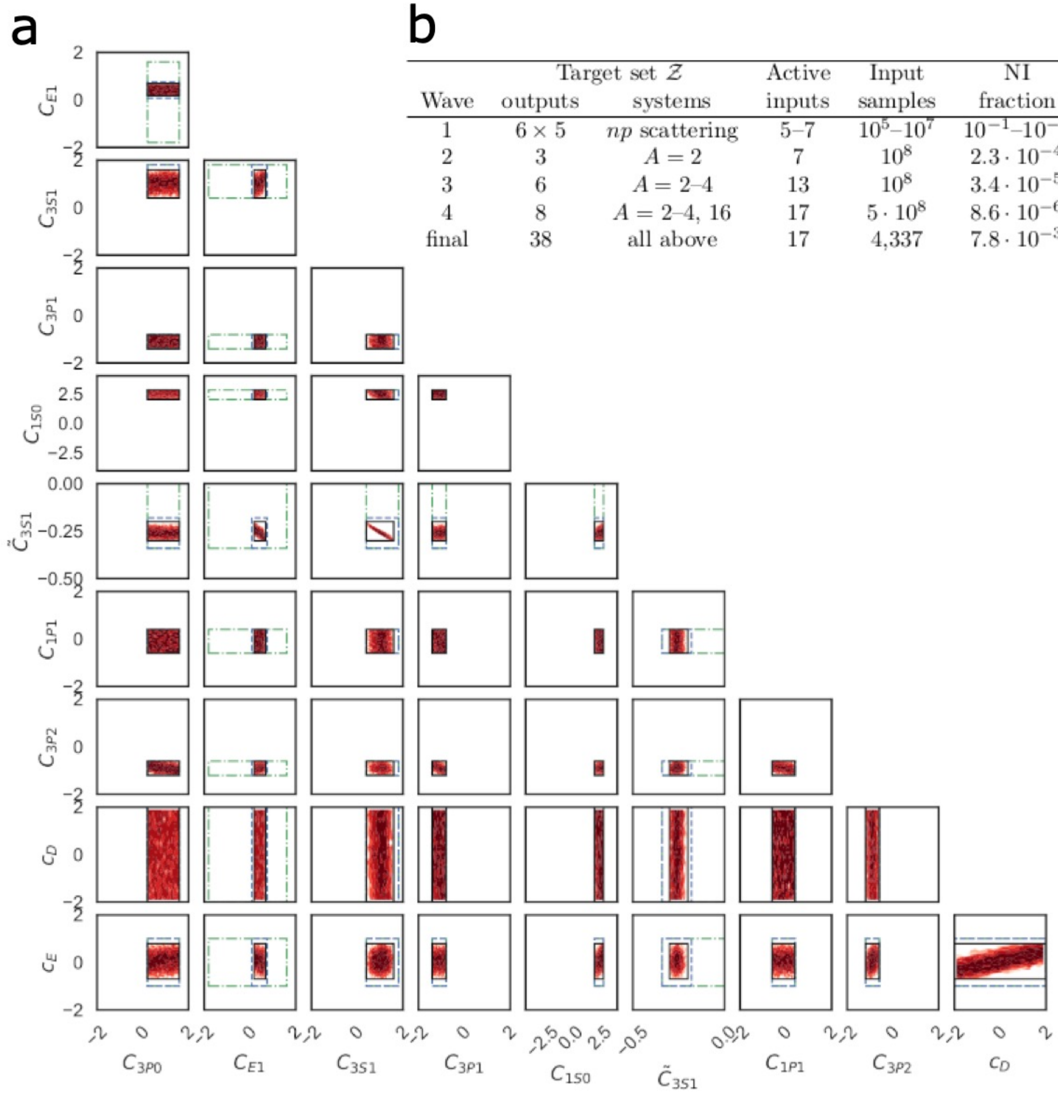
Empirical Gaussian fit



Non-implausible interactions

- Sequentially rule out the possibility:



b

Wave	Target set $\mathcal{Z}$ outputs	Target set $\mathcal{Z}$ systems	Active inputs	Input samples	NI fraction	Proportion space NI
1	6×5	np scattering	5–7	10^5–10^7	10^{-1}–10^{-4}	$3.2 \cdot 10^{-6}$
2	3	$A = 2$	7	10^8	$2.3 \cdot 10^{-4}$	$3.0 \cdot 10^{-7}$
3	6	$A = 2$–4	13	10^8	$3.4 \cdot 10^{-5}$	$9.2 \cdot 10^{-8}$
4	8	$A = 2$–4, 16	17	$5 \cdot 10^8$	$8.6 \cdot 10^{-6}$	same
final	38	all above	17	4,337	$7.8 \cdot 10^{-3}$	same

Error assignments

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History-matching observables						
Observable	z	$\varepsilon_{\rm exp}$	$\varepsilon_{\rm model}$	$\varepsilon_{\rm method}$	$\varepsilon_{\rm em}$	PPD
$E(^2\mathrm{H})$	-2.2246	0.0	0.05	0.0005	0.001%	$-2.22^{+0.07}_{-0.07}$
$R_{\rm p}(^2\mathrm{H})$	1.976	0.0	0.005	0.0002	0.0005%	$1.98^{+0.01}_{-0.01}$
$Q(^2\mathrm{H})$	0.27	0.01	0.003	0.0005	0.001%	$0.28^{+0.02}_{-0.02}$
$E(^3\mathrm{H})$	-8.4821	0.0	0.17	0.0005	0.01%	$-8.54^{+0.34}_{-0.37}$
$E(^4\mathrm{He})$	-28.2957	0.0	0.55	0.0005	0.01%	$-28.86^{+0.86}_{-1.01}$
$R_{\rm p}(^4\mathrm{He})$	1.455	0.0	0.016	0.0002	0.003%	$1.47^{+0.03}_{-0.03}$
$E(^{16}\mathrm{O})$	127.62	0.0	1.0	0.75	0.5%	$-126.2^{+3.0}_{-2.8}$
$R_{\rm p}(^{16}\mathrm{O})$	2.58	0.0	0.03	0.01	0.5%	$2.57^{+0.06}_{-0.06}$
Calibration observables						
Observable	z	$\varepsilon_{\rm exp}$	$\varepsilon_{\rm model}$	$\varepsilon_{\rm method}$	$\varepsilon_{\rm em}$	PPD
$E/A(^{48}\mathrm{Ca})$	-8.667	0.0	0.54	0.25	—	$-8.58^{+0.72}_{-0.72}$
$E_{2^+}(^{48}\mathrm{Ca})$	3.83	0.0	0.5	0.5	—	$3.79^{+0.86}_{-0.96}$
$R_{\rm p}(^{48}\mathrm{Ca})$	3.39	0.0	0.11	0.03	—	$3.36^{+0.14}_{-0.13}$
Validation observables						
Observable	z	$\varepsilon_{\rm exp}$	$\varepsilon_{\rm model}$	$\varepsilon_{\rm method}$	$\varepsilon_{\rm em}$	PPD
$E/A(^{208}\mathrm{Pb})$	-7.867	0.0	0.54	0.5	—	$-8.06^{+0.99}_{-0.88}$
$R_{\rm p}(^{208}\mathrm{Pb})$	5.45	0.0	0.17	0.05	—	$5.43^{+0.21}_{-0.23}$
$\alpha_D(^{48}\mathrm{Ca})$	2.07	0.22	0.06	0.1	—	$2.30^{+0.31}_{-0.26}$
$\alpha_D(^{208}\mathrm{Pb})$	20.1	0.6	0.59	0.8	—	$22.6^{+2.1}_{-1.8}$

Error assignments

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