Lattice QCD computation of inclusive processes

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1. Project Purpose

In Quantum Chromodynamics (QCD), the "inclusive processes" mean the transitions when they are summed over the processes going to all possible final states. For instance, the inclusive semi-leptonic B meson decay refers to all processes involving underlying quark-level process b→q (plus lepton and neutrino) that includes specific hadronic final states (D, D* meson or pion depending on the quark flavor) as well as multi-body states such as D+pion(s). So far, theoretical calculations of the rate of such processes have been done using perturbation theory and operator product expansion (OPE), but their uncertainty is hard to quantify due to potential non-perturbative corrections as well as its underlying assumption (quark-hadron duality). Fully non-perturbative computation had been long awaited since experimental data are available, but practical method was proposed only recently by the authors including the PI of this project.

In particular, there is a long-standing serious problem in the semi-leptonic B meson decays. Namely, the determinations of the Kobayashi-Maskawa matrix elements $|V_{cb}|$ and $|V_{ub}|$ using the inclusive processes are in conflict with the corresponding determinations using corresponding exclusive processes (that specifies the final states), for which hadronic effect is precisely computed using the first-principles calculation of lattice QCD. The precision of $|V_{cb}|$ and $|V_{ub}|$ determinations is limited due to this disagreement. This is an obstacle that one has to overcome before using the B factory data for more precise test of the Standard Model. Whether the disagreement is due to theory or to experiment, or to inclusive or to exclusive, is not known. Full theoretical understanding of the inclusive decay rate is of paramount importance.

The newly proposed method to compute the inclusive decay rate is based on lattice QCD. In standard applications of lattice QCD, one chooses specific initial state and final state, and they are isolated by taking the long time-separation limit on the Euclidean lattice. To compute the inclusive rate in this direction, one has to compute the amplitudes to all possible final state one by one, many of which are multi-particle states. Such calculation is possible only for two-body final state (and three-body states in some cases), but the method is quite cumbersome and does not exist for more complicated final states. The new method overcome this difficulty by treating matrix elements with two current insertions. The two currents are displaced in Euclidean time direction and all possible states can propagate between them. The idea is to reconstruct the decay rate combining lattice matrix elements with varying time separations using a technique of Chebyshev approximation. In a paper [P. Gambino and S. Hashimoto, Phys. Rev. Lett. 125, 032001 (2020)], a prototype calculation was presented for a setup where quark masses are different from the real world and the feasibility of the method was demonstrated. This opens vast new applications of lattice QCD, including B meson decays as well as inelastic lepton-nucleon scatterings.

The purpose of this project was to push this new method one-step forward toward the solution of the puzzle between the inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$. In order to verify the method quantitatively, we need to compare the calculation with existing experimental measurements. That can be done using D mesons (charmed mesons). The D meson semi-leptonic decays are experimentally measured by the BESIII experiment, and the corresponding Kobayashi-Maskawa matrix elements $|V_{cd}|$ and $|V_{cs}|$ are well known, so that the process can be used for a test of the theoretical method to compute the rate.

2. Results

We computed the inclusive semi-leptonic decay rate of Ds meson using the method mentioned above. The lattice calculation is performed based on the ensembles we generated in the past, for which we have already computed the exclusive decay form factors of various channels. We have three lattice spacings corresponding to the lattice cutoff 2.5 GeV, 3.6 GeV and 4.5 GeV, with which the continuum limit can be estimated. The sea and valence quarks are described by the domain-wall fermion formulation on the lattice, for which the chiral symmetry is precisely preserved and the discretization of O(a) is prohibited. The computation of the necessary hadronic four-point function has to be carried out for various decay channels and for various parameters.

With the MCRP resources, the computation of the coarsest lattice with lattice cutoff 2.5 GeV was performed. In order test the saturation of the initial state Ds meson, different separations of source time-slice and the position of the inserted current was tested.

Using the data obtained in this calculation, we can evaluate the integral over the final-state energy. And then the integral over the final-state momentum gives the total decay rate, which can be compared with the experimental data. The analysis of the results is ongoing, and a part of the results is planned to be presented at summer conferences in 2023.

3. Roles of the MCRP and its significance

The machine power provided by Wisteria was essential to perform the computation required in this project.

4. Future plan

The inclusive rate of D meson is interesting as itself, i.e. adding another confirmation of the Kobayashi-Maskawa matrix elements $|Vcs|$ and $|Vcd|$, but the real value of the work is not there. The verification of the method of inclusive lattice calculation with the experimental data paves the way toward the application for the study of $|Vcb|$ and $|Vub|$. They are computationally more demanding because of the heavy initial quark, for which the extrapolation of the mass is necessary, and of the larger phase space of the final states, for which larger momenta have to be inserted. Since the tension between the inclusive and exclusive determinations is a major obstacle toward the precise test of the Standard Model, the stakes are high. The experimental project to precisely measure the inclusive rate is running at KEK, the SuperKEKB/Belle II project, and the associated theoretical research is highly motivated. We plan to go on in this direction.

5. Publications and conference presentations

The analysis of the results is still on-going and no publication/presentation has been made so far.

(1) Journal papers

(2) Presentations

(3) Others

Supercomputer	Use	Allocated resources*

		Initial resources	Additional resources
Cygnus	No		
Wisteria/BDEC-01	Yes	191,250	
*in units of node-hour product			